

**THE STRESS AND STRAIN ANALYSIS FOR  
 A CYLINDRICAL HORIZONTAL SHELL AND  
 ESTABLISHMENT OF THE POSITION FOR SUPPORTS**

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**Abstract**

In the paper, we present a finite element analysis, for the state of deformation, at a part of a horizontal cylindrical vessel. The purpose of this analysis is to establish the optimal position of the supports, for a minimal state of stresses and deformations.

**1. Introduction**

It is considered a horizontal cylindrical vessel (figure 1), which has the length greater than diameter. The vessel is filled with a liquid with the specific mass  $\gamma$ . The fluids actuate to the wall of vessel with a static internal pressure, in the radial direction; the value of pressure into a point is proportionally with the depth of the point. The vessels stay by the supports with an adequate form; theoretical, the contact between vessel and supports is an arc of circle.

The problem in discussion cannot be solved with the calculus relations known from the strength of materials, [1].

In this paper, the authors give an example for solving the problem with a finite element analysis.

The solution is obtained for a particular case, respectively for a steel vessel, with radius  $R = 1\text{m}$  and the thickness of the wall of 20 mm; the vessel has the length  $L = 12\text{m}$  and it is filled with a liquid who have  $\gamma = 10^{-5}\text{N/mm}^3$ .

**2. Establishment of the load on vessel**

To realise the finite element analysis it is necessary that the loads which act in the mesh nodes to be concentrated. This loading can be determined by a preliminary

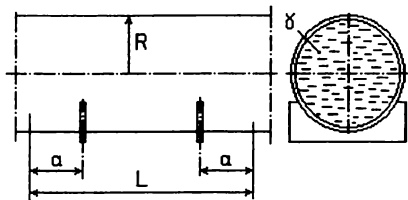


Figure 1

calculus, in which we consider that on the wall of vessel acting a variable pressure like direction and size too.

In the next sequences is presented, first, the solution of problem in the general case, after than this is particularised to the studied case.

It is considered (figure 2) a part of the vessel wall, delimited by two transversal sections at the distance  $t$  and by the vertical symmetry plan. In the scheme of calculus is represented the intensity of the distributed load,  $q(\alpha)$ , which is calculated

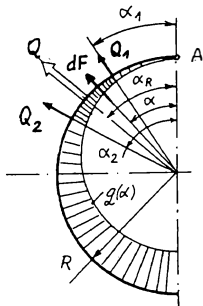


Figure 2

depending on the pressure  $p(\alpha)$  with the relation

$$q(\alpha) = t \cdot p(\alpha) \quad (1)$$

Admitting that the vessel is full charge with liquid and that the pressure exercised by the liquid is produced only by his mass, the variation law of the pressure become:

$$p(\alpha) = \gamma \cdot R \cdot (1 - \cos \alpha) \quad (2)$$

It is considered the loading which acts in the delimited zone by the angles  $\alpha_1$  and  $\alpha_2$ ; we formulate the next two problems:

(P1) - it must be determined the resultant force

$Q$  of the distributed load (its size and its angle  $\alpha_R$ );

(P2) - it must be replaced the distributed loading with two concentrate forces,  $Q_1$  and  $Q_2$ , placed at the end of the zone.

The result of the second problem (P2) is necessary in the finite element analysis. The solution of the first problem (P1) gives a more simple expression for the results of the second problem (P2).

For to solve these problems we include the condition that the forces  $Q$  or ( $Q_1$ ,  $Q_2$ ) which replace the distributed load, are equivalent with that, which means it must have:

- the same projection on a determined direction (e.g. vertical direction);
- the same moment in rapport with a given point (e.g. in rapport of A point).

The a) and b) conditions lead, respectively, to:

$$\int_{\alpha_1}^{\alpha_2} q(\alpha) \cos \alpha \cdot R \cdot d\alpha = Q \cos \alpha_R = Q_1 \cos \alpha_1 + Q_2 \cos \alpha_2 \quad (3)$$

$$\int_{\alpha_1}^{\alpha_2} q(\alpha) \sin \alpha \cdot R^2 \cdot d\alpha = Q \sin \alpha_R = Q_1 R \cdot \sin \alpha_1 + Q_2 R \cdot \sin \alpha_2 \quad (4)$$

Taking into account the relations (1) and (2), the solution of the systems equations which derived from relations (3) and (4), leads to the next results:

$$Q = \gamma R^2 t \sqrt{I_1^2 + I_2^2}; \quad \tan \alpha_R = \frac{I_1}{I_2} \quad (5)$$

$$Q_1 = Q \frac{\sin(\alpha_2 - \alpha_R)}{\sin(\alpha_2 - \alpha_1)}; \quad Q_2 = Q \frac{\sin(\alpha_R - \alpha_1)}{\sin(\alpha_2 - \alpha_1)} \quad (6)$$

in which:

$$I_1 = \int_{\alpha_1}^{\alpha_2} \sin \alpha \cdot (1 - \cos \alpha) \cdot d\alpha \quad ; \quad I_2 = \int_{\alpha_1}^{\alpha_2} \cos \alpha \cdot (1 - \cos \alpha) \cdot d\alpha \quad (7)$$

The solutions of these integrals lead to the following results:

$$I_1 = \cos \alpha_1 - \cos \alpha_2 - \frac{1}{4} \cos 2\alpha_1 + \frac{1}{4} \cos 2\alpha_2 \quad (8)$$

$$I_2 = \sin \alpha_2 - \sin \alpha_1 - \frac{1}{4} \sin 2\alpha_2 + \frac{1}{4} \sin 2\alpha_1 - \frac{1}{2} (\alpha_2 - \alpha_1) \quad (9)$$

### 3. Calculus of the loads in nodes

To realise the finite element analysis, the length  $L = 12$  m of the vessel was divided in 12 equal parts; the circumference was divided in 24 sectors, for each of them corresponding a  $15^\circ$  centre angle. In this way it was realised a mesh with 288 elements. Because the type of finite element which was utilised has intermediary nodes at the middle of each side, the loads which were obtained from the liquid pressure were introduced in 25 transversal sections situated at the distance  $t = 0.5$  m. In each of these sections act 12 concentrated forces like in figure 3. They was calculated with the relations (5), ... ,(9) and it obtained the values show in Table 1. On the end sections, the forces have values equal with half of the values show in Table 1.

Table 1

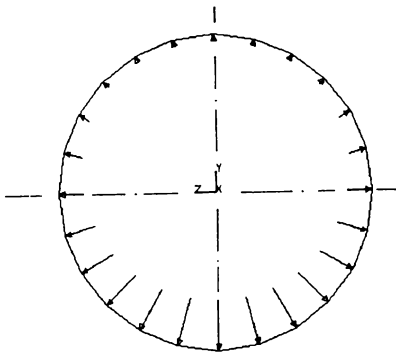


Figure 3

Nod <i>i</i>	$Q_i$ N
1	7.528
2	52.131
3	182.900
4	390.924
5	662.026
6	977.231
7	1316.528
8	1655.318
9	1971.023
10	2242.125
11	2450.150
12	2580.917
13	2625.509

### 4. The finite element analysis

The discretisation of the analyzed model, was made considered 12 elements on the length of the vessel and 24 elements on its circumference. The element used in the

analyse was QTS8, these being included in the class of the thin shell in 3-dimensions; this class is specific for the program of analysis with finite elements, LUSAS ver. 11.1.

The position of supports on the circumference was considered for an angle of 150 degree, respectively for a number of 10 elements.

In the analysis we consider that the material of vessel is an thermo-resistant steel with the next characteristics: Young modulus -  $1.95 \cdot 10^5$  MPa, Poisson ratio - 0.28, specific weight -  $7.85 \cdot 10^{-5}$  N/mm.

For the establishment of the best position of supports, from the point of view of stresses and displacements, we consider at the beginning, for them position, a value of  $a = 0.208L$ . This starting value was chosen from the theory of bending, applied at the straight beams. In the case of a straight beam, with simple supports, charged with a uniform distributed load, the position of supports must be at the distance of  $0.2119L$  from the extremities, so that the rotation of the ending sections to be zero.

### 5. Results. Conclusions

After we made some other analysis, in which was modified the  $a$  parameter between the values  $0.166L$  and  $0.33L$ , we choose  $a = 0.25L$ .

For this value of  $a$ , we obtain the best results for the stresses and displacements, results which are presented in the figures 4,....,8.

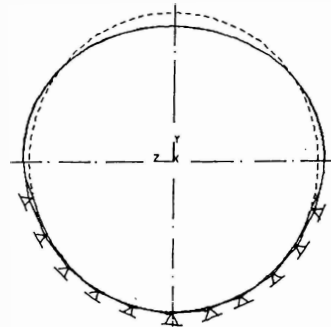
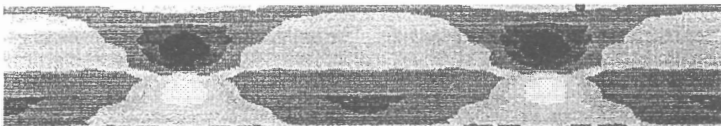


Figure 4 The deformation of the vessel in the transversal section in which the supports stay



Figure 5 The shape of the deformed vessel

Figure 6 The distribution of the normal stress on the longitudinal direction



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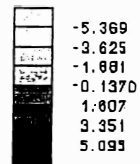




Figure 7 The distribution of the maximum principal stress

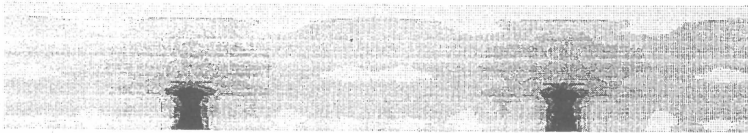
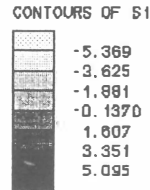
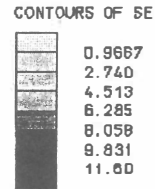


Figure 8 The distribution of the Von Mises equivalent stress



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