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SPECKLE STRAIN GAUGE IN THE IMAGE FIELD FOR SMALL OBJECT DEFORMATION

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Summary. The contributed paper gives a brief survey of theory of small deformation measurement by means of correlation of speckle fields. A speckle field is investigated in the free-space geometry and in the image field of displeyed optic system. Both of theoretical approaches are in conclusion of the paper evaluated and they are mutually compared from view of stress measurement.

1.INTRODUCTION

The effect of speckle is a resulting expression of an interference of elementary coherent beams of radiation, e.g. of radiation being diffusively scattered by scattering surface of object. The intensity field of radiation, being influenced in this way, has a random character and it is possible to work with this field as with randon event. When the investigated object was deformed (zone of so-called small deformations), then the tensor of object elementary surface deformation can be determined from mutal shift of speckle fields, that characterized two states of object deformation (e.g. before and after deformation). To this it can be utilized the experimental arrangement in so called optically free-space geometry, i.e. the arrangement without utilization of optic system in the direction of speckle field observation. The theoretical description has been made by I. Yamaguchi [1], [2]. This approach, with a view to determination of stress components of object elementary surface I have presented on previous EAN conferences [3], [4].

If we use to observation of a speckle field (for detection of field by means of CCD element) a display optical system, then we obtain at evolution of the tensor of object element surface deformation rather other results. The presented paper deals with this problematics.

2.CORRELATION OF INTENSITIES

If we denote according Fig.1 the location of object point P_f with position vector **f** being located in plane (f_x, f_y) of cartesian coordinate system $O_f(f_x, f_y, f_z)$, by $\mathbf{a}(\mathbf{f}, t_i)$ - the vector of displacement of object surface point P_f into point $P_{f'}$, the complex amplitude of monochromatic wave in plane (f_x, f_y) we denote with u_i and corresponding complex amplitude in plane (g_x, g_y) we denote with U_i, then the intensity being observed in point P_g is given with relation $I_i(\mathbf{g}) = |U_i(\mathbf{g})|^2$. Further, if two different displacements of object point P_f into points P_{fi}, being characterized with displacement vectors $\mathbf{a}_i$, we denote with indexes i=1,2, then the cross correlation function of observed spatially varying intensities around mean value of intensity $\Delta I_i = I_i(\mathbf{g}) - \langle I_i(\mathbf{g})\rangle$, is defined with last expression in relation

$$\langle I_i(\mathbf{g}_1)\cdot I_2(\mathbf{g}_2)\rangle = \langle|U_1(\mathbf{g}_1)|^2\rangle\cdot\langle|U_2(\mathbf{g}_2)|^2\rangle + |\langle U_1(\mathbf{g}_1)\cdot U_2^*(\mathbf{g}_2)\rangle|^2, \qquad (1)$$

where U^*_2, is quantity, being complex conjugated with U_2, and thus it is valid that $\langle\Delta I_1(\mathbf{g}_1)\cdot\Delta I_2(\mathbf{g}_2)\rangle = |\langle U_1(\mathbf{g}_1)\cdot U_2^*(\mathbf{g}_2)\rangle|^2$, let us say for $\mathbf{g}_1 = \mathbf{g}$ and $\mathbf{g}_2 = \mathbf{g} + \Delta\mathbf{g}$ it is valid

$$\langle \Delta I_1(\mathbf{g}) \cdot \Delta I_2(\mathbf{g}+\Delta\mathbf{g})\rangle = \left|\langle U_1(\mathbf{g}) \cdot U_2^*(\mathbf{g}+\Delta\mathbf{g})\rangle\right|^2. \tag{2}$$

3.IMAGE FIELD

If we now place into direction of observation an optical system according to Fig.2, then with utilization of [1],[2], the cross correlation of amplitudes is given by relation

$$\begin{aligned}
&\langle U_1(\mathbf{g},Z) \cdot U_2^*(\mathbf{g}+\Delta\mathbf{g},Z)\rangle = \\
&= \exp\left\{jk\left[\tfrac{\mathbf{g}\cdot\Delta\mathbf{g}}{L_0'+Z} + \mathbf{I}_c(\mathbf{f}_g,0)\cdot \mathbf{a}(\mathbf{f}_g,0)\right]\right\} \cdot \int P^*(\mathbf{s})\cdot P\left(\mathbf{s}+L_0[\nabla \mathbf{I}_c(\mathbf{f},0)\cdot \mathbf{a}]_{\mathbf{f}_g}\right) \\
&\times \exp\left\{jk\tfrac{\mathbf{s}}{L_0'+Z}\cdot[\Delta\mathbf{g}-\mathbf{A}(\mathbf{g},Z)]\right\} d^2\mathbf{s},
\end{aligned} \tag{3}$$

where k is wave number; Z is the distance from scanning plane to Gaussian image plane, whereat it is valid that $1/L_0+1/L_0' = 1/F$ (Fig 2); F is focal length of optical system; $\mathbf{I}_c(\mathbf{f},\mathbf{g}) = \mathbf{I}_s(\mathbf{f}) + \mathbf{I}(\mathbf{f},\mathbf{s}), \mathbf{I}_s(\mathbf{f}) = \mathbf{L}_s/L_s$ is the unit vector of direction of illumination; $\mathbf{I}(\mathbf{f},\mathbf{s}) = \mathbf{L}(\mathbf{f},\mathbf{s})/L(\mathbf{f},\mathbf{s})$ is the unit vector of direction of observation, $P(s)$ is the pupil function of optical system.

The maximum of correlation will come, if in relation (3) it will be $\Delta\mathbf{g} = \mathbf{A}$. The expression $\mathbf{A}$, being written in components, is

$$\begin{aligned}
A_x &= m(a_x\cos\theta_R + a_z\sin\theta_R) \\
&-\tfrac{Z}{L_0'}\left[m(a_x\cos\theta_R + a_z\sin\theta_R) - a_x\left(\tfrac{L_0\cos^2\theta_s}{L_s\cos\theta_R} + \cos\theta_R\right) + a_z\left(\tfrac{L_0\cos\theta_s\sin\theta_s}{L_s\cos\theta_R} + \sin\theta_R\right)\right] \\
&-\tfrac{ZL_0}{L_0'}\left[\varepsilon_{xx}\left(\tfrac{\sin\theta_s}{\cos\theta_R} + \tan\theta_R\right) - \Omega_y\left(\tfrac{\cos\theta_s}{\cos\theta_R} + 1\right)\right]
\end{aligned} \tag{4}$$

$$\begin{aligned}
A_y &= ma_y - \tfrac{Z}{L_0'}\{ma_y - a_y\left(\tfrac{L_0}{L_s}+1\right) + L_0[\varepsilon_{xy}(\sin\theta_s + \sin\theta_R) \\
&-\Omega_x(\cos\theta_s+\cos\theta_R) - \Omega_z(\sin\theta_s + \sin\theta_R)]\ \}
\end{aligned} \tag{5}$$

where θ_S and L_s are angle and distance of the illumination source (Fig.2), θ_R is the angle of observation (Fig.2) and $m = -L_0'/L_0$ is magnification. The components of deformation tensor of the illuminated element of object surface, being investigated, and that the components of translation, rotation and elastic deformation, are $(a_x,a_y,a_z),(\Omega_x,\Omega_y,\Omega_z)$ and $(\varepsilon_{xx},\varepsilon_{xy},\varepsilon_{yy})$. At the same time it is valid that

$$(a_x,a_y,a_z) = [a_x(0),a_y(0),a_z(0)], \tag{6}$$

$$(\Omega_x,\Omega_y,\Omega_z) = \left\{\left(\tfrac{\partial a_z}{\partial y}\right)_0, -\left(\tfrac{\partial a_z}{\partial x}\right)_0, \tfrac{1}{2}\left[\left(\tfrac{\partial a_y}{\partial x}\right)_0 - \left(\tfrac{\partial a_x}{\partial y}\right)_0\right]\right\}, \tag{7}$$

$$\varepsilon_{xx} = \left(\tfrac{\partial a_x}{\partial x}\right)_0, \varepsilon_{yy} = \left(\tfrac{\partial a_y}{\partial y}\right)_0, \varepsilon_{xy} = \varepsilon_{yx} = \tfrac{1}{2}\left[\left(\tfrac{\partial a_x}{\partial y}\right)_0 + \left(\tfrac{\partial a_y}{\partial x}\right)_0\right]. \tag{8}$$

Maximum of cross correlation function of recorded speckle field intensities due to object deformation is given with relation

$$\langle \Delta I_1(\mathbf{g},Z).\Delta I_2(\mathbf{g}+\Delta\mathbf{g},Z)\rangle = \left|\int P^*(\mathbf{s}).P(\mathbf{s}+\mathbf{A}_P)d^2\mathbf{s}\right|^2, \tag{9}$$

where

$$\mathbf{A}_P = L_0[\mathbf{I}_c(\mathbf{f}, 0) \cdot \mathbf{a}]_{\mathbf{f}g}. \qquad (10)$$

The equation (9) says that correlation maximum is proportional to square of modulation transfer function of optical system, where a spatial frequency is proportional to speckle displacement in plane of pupil. However, the effect of speckle correlations in image plane depends on speckle displacement in plane of pupil, with given size of it (the pupil). This effect is in addition to foregoin, influenced with optical aberrations [5], [6].

4.OPTICAL STRAIN GAUGE

At arrangement in optical free space geometry [4] it is advantageously used for measurement of elastic deformation two arrangements, according to Fig.3. They are either double symetrical illumination with one CCD scanner, or double symetrical scanning with single illumination. If we save these approaches, whereat the optical system is always placed into direction of observation, then we obtain conditions for determination of elastic deformation components which are rather differnt than the ones at arrangement in optical free-space geometry. Then at solution of a concrete problem at measurement of elastic deformation components we obtain a widened scale of possible arrangements, sensitivity and measuring accuracy. We obtain then

$$\Delta A_x = A_x(\theta_s, \theta_R) - A_x(-\theta_s, \theta_R) = -\frac{2Z}{L_0'}\left(a_z \frac{L_0 \cos\theta_s \sin\theta_s}{L_s \cos\theta_R} + L_0 \varepsilon_{xx} \frac{\sin\theta_s}{\cos\theta_R}\right) \qquad (11)$$

By comparison of relation (11) with the arrangement in optical free space geometry [4] we obtain the same relation, only differed with coefficient Z/L_0 . In case of collimated beam of illumination ($L_0 \langle\langle |L_s|$), we obtain the relation (11) in form

$$\Delta A_x = -2\frac{Z}{L_0'} L_0 \varepsilon_{xx} \frac{\sin\theta_s}{\cos\theta_R} = 2\frac{Z}{m}\frac{\sin\theta_s}{\cos\theta_R}\varepsilon_{xx}, \qquad (12)$$

when the difference is given again by coefficient Z/L_0. In the case of a symetrical arrangement with $\theta_R = 0$, we obtain advantageously

$$\Delta A_x = 2\frac{Z}{m}\varepsilon_{xx}\sin\theta_s. \qquad (13)$$

It follows from relations (11), (12), (13) that at utilization of optical system this method is insensitive at Z=0, thus in image plane. On the contrary, the sensitivity of this method can be influenced expresively with the choise of coefficient Z/m, thus with selection of optical system and with geometrical arrangement of experiment.

At the arrangement according to Fig.3b (two directions of observation) in plane (x,z) we are to make records of distributed speckle intensities on both CCD scanners, always before and after deformation. Then we obtain

$$\begin{aligned}\Delta A_x &= A_x(\theta_s, \theta_R) - A_x(\theta_s, -\theta_R) = \\ &= 2ma_z \sin\theta_R - \frac{Z}{L_0'}(2ma_z \sin\theta_R + 2a_z \sin\theta_R + 2L_0\varepsilon_{xx}\tan\theta_R).\end{aligned} \qquad (14)$$

By comparison of relation (14) with arrangement in optical free space geometry [4], we ascertain that both third and fourth expressions are influenced again with coefficient Z/L_0' only, but they appear two new members that influence coponent a_z of displacement vector **a**.

The arrangement according to Fig.3b doesn't depend on angle θ_S, thus on angle position of illuminating source. For $Z \langle\langle |L_0'|$ the expression (14) has form

$$\Delta A_x = 2ma_z \sin\theta_R \tag{15}$$

and the arrangement is convenient to measurement of component a_z of displacement vector **a**. For $Z = |L_0'|$ it holds

$$\Delta A_x = -2a_z \sin\theta_R - 2L_0\varepsilon_{xx}\tan\theta_R, \tag{16}$$

i.e. the arrangement is analogical as at arrrangement in optical free space geometry [4]. For $Z \rangle\rangle |L_0'|$ the relation (14) has form

$$\Delta A_x = -2\left(\frac{Z}{L_0'} - 1\right) ma_z \sin\theta_R - \frac{Z}{L_0'}(2a_z \sin\theta_R + 2L_0\varepsilon_{xx}\tan\theta_R) \tag{17}$$

where the expression in parentheses at second member of equation corresponds with arrangement in optical free-space geometry. If we further elect $|a_z| \langle\langle |\varepsilon_{xx}| . \frac{L_o}{\cos\Theta_R}$, then the members with a_z are negligible and the relation (17) has form

$$\Delta A_x \cong 2Z\frac{1}{m}\varepsilon_{xx}\tan\theta_R, \tag{18}$$

which means that the method is $(-Z/mL_0)$ times more sensitive then at arrangement in optical free-space geometry.

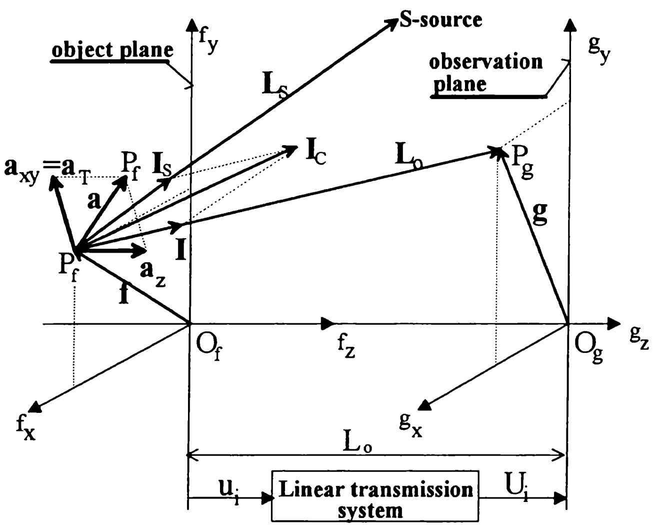

Fig.1 Coordinate systems for derivation of speckle displacement in the free-space geometry.

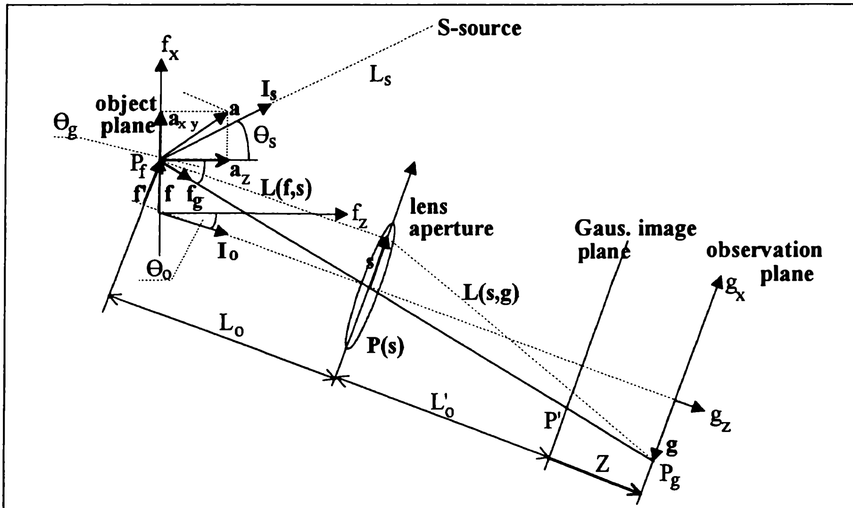

Fig.2 Coordinate systems for derivation of speckle displacement in the image field.

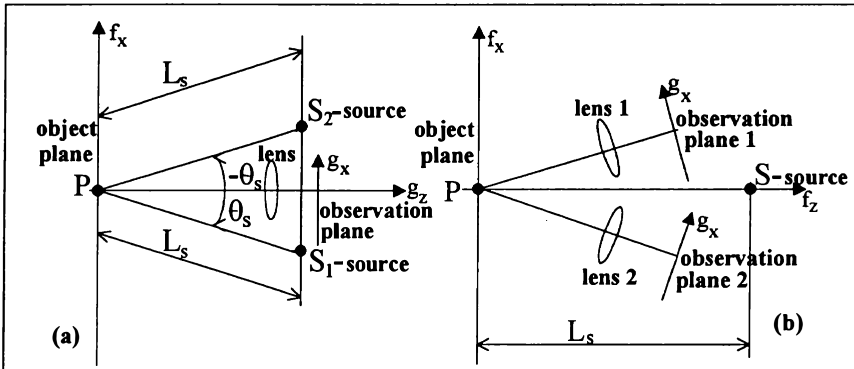

Fig.3 Two configurations of speckle strain gauge. (a) Dual beams; (b) double sensors.

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