EAN '97

35th International Conference on
Experimental Stress Analysis
June 4 - 6, 1997, Olomouc, Czech Republic

STRESS STATE IN ODING TYPE RING, DETERMINED ANALYTICALLY, NUMERICALLY AND EXPERIMENTALLY

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Abstract

This paper presents original relations established by authors in order to design Oding type rings used in stress relaxation test. Stress distribution in the active zone of the ring can be determined by three methods: analytically, numerically (finite elements analysis) and experimentally (strain gauge measurement)

1. Introduction

As it is known, the test specimen most frequently used in stress relaxation experiment, [5], is the Oding ring; its *active zone* has a rectangular variable cross section and its constitutes a fully stressed curve bar; many researches use the Oding ring with its original dimensions: $\rho = 26.8$ mm, $e_1 = 0.7$mm, $e_2 = 0.7$ mm, $R_1 = 25$ mm, $R_2 = 28.6$ mm.

Papers [3] and [4] show relations that allow designing a "Oding type ring" of different dimensions than the original. Two main dimensions are to be established first of all: ρ - the radius of curvature and H_0 - the minimum height of the cross section. The adimensional parameter can be determined:

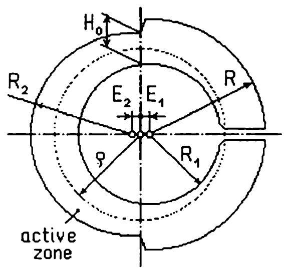

Figure 1 Oding ring

$$t_0 = \frac{H_0}{2\rho} \tag{1}$$

and the coefficients:

$$\begin{aligned} e_1 &= 0.648578\,t_0^2 + 0.388947\,t_0 + 0.0003909507 \\ r_1 &= 0.229937\,t_0^2 - 1.025755\,t_0 + 1.000248 \\ e_2 &= 0.342595\,t_0^2 + 0.399208\,t_0 + 0.0001372968 \\ r_2 &= 0.0959791\,t_0^2 + 1.015504\,t_0 + 0.999984 \end{aligned} \tag{2}$$

which help establishing the constructive dimensions:

$$E_1 = e_1\rho, \quad R_1 = r_1\rho, \quad E_2 = e_2\rho, \quad R_2 = e_2\rho \tag{3}$$

This paper checks the above relations in the particular case of a ring with ρ = 24.5 mm şi H_0 = 4 mm; this consists of the determination of the stress distribution in the active zone of the ring in order to find out whether this zone is a fully stressed bar; the stresses are determined analytically (with the relations already known from the strength of materials), numerically (finite elements analysis) and experimentally (strain gauge measurement).

2. The analytic determination of the stress distribution in the Oding type ring

The adimensional parameter t_0 , (1), for the studied ring, has the value $t_0 = 4/(2 \cdot 24.5) = 0.0816$; considering (2) and (3):

$$E_1 = 0.8933 \text{ mm}, \quad R_1 = 22.492 \text{ mm},$$
$$E_2 = 0.8577 \text{ mm}, \quad R_2 = 26.546 \text{ mm} \tag{4}$$

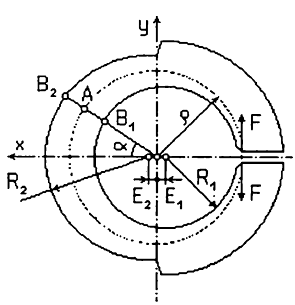

Figure 2

The active zone of the ring is a curve bar the axis of which is an circle arch with the radius ρ. The cross section of the bar is rectangular: the breadth G = 5 mm is constant but the height is variable. In a current section, determined by the angle α, figure 2, the height $H(\alpha)$ is equal with the segment $B_1 B_2$.

B_1 , B_2 are the cross points of the straight line $y = x \cdot \tan\,\alpha$ with the circles that have the equations:

$$(x+E_1)^2+y^2 = R_1^2$$
$$(x-E_2)^2+y^2 = R_2^2 \tag{5}$$

The co-ordinates of the points B_1 and B_2 can be determinated and then the values:

$$H_1(\alpha) = AB_1 , H_2(\alpha) = AB_2 , \; H(\alpha) = H_1(\alpha) + H_2(\alpha). \tag{6}$$

They are listed in Table 1 for α at each 10°. The table also presents the

Table 1

α gr	$H_1(\alpha)$ mm	$H_2(\alpha)$ mm	$H(\alpha)$ mm	$H_1(\alpha) - H_2(\alpha)$ mm
0	2.901257	2.904001	5.805258	0.0027
10	2.888220	2.890552	5.778771	0.0023
20	2.849457	2.850653	5.700110	0.0012
30	2.786004	2.785625	5.397191	- 0.0003
40	2.699583	2.697608	5.571629	- 0.0019
50	2.592550	2.589484	5.182034	- 0.0031
60	2.467885	2.464751	4.932636	- 0.0032
70	2.329115	2.327406	4.656521	- 0.0018
80	2.180238	2.181786	4.362024	0.0015
90	2.025642	2.032427	4.058069	0.0068

difference $H_1(\alpha) - H_2(\alpha)$ in order to show that the point A is situated at the middle of the segment $B_1 B_2$.

Practically, the ring is realised with less dimensional precision than these given by the values calculated with (4). It is built with: $E_1 = 0.89$ mm, $R_1 = 22.5$ mm, $E_2 = 0.86$ mm, $R_2 = 26.5$ mm. Thus the values listed in Table 1 change as shown in Table 2. It is these latter values that help calculating the stresses in the active zone of the ring.

Table 2

α gr	$H_1(\alpha)$ mm	$H_2(\alpha)$ mm	$H(\alpha)$ mm	$H_1(\alpha) - H_2(\alpha)$ mm
0	2.889999	2.860001	5.75	0.02999
10	2.87701	2.846513	5.723523	0.03049
20	2.838387	2.806502	5.644889	0.03188
30	2.775164	2.741294	5.516459	0.03387
40	2.689053	2.653033	5.342086	0.03602
50	2.582413	2.544607	5.127020	0.03781
60	2.458206	2.419533	4.877739	0.03867
70	2.319947	2.281813	4.601760	0.03813
80	2.171624	2.135801	4.307425	0.03582
90	2.017609	1.986042	4.003651	0.03157

The normal stress in the cross section α of the ring has extreme values in B_1 and B_2; the extreme values are calculated, [2], with the following relations:

$$\sigma_{B_1}(\alpha) = \sigma_{\text{int}}(\alpha) = \frac{1}{A(\alpha)}\left(N(\alpha) + \frac{M(\alpha)}{\rho} + \frac{M(\alpha)}{\rho \cdot k(\alpha)} \cdot \frac{-H_1(\alpha)}{\rho - H_1(\alpha)}\right), \tag{7}$$

$$\sigma_{B_2}(\alpha) = \sigma_{ext}(\alpha) = \frac{1}{A(\alpha)}\left(N(\alpha) + \frac{M(\alpha)}{\rho} + \frac{M(\alpha)}{\rho \cdot k(\alpha)} \cdot \frac{H_2(\alpha)}{\rho + H_2(\alpha)}\right), \tag{8}$$

where $k(\alpha)$ represents the *shape coefficient* (adimensional), $A(\alpha) = G \cdot H(\alpha)$, mm^2, the transversal section value; $N(\alpha)$ and $M(\alpha)$ represents the axial force and the bending moment. These values are calculated with the relations:

$$k(\alpha) = -1 + \frac{1}{2\,t(\alpha)} \ln \frac{1 + t(\alpha)}{1 - t(\alpha)}, \tag{9}$$

$$t(\alpha) = \frac{H(\alpha)}{2\rho}.$$

$$N(\alpha) = F\cos\alpha$$

$$M(\alpha) = -FR(1 + \cos\alpha)$$

The calculus of stresses is done for $F = 150$ N. The results are shown in Table 3. The stress in the points on the interior contour of the ring has the average value of $299.83 \text{ MPa} \approx 300 \text{ MPa}$.

Table 3

α gr	$\sigma_{ext}(\alpha)$ MPa	$\sigma_{int}(\alpha)$ MPa
0	-240.89	296.29
5	-241.01	296.37
10	-241.40	296.62
15	-242.03	297.03
20	-242.90	297.59
25	-244.00	298.27
30	-245.31	299.06
35	-246.80	299.92
40	-248.45	300.80
45	-250.22	301.67

α gr	$\sigma_{ext}(\alpha)$ MPa	$\sigma_{int}(\alpha)'$ MPa
50	-252.05	302.46
55	-253.90	303.09
60	-255.69	303.48
65	-257.33	303.53
70	-258.72	303.10
75	-259.74	302.06
80	-260.24	300.24
85	-260.05	297.45
90	-258.97	293.47

The average stress on the interior of the ring was calculated to the stresses in 91 sections of the ring (for α = 0°, 1°, ... 90°). The stresses with the biggest error are 303.56 MPa (at α = 63°) and 293.472 MPa (at α = 90°), therefore with deviations of +1.24 % and of -2.12 %, respectively.

3. Stress analysis with finite elements

Because of the symmetry, the analysis was done only for a half of the ring; the stresses in the active zone and in the load application zone are studied, [1]. Quadrilateral finite elements with 8 nodes are used. The total number of elements was 684 and of nodes 2223.

Figure 2 presents the meshed model and the field of isostatics. On the interior contour is situated the isostatic of 281 MPa.

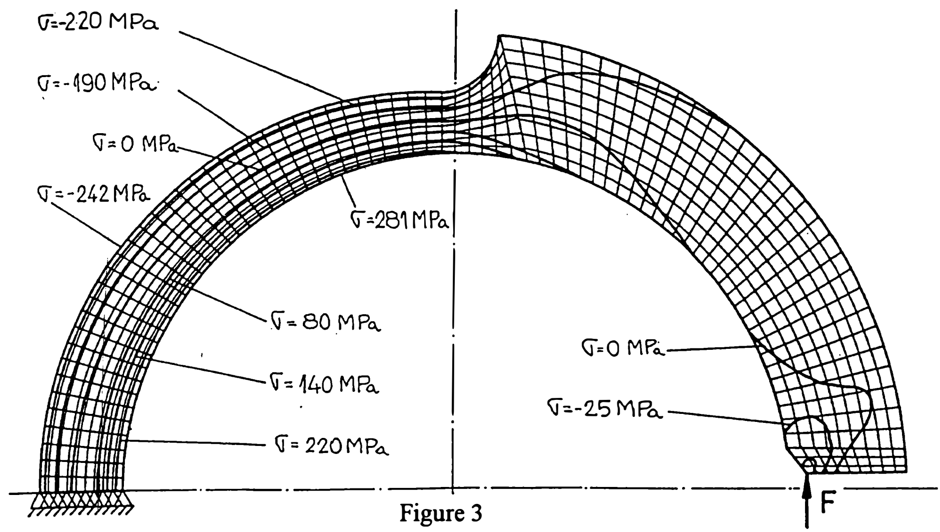

Figure 3

4. The experimental stress determination on the interior contour

The strain gauge measurement are used. Four strain SM 120 (Microtechna Praga) were soldered on the interior of the ring. Introducing a wedge that caused a supplementary span of 1.92 mm helped creating a loading force 150 N.

The average measured stress was of 311 MPa.

5. Conclusions

Relations (2) and (3), demonstrated in the papers [3] and [4], allow an accurate designing of "Oding type" rings, with different dimensions than these of the original ring. All check methods show a relatively constant value of the stress on the interior contour. The value differences in the three methods come from errors specific to each. The finite elements analysis shows a field of stresses specific to a fully stressed bar.

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