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STRESS ANALYSIS APPLICATION USED FOR A COMPLEX ASSESSMENT OF THE FATIGUE STRENGTH OF RUBBER MIXER

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Resumé: *An experimental stress analysis of the shaft of a mixer for India rubber mixing is presented in the paper. Using strain gauges, determined were internal forces and moments in three investigated cross-sections of the shaft. There appeared five characteristic periods in the course of the mixing process. Using an elementary analytic model, loads acting on the rotor tips were computed for all the periods. Assessed were then the bending moment distributions in the rotor, as well as the stress distribution in all the critical points of the rotor blades. Such obtained internal forces and moments are to be used as input values for the subsequent mixer FEM amalysis.*

Introduction:

Process of homogenization of a rubber mixture having defined properties is one of the first technologic operations carried out during the tyre manufacture. For this purpose a special mixer is used. The machine consists of two parallel cylindrical chambers containing two asynchronous counter-rotating shafts, each of them having two spiral blades with a different pitch. A dose of raw rubber is mixed with required ingredients for about 2 minutes. During this process, the blades are exposed to high mechanical, abrasive and thermal loading which resulting in certain damages: usually caused by fatigue cracking in the high stress concentration regions, especially at the blade tips with hard surfacing.

After consultations with the producer, a plan of necessary work to be done at the CTU, Fac. of Mech. Eng., which including the bellow listed items of the problem solution:

1. Experimental assessment of forces and moments acting on the mixer shaft under operating conditions.
2. FEM stress analysis of various types of blades.
3. Fatigue tests of the shaft material sample containing the hard surfacing.
4. Theoretical prediction of the fatigue lifetime of the machine components and the assessment of the potential positive effects caused by the accomplished improvements.

This paper describes the solution of the first partial task of the problem.

The proposed measurements of forces and moments acting on the mixer shaft, by using strain gauges, were carried out on a machine working in the firm of MITAS Praha a.s. The capacity of the mixer chambers was *250 l* while the shaft rotated with the nominal angular speed of *30 rpm*. The measurements were executed in cooperation with SVUSS Praha a.s.

Measurement preparation and instrumentation

The strain gauges were possible to be placed only on several parts of the shaft, i.e. on the accessible sections, having the length of about 125 mm, situated on the both sides of the shaft between the working chamber and the bearings. During the operation, the shaft was rotating at the speed of 24 rpm, i.e. having a slip with respect to the parallel shaft speed being 30 *rpm*.

The signal transmission from the measured shaft was executed by means of brushes and rings. This technically optimal method did not cost much and was not time-consuming. On the rotating shaft, placed also were the power source, amplifier and optical rotation counter in order to assess the blade angular displacement with respect to the vertical plane.

To determinate the axial force, torque and bending moment transferred by the shaft, together with the bending moment gradient in the measured shaft sections, chosen were three cross-sections marked A,B,C (Fig. 1) for the strain gauge installations. For the torque measuring in the cross-section A, installed were two strain gauge crosses, having the 45° orientation, connected to full bridge. The Vishay MM foil strain gauges of the type EA 06-125 TW 120 were used. The axial force and bending moment measuring was accomplished by placing three axial strain gauges at 120° on the shaft circumference. The used strain gauges were by Hottinger Baldwin Messtechnik, type 3/120 LY 11, having defined thermal curves for correcting possible temperature changes during measurement. Parallel to the deformations, measured were the temperature of both the gland box flange and the shaft itself, here close to the strain gauges, using the G-600-A measuring resistors by EKOREG s.r.o., Ustí nad Labem.

For the strain gauge signal recording, placed directly on the shaft was a DC feeding which amplified the signals as well. The amplification reached up K=200. Measuring equipment consisted of the amplifier, the A/D converter and the measuring computer (Cardstar PC 486). The shaft and flange temperatures were measured by using the exchange UPM60.

Theoretical stress analysis of the mixer shaft

Numerical investigation of the force distribution over the mixer shaft is a very difficult task of the non-linear theory of viscous liquid mixing. Therefore, when designing the computational model, considerably simplified and idealized conditions were assumed, which had been given gradually more precision to, with respect to the experimentally and numerically (FEM) obtained data.

During mixing, the working surfaces of the mixer blades are loaded by distributed forces of unknown function. Considered was the simplest computational model, which substituting the load by a concentrated force F acting approximately in the middle of the blade length and in the distance of $R=250\ mm$ from the mixer shaft axis, i.e. in the centre of gravity of an idealized distributed load having a triangular shape. The force resolvings are according to the drawing documentation shown in **Fig. 1**. The longer left blade (L) is loaded by the force N_L and the shorter right blade (P) by the force N_P. These both forces are resolved into the axial component O and in the component F being perpendicular to it. The magnitudes of the normal forces N_L, N_P are supposed to be determined by the mixture properties for a given mixing moment. The force ratio N_L / N_P is then the investigated unknown variable.

The axial force, exerting on the shaft as a resultant force in axial direction, was possible to determine experimentally. When resolving the forces acting on the blades, the following expression were derived:

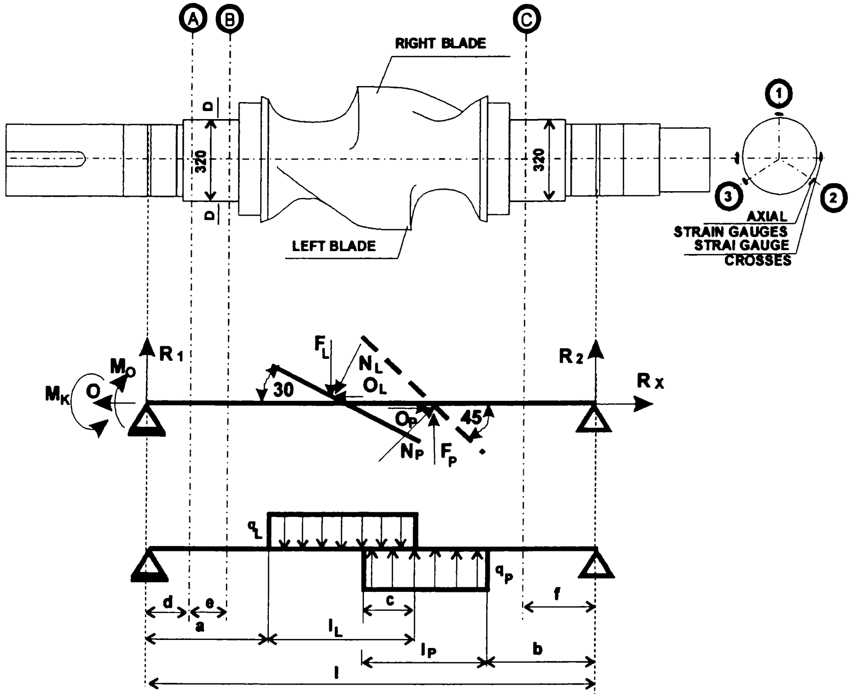

Fig. 1 Scheme of the mixer shaft with blades and the beam model

$$O = O_P - O_L = -\frac{1}{\sqrt{2}} F_P\left(\frac{N_L}{N_P}\right) + \frac{\sqrt{2}}{\sqrt{3}} F_L\left(\frac{N_P}{N_L}\right) = F_P\left(-\frac{1}{\sqrt{2}}\frac{N_L}{N_P} + 1\right) \qquad (1)$$

Torque, which can be derived from measuring as well, is given by the expression:

$$M_K = \left(F_P + F_L\right)R = F_L R\left(\frac{\sqrt{3}}{\sqrt{2}}\frac{N_L}{N_P} + 1\right) \qquad (2)$$

Solving the equations (1) and (2), the requested ratio N_L / N_P of mixing forces acting on the blades was obtained in the form

$$\frac{N_L}{N_P} = \sqrt{2}\,\frac{M_K - O \cdot R}{M_K + \sqrt{3} \cdot O \cdot R} = \sqrt{2} \cdot \frac{1 - \dfrac{O \cdot R}{M_K}}{1 + \dfrac{\sqrt{3} \cdot O \cdot R}{M_K}} \qquad (3)$$

whose concrete values can be computed for the known, experimentally determined, quantities O and M_k.

A beam model was used (Fig. 1) for analytical approach to the mixer bending stress determination. The beam is supposedly subjected along the blades to uniformly distributed load $q_L=F_L/l_L$, $q_P=F_P/l_P$. The documentation shows the dimensions as follow: $l=1638$ *mm*, $a= 416$ *mm*, $b= 416$ *mm*, $c= 124$ *mm*, $d=156$ *mm*, $e=120$ *mm*, $f=276$ *mm*, $l_L=580$ *mm*, $l_P=350$ *mm*.

Computed can be magnitudes of the reaction R_1 and the support moment M, which acting in the support 1, from the bending moment values experimentally determined in sections A and B, i.e. in the segment, where the moment is linearly distributed. Applying the moment equilibrium equations with respect to the supports 1 and 2, respectively, the reaction magnitudes in those supports were obtained. Using experimentally found values of the ratio N_L / N_P (3) and the reactions exerting in the support 1 (i.e. R_1 and M), computed were uniformly distributed load of blades from the following equations:

$$q_L = \frac{R_1 l + M}{l_L\left(b + l_P - c + 0{,}5 l_L\right) - \left(\frac{q_P}{q_L}\right) l_P\left(b + 0{,}5 l_P\right)}, \quad \frac{q_P}{q_L} = \sqrt{\frac{2}{3}\frac{l_L}{l_P}\frac{N_P}{N_L}} \tag{4}$$

By this procedure, determined was the loading of the beam model which allows to derive the bending moment distributions in each of the beam segments. The curves of these bending moments were computed and some typical segments of the experimentally obtained, time-depending, working load are plotted in **Fig. 2**.

Analysis of measurement and results

The measurement provided 27 data files which were utilized for the bending and axial analyses and 7 files for the torque analysis. Each file contained 120 000 samples (sampling frequency being 500 samples per second). The total number of all the data measured reached figure of 4 080 000. The data were filtered and reduced to an equal number of values and then statistically evaluated to obtain a typical mixing process. Using experimentally obtained deformations, performed were computations yielding:

a) the strain ε_{MO} corresponding to to the bending moments;
b) the strain ε_O corresponding to the axial force;
c) the angle φ determining the bending moment plane position [1].
A similar procedure was applied in all the sections A,B and C.

The whole process showed several typical phases in the measured data distribution, (see **Fig. 3**):

I. Rise of torsional moment from the unloaded stage to the full load (rot range 0-5).
II. Work region of the maximum load (mixing the tough mixture into a dough, rot range 6-18).
III. Intensive load release (the mixture warming-up reducing the mixing resistance, rot range 19-26).
IV. Slight stress increase (by adding the filler and other components, rot range 27-38).
V. Phase of mixing resistance caused by the dough mixture (until the chamber discharging).

The twisting stress reached its peak $\tau_{max} = 17.2$ *MPa* in the phase II. After averaging, the peak was reduced to $\tau = 13$ *MPa*, which dropped *to* $\tau_{max} = 6\text{-}8$ *MPa* in the final phase *V*. Assessed was, when computing the equation (3), that, in the initial phase of mixing *I* and in the final stable phase *V*, the ratio N_L / N_P reached closely the value of *1,05*, which confirmed the

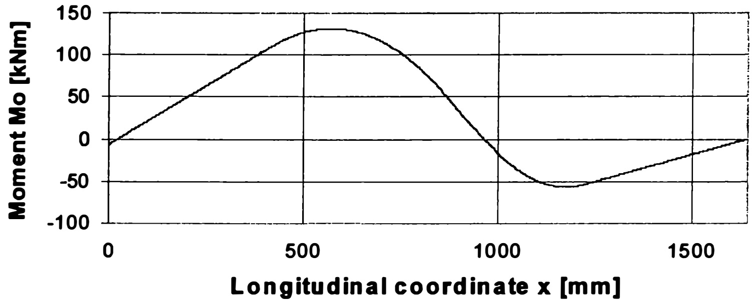

Fig. 2 Distribution of the computed bending moments

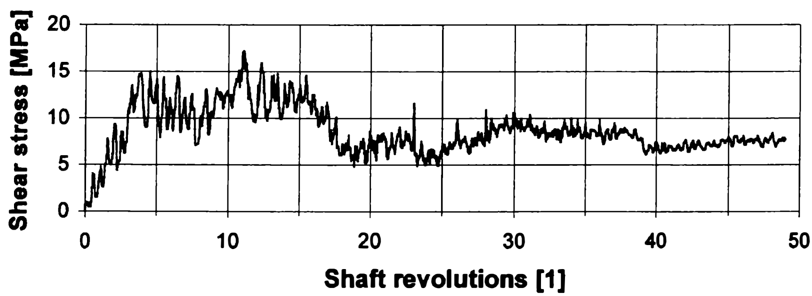

Fig. 3 Time dependence of the mixer shaft shear stress

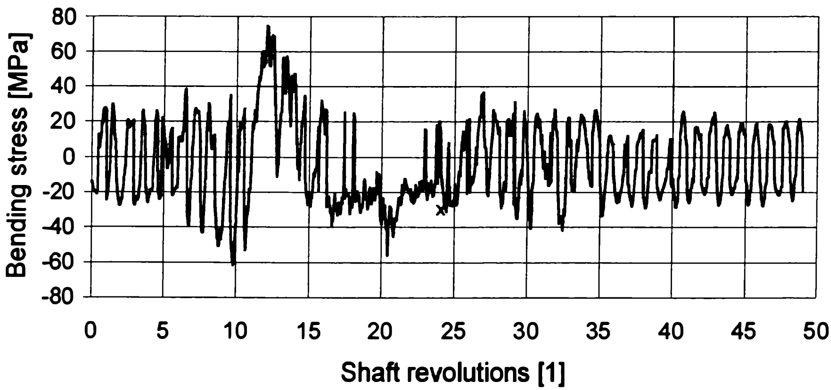

Fig. 4 The bending stress magnitudes on the blade

force balance on the blades. In the operating phase *II,* the ratio N_L / N_P (being *0,6*) was smaller. It means that the longer blade was subjected to a smaller normal force than the right shorter blade. This can be explained by the pressure of the working rammer in this phase. Because the longer blade has a smaller haunch angle, the rammer may induce higher back pressure (acting against mixing resistance) on it than on the other blade.

The highest stress value in tension (being *13,5 MPa*) was reached in the section *B* during the phase *II*, which corresponds with the axial force of *790,1 kN*, while in the stable phase *V*, the stress reached only the value of *4,5 MPa* which corresponding to the force of *271, 5kN.*

The highest bending stress peak on the blade reached the value of *75 MPa*, while in the stable phase there acted its average of *±22 MPa* , see **Fig. 4**. The time dependence of the bending moment track position φ showed, that, for $\varphi = \pm\ 90°$, the point *1* of the first strain agauge passed either the upper or the lower positions of the shaft (likewise the starting point of the long blade spiral). Consequently, the shaft was bent in the cross-section *B* in the horizontal plane. To make clearer the position of the shaft bending plane, the relative angle φ, with respect to the rotating shaft, has been recounted to the angle β, which corresponded to the horizontal plane of the machine frame. Its course was changing substantially during the initial phase. And, in the final phase of mixing, when the force relations being already stable, the shaft bent under the average angle $\beta = -18°$ (in the cross-section *B*) and under the angle $\beta = +18°$ (in the cross-section C). It concerned the unsymmetric bending with respect to various planes. This fact was not considered in the simplified beam plane model and thus the load results, obtained according to this model, are slightly on the safe side.

Conclusion

For a typical mixing cycle, carried out were an experimental stress analysis of a mixer shaft (applying strain gauges), together with an analytical approach (using a simplified beam model), aiming at the assessment of a basic time-dependent spectrum of the forces and moments exerting in the shaft (produced by tension, bending and torque). Using such obtained results, assessed was the stress distribution acting in the measured shaft cross-sections as well as its extrapolation on the mixer blade. The computed internal forces and moments, exerting in the measured shaft cross-sections, will be used as outgoing values for the FEM mixer blade model which has been just designed

References:

[1] RŮŽIČKA, M.;MICHALEC JIŘÍ: Experimentální analýza namáhání hřídele hnětače 250 l. Zpráva FS ČVUT,Praha 1996.

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