EAN '97

35th International Conference on
Experimental Stress Analysis
June 4 - 6, 1997, Olomouc, Czech Republic

A MATHEMATICAL MODEL FOR THE STUDY OF RESIDUAL STRESSES USING THE DISPLACEMENTS MEASURED DURING PLASMA NITRIDING

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Abstract

The paper presents a new method for the study of the phenomenon of residual stresses generation during the nitriding process. This method is based on the measurement of the bending deformations which occur in an asymmetrically-nitrided specimen during a complete cycle of thermochemical treatment. A mathematical model is introduced for the study of the influence of the thermal relaxation phenomenon on the process of residual stresses generating, starting from the stress and strain state obtained as a consequence of the specific volume difference between the resulted phases and the basic material. The method allows an *in situ* experimental ascertaining of the mathematical models which describe the residual stress generating phenomenon during both gas and plasma nitriding.

1. Introduction

Nitriding thermochemical treatment causes, in the treated parts, a stress state characterized by large compression residual stresses within the nitrided layer and by lower tensile residual stresses in the core.

From the results obtained by most of the researchers are may conclude that residual stresses generating during nitriding has the following causes:

* *during the maintaining period,*

a) deformation incompatibility as a consequence of the specific volume differences between the formed phases and the basic material, corresponding to the nitriding temperature;

b) reduction of the residual stresses as a consequence of thermal relaxation phenomena;

* *during the cooling stage,*

c) deformation incompatibility, caused by the thermal expansion differences between the formed phases and the basic material.

Considering the major effect of residual stresses, from nitrided parts, on mechanical properties and especially on fatigue life, the studies regarding an as comprehensive as possible understanding of both the causes and mechanisms which generate them have been lately intensified. Sustained efforts have been also developed for modeling and predicting by means of calculations [2], [3], [7], both the residual stresses and the fatigue life of treated parts.

Also, it is important to develop and improve the methods for experimental determination of residual stresses both after and during (in situ) the nitriding process.

For the investigation of the phenomenon of residual stresses generating during the nitriding process, the *in situ* techniques are extremely useful and necessary. Until now only the X - ray diffraction method has been employed in this purpose [4], [5], in the case of gas nitriding. The main disadvantage of this method consists in the fact that it allows only the local analysis of the superficial compound layer, that is only the residual stresses from the analysed phase are detectable. In addition, this method cannot be applied in to case of plasma nitriding.

More complete information may offer, in this case, the *"global"* methods, based on measuring the deformations produced by residual stresses. It has been experimentally - observed that, if nitriding has performed asymmetrically, bending deformations would occur within the treated parts. Flexure occurs in such a way that the nitrided portion forms the convexity of the parts. The values of above deformations may offer information regarding both the occurrence and evolution of residual stresses during the nitriding process. This principle represents the fundamentals of the researching method introduced by the authors for the study of the phenomenon of residual stresses generating during the plasma nitriding process, and detailed in the following chapter.

2. Principle of the Method

The purpose of the researches was to elaborate a methodology for experimental ascertaining of the mathematical models which describe the phenomenon of residual stresses generating, based on the measurement of bending deformations which occur in an asymmetrically - nitrided specimen, during a complete cycle of treatment.

The following *hypotheses* are assumed:

- the bending behaviour of the specimen is approximated to that of a bar;
- the geometrical axis of the bar is deformed after a circular arc (which was experimentally - ascertained).

The specimen, with rectangular cross - section is considered as being embedded at one end and free at the other one. All of the specimen surfaces, excepting one face, are protected against nitriding.

Because of asymmetrical nitriding, the residual stresses, which develop in the nitrided layer of the unprotected surface, will bend the specimen.

Assuming that the bar deforms after a circular arc, Figure 1, it follows that the *specific rotation* ω is the same for all the cross - section, being equal to the *curvature* $1/\rho$:

$$\omega = \frac{d\varphi}{dx} = \frac{\varphi}{x} = \frac{\varphi_{max}}{l} = \frac{1}{\rho} \qquad (1)$$

were φ - rotation of any cross-section located at a distance x from the fixed end of the specimen; φ_{max} - rotation of the free end cross-section; l - specimen length; ρ - curvature radius of the deformed bar.

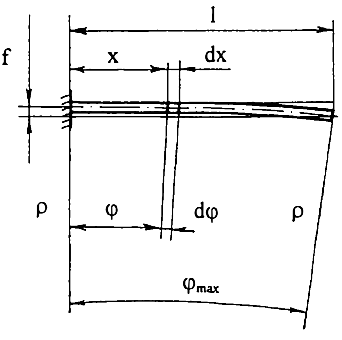

Figure 1. Rotation of specimen cross - section

According to Figure 1, deflection f may be geometrically-expressed at a distance l, as:

$$f = \rho\,(1 - \cos\varphi_{max}) \qquad (2)$$

By replacing in (2) the curvature radius ρ and the maximum rotation φ_{max}, derived from (1) as a function of specific rotation ω, the deflection f becomes:

$$f = \frac{1-\cos(\omega \cdot l)}{\omega} \tag{3}$$

The ***specific rotation*** ω may be determined after covering the following steps:

- the strains $\varepsilon(y)$ are geometrically - calculated over the height of the cross - section, at some moment t, of the process;

When establishing the expression for $\varepsilon(y)$ it is noticeable that nitriding causes a lattice distortion upon the superficial layer which is partially prevented by the basic material. Assuming the Bernoulli hypothesis, it follows that the cross - section remains plane and rotates as compared to the initial position. By analysing the deformations on the entire cross - section, the expression for $\varepsilon(y)$ is derived which also includes the specific rotation $\omega(t)$ of the cross - section.

- the stress expression $\sigma(y)$ are written, for the fibres located on the ordinates y;

These expressions are different in the two cross - section areas: nitrided layer and basic material. The functions $\sigma(y)$ are determined based on $\varepsilon(y)$, considering either the elastic either the elasto - plastic behaviour of the material.

- the equivalence equations are written on the specimen cross - section, considering that both the forces and the momenta are zero on any cross - section:

$$N = \int_A \sigma(y)\, dA = 0 \tag{4}$$

$$M = \int_A \sigma(y) \cdot y\, dA = 0 \tag{5}$$

By solving the system formed by equations (4) and (5) both specific relation $\omega(t)$ and the position of cross -section neutral axis are determined.

Knowing the value of $\omega(t)$, the deflection $f(t)$ may be calculated with relationship (3).

On the other hand, determining the position of cross - section neutral axis allows to settle the expressions $\varepsilon(y)$ and $\sigma(y)$.

By comparing the calculated values of the deflections f, to those experimentally - determined during nitriding, different mathematical models may be ascertained, which describe the phenomenon of residual stresses generating.

Within reference [1], we has established the relationship for determining both the stress state and the curvature of the asymmetrically - nitrided specimen, as a function of the variation of the both the specific volume and Young modulus through the nitrided layer thickness. The calculation has been achieved assuming that both the basic material and the phases formed during nitriding behave in a linear - elastic way.

In the following, the creep effect is analysed upon the process of residual stresses generating, during nitriding.

3. A Mathematical Model for the Study of the Influence of Thermal Relaxation Phenomenon on the Residual Stresses Generating Process in Nitrided Layers

Considering that nitriding temperatures are ranged between 400 and 580 °C, [2], [6], and that maximum stresses, resulting in the nitrided layer area as a consequence of the specific volume difference between the formed phases and the basic material, are at the level of hundreds of MPa, it is obvious that the *accumulation of plastic deformations* by means of the ***creep*** phenomenon may considerably reduce the residual stress level.

Let $\sigma(y)^{\Delta V}$, Figure 2(a), be the stress state within the specimen, caused by the deformation incompatibility as a consequence of the increase of specific volume in the nitrided layer [1].

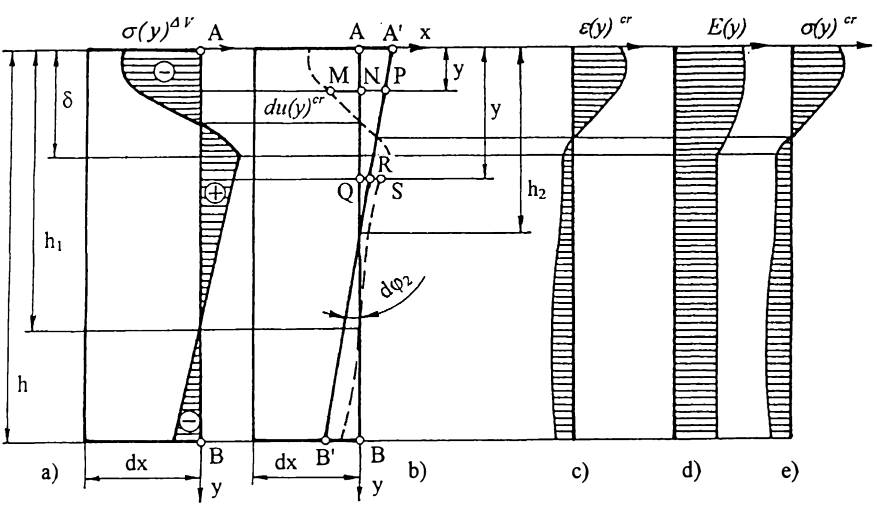

Figure 2. Deformation of specimen cross-section, if creep phenomenon is taken into account, and diagrams of the functions $\varepsilon(y)^{cr}$, $E(y)$, $\sigma(y)^{cr}$

Due to the creep phenomenon, the longitudinal fibres tend to change their length, depending upon both the stress and the temperature level *T*, reached after time *t*.

In Figure 2(b), with dashed line the elementary displacements $du(y)^{cr}$ are shown, which are caused by the plastic deformation accumulation , in a Δt period of time, during which $\sigma(y)^{\Delta V}$ stresses are considered constant, providing creep would be free within each fibre.

For the plastic strains, caused by creep, the following functions are considered:

$$\varepsilon(y)^{pl} = F_1(\sigma(y)^{\Delta V}, t, T), \quad \text{for } y \in [0, \delta] \tag{6}$$

$$\varepsilon(y)^{pl} = F_2(\sigma(y)^{\Delta V}, t, T), \quad \text{for } y \in [\delta, h] \tag{7}$$

The functions $F_1(\sigma(y), t, T)$ and $F_2(\sigma(y), t, T)$ must be experimentally -determined by means of special tests. Generally, in literature there are not such results because nitriding steels are not used at high temperatures which cause the occurrence of creep.

The experimental tests, performed for determining the functions $F_1(\sigma(y), t, T)$ and $F_2(\sigma(y), t, T)$ are on short duration, as compared to the period of a nitriding cycle. Determining the function $F_2(\sigma(y), t, T)$ may be achieved by creep tests performed on usual specimens while establishing the creep behaviour of the nitrided layer constitutes the privilege of micromechanics laboratories.

For the elementary displacements, $du(y)^{cr}$, the following functions result:

$$du(y)^{cr} = F_1(\sigma(y)^{\Delta V}, t, T) \cdot dx, \quad \text{for } y \in [0, \delta] \tag{8}$$

$$du(y)^{cr} = F_2(\sigma(y)^{\Delta V}, t, T) \cdot dx, \quad \text{for } y \in [\delta, h] \tag{9}$$

If Bernoulli's plane sections hypothesis is accepted, it follows that the points of the cross - section *AB*, which would imaginarily lay on the $du(y)^{cr}$ curve, must belong to the segment *A'B'*.

Since the stress state, $\sigma(y)^{\Delta V}$, is described by two functions, upon the cross -section of the specimen, it followe that strains as well must be analysed upon the two characteristic portions of the cross - section.

- The elastic strains of the fibres from the nitrided layer zone, due to the creep phenomenon occurring for $y \in [0, \delta]$, Figure 2(b), will be:

$$\varepsilon(y)^{cr} = \frac{MN + NP}{dx - MN} \tag{10}$$

Bat $MN = d\,u(y)^{cr} = F_1(\sigma(y)^{\Delta V},\ t,\ T)\cdot dx,$ according to relationship (8) and
$NP = (h_2 - y)\cdot tgd\varphi_2 \cong (h_2 - y)\cdot d\varphi_2,$ according to Figure 2(b).

By replacing *MN* and *NP* in relationship (10) and by simplifying the fraction with *dx*, it follows that:

$$\varepsilon(y)^{cr} = \frac{F_1(\sigma(y)^{\Delta V},t,T) + (h_2 - y)\cdot\dfrac{d\varphi_2}{dx}}{1 - F_1(\sigma(y)^{\Delta V},t,T)} \tag{11}$$

The parameter $$\omega^{cr} = \frac{d\varphi_2}{dx} \tag{12}$$

represents the *specific rotation* of the cross - section, as a consequence of creep. It has the same value upon all the cross - sections if the hypothesis that the geometrical axis of the bar is deformed after a circular arc is accepted.

By replacing (12) into (11) it follows:

$$\varepsilon(y)^{cr} = \frac{F_1(\sigma(y)^{\Delta V},t,T) + (h_2 - y)\cdot\omega^{cr}}{1 - F_1(\sigma(y)^{\Delta V},t,T)} \tag{13}$$

- For the basic material zone, $y \in [\delta,\ h]$, the elastic strains of the fibres will be:

$$\varepsilon(y)^{cr} = \frac{QS - QR}{dx + QS} \tag{14}$$

Bat $QS = du(y)^{cr} = F_2(\sigma(y)^{\Delta V},\ t,\ T)\cdot dx,$ according to relationship (9) and
$QR = (h_2 - y)\cdot tgd\varphi_2 \cong (h_2 - y)\cdot d\varphi_2,$ according to Figure 2(b)

By replacing the segments *QS* and *QR* into relationship (14) and by simplifying with *dx*, it follows that:

$$\varepsilon(y)^{cr} = \frac{F_2(\sigma(y)^{\Delta V},t,T) - (h_2 - y)\cdot\omega^{cr}}{1 + F_2(y)^{\Delta V},t,T)} \tag{15}$$

Since, to above reasoning starts from an equilibrium position, it follows that after the development of the creep process, as well the variation of the internal forces must be in equilibrium, therefore the following equations must be true:

$$\int_0^\delta \sigma(y)^{cr}\,dy + \int_\delta^h \sigma(y)^{cr}\,dy = 0 \tag{16}$$

$$\int_0^\delta \sigma(y)^{cr}\cdot y\,dy + \int_\delta^h \sigma(y)^{cr}\cdot y\,dy = 0 \tag{17}$$

were $$\sigma(y)^{cr} = E(y)\cdot\varepsilon(y)^{cr} \tag{18}$$

By solving the system formed by equations (16) and (17), both the specific rotation ω^{cr} and the ordinate h_2 , from Figure 2(b), may be determined, therefore the variations may be calculated for the deflections f^{cr} by means of relationship (3) and for the stress σ^{cr} .

By overlapping the effects, it follows that both the stresses $\sigma(y)$ and the bending deformations f, occurring in the specimen at a moment t of the nitriding process during its maintaining period, must be:

$$\sigma(y) = \sigma(y)^{\Delta V} + \sigma(y)^{cr}, \tag{19}$$

$$f = f^{\Delta V} + f^{cr} \tag{20}$$

were $\sigma(y)^{\Delta V}$ and $f^{\Delta V}$, are calculated according to [1].

The calculated bending deformation f may be compared to that experimentally-determined at the same moment t of the nitriding process.

4. Conclusions

1) Based on the above - introduced mathematical model, an automatic computation program may be conceived which will determine both the stress state upon the specimen thickness and the bending deformation *f* for different moments of the nitriding process. The total duration of the process is divided into a convenient number of steps (time intervals) and the calculation algorithm requires that within each step, the following stage must be carried out:

a) Both the stresses on the specimen thickness and the bending deformation are determined as a consequence of the volume increase, caused by nitriding [1];

b) The variation of both the stress state and the bending deformation is calculated as a consequence of the creep process. Within each longitudinal fibre of the specimen an elementary creep process is assumed. This process is described by a family of functions which supply the values of strains, different from one fibre to another, corresponding to the stresses from the respective fibres, as determined in stage a).

c) By overlapping the effects determined by the stage a) and b), both the stresses and the bending deformation are obtained at the end of the corresponding time interval under study. These values represent the initial data for the next calculation step.

2) By employing the above - introduced method and the calculation relationships, derived for deformations and stresses, different mathematical models which describe the residual stresses generating phenomenon during nitriding may be experimentally *in situ* ascertained. In this purpose, a special laboratory installation has been designed and built for plasma nitriding.

3) For the cooling stage of the nitriding cycle, the same reasoning must be followed but when deriving the calculation relationships both the phenomenon of thermal relaxation of stresses and the difference of thermal expansion coefficients between the formed phases and the basic material must be taken into account.

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