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PROCESSING METHOD OF EXPERIMENTAL DATAS OF TIME CONCRETE STRENGTH RELATIONSHIP IN THE TIME T TO MATHEMATICAL DIMENSIONAL CORRECT DEPENDENCE

METODIKA SPRACOVANIA EXPERIMENTÁLNYCH DÁT RASTU PEVNOSTI BETÓNU SLEDOVANÉHO V ČASE T DO MATEMATICKEJ DIMENZIONÁLNE SPRÁVNEJ ZÁVISLOSTI

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Processing method of changeable concrete strength into mathematical non-dimensional correct dependence. Dimensional analysis as a base for derivation of non-dimensional arguments. Criterial equation as mathematical model for experimental or theoretical datas processing to searched dependence. Numerical application. The method processing mathematical dependence is based on dimensional analysis. The method uses the SI units and transfers all dimensional arguments to the non-dimensional arguments. The resulting dependence is approximately determinated from initial experimental datas. The propriety of the obtained dependence is verified by the correlation index.

1. General rules for cube strength calculation

The basic characteristic of concrete is its compressive strength. It is determinated on cubes or cylinders by standard experimental test. For the classification of concrete, the compressive strength, determined on 150 mm cubes at the age of 28 days was used. According to standard [1], cube strength was determined by formula:

$$R_{c,cu} = \frac{F}{A} \tag{1}$$

where F is ultimate force [N],

A - compressed cross-section area of tested specimen [mm^2].

The cube strength is affected by various factors, such as: concrete mixture proportion, properties of concrete components, proportion of tested specimens, workability of the fresh concrete, admixtures, bulk density of hard concrete, age of concrete, temperature and humidity of environment, etc. In generality, each factors can be marked as: a_1 , a_2 , ... a_n . From this point of view, cube strength is function of "n" variable factors:

$$R_{c,cu} = f\,(a_1 , a_2 , \dots a_n) \tag{2}$$

Mathematical model was used for mathematical analyse.

2. Mathematical processing

2.1 Mathematical processing method

If we want to make dependence between changable bulk density (ρ_v) and cube strength ($R_{c,cu}$), we need select factors, which are constant in experiment, such as:

- standard dimension of cube a,
- bulk density ρ_v,
- age of concrete t,
- unit strength value of concrete $\overline{R}_b$ – 1,0 MPa

Then formula is:

$$R_{c,cu} = f\,(\rho_v, a, t, \overline{R}_b \tag{3}$$

Selected quantities formed into non-dimensional argument by using basic SI units:

length unit	[m]	[L]	
weight	[kg]	[M]	(4)
time	[s]	[T]	
temperature	[°C]	[θ]	

Selected quantities don't include temperature effect, hence we leave it out.

2.2 Processing of non-dimensional arguments

All quantities can be transcribe by their basic physical units:

bulk density	ρ_V [$kg.m^{-3}$]	[$kg.m^{-3}$]	[$M\ L^{-3} T^{\circ}$]	
edge of cube	a [m]	[m]	[$M^{\circ}\ L\ T^{\circ}$]	(5)
time	t [s]	[s]	[$M^{\circ}\ L^{\circ}\ T$]	
cube strength of concrete	$R_{c,cu}$ [Pa]	[$kg.m^{-1}.s^{-2}$]	[$M\ L^{-1}\ T^{-2}$] .	

The left side of formula (3) must be processed as the complex NDA. It is marked as K and its form is

$$K = \rho_v^{x_1} . a^{x_2} . t^{x_3} . R_b^{x_4} \qquad (6)$$

So as we could define unknown roots $x_1 + x_2 \dots x_4$ exponents over each base „M, L, T" must be used. For their calculation we can use this homogeneouse linear system of equations:

$$\begin{array}{lllll} x_1 & +0 & +0 & +x_4 & = 0 \\ -3x_1 & +x_2 & +0 & -x_4 & = 0 \\ 0 & +0 & +x_3 & -2x_4 & = 0\,. \end{array} \qquad (7)$$

System of equations (7) has ∞^2 solutions. From fundamental solution of system (7), this roots can be used: $x_1 = -\frac{1}{2}$; $x_2 = -1$; $x_3 = 1$; $x_4 = \frac{1}{2}$. Then the formula (6) changes to:

$$K = \rho_c^{\frac{1}{2}} . a^{-1} . t . R_b^{\frac{1}{2}} = \frac{t}{a}\sqrt{\frac{R_b}{\rho_v}} \;;\; \text{mark as } \pi_1 = \frac{t}{a}\sqrt{\frac{R_b}{\rho_v}} . \qquad (8)$$

Physical dimension check:

$$K\left[\frac{s}{m}\right] . \sqrt{\frac{kg}{ms^2} . \frac{m^2 kg}{\cdot}} = \left[\frac{s}{m}\right] . \left[\frac{m}{s}\right] = [-] . \qquad (9)$$

The NDA, of the left side of formula (6), was found by the same procedure, and it is marked as π_O

$$\pi_o = \frac{R_{c,cu}}{R_b} . \qquad (10)$$

By using fonnulas (8) and (10) the formula (6) to the general - criterial equation can be rewrited:

$$\pi_o = f(\pi_1) \Rightarrow \frac{R_{c,cu}}{R_b} = f\left(\frac{t}{a}\sqrt{\frac{R_b}{\rho_v}}\right) \qquad (11)$$

This fonnula (11) is mathematical model which expresses the dependence of change of bulk density to cube strength.

2.3 Numerical application

In this application we will show the practical usage of the model (11). To determine functional dependence - fitting curve, the results from experimental measurements obtained on thirty cubes were used.

The edge was a = 0,10 m. They were made from the same concrete mixture. Average bulk density $\rho_v = 2252\ kg.m^{-3}$. The experimental tests included 3 cubes, there were tested in time intervals according to table 1. This table also showed the compressive concrete strength $R_{c,cu}$.

For calculation π_1 from formula (8) we used quantity ρ_q

$$\rho_q = 10\ \rho_v \qquad (12)$$

where ρ_v is bulk density of concrete

ρ_q - unit weight of concrete

Table 1

Number of measurements	Measured values				Theoretical values obtained by approximation
	age of concrete	cube strength of value	Initial datas		
	t [days]	$R_{c,cu}$ [MPa]	π_1 [-]	π_o [-]	$R_{c,cut}$ [MPa]
1	14	13,0	932,918	13,0	13,95
2	20	16,2	1 332,74	16,2	15,20
3	28	17,0	1 865,84	17,0	16,43
4	50	19,1	3 331,85	19,1	18,83
5	75	20,6	4 997,78	20,6	20,71
6	90	21,3	5 997,33	21,3	21,61
7	120	22,6	7 996,45	22,6	23,13
8	140	23,3	9 329,19	23,3	23,98
9	180	26,6	11 994,70	26,6	25,44
10	365	29,6	24 322,52	29,6	30,05

$$\pi_1 = \frac{t}{a}\sqrt{\frac{\overline{R}_b}{\rho_q}}\ ; \quad \pi_o = \frac{R_{c,cu} t_i}{\overline{R}_b}\ ; \quad \pi_o = f(\pi_1);$$

$a = 0,10\ m$; $\rho_q = 10\ \rho_v = 22\ 520\ kg.m^{-3}$

$\overline{R}_b = 1,0\ MPa = 10^6\ N.m^{-2} = 10^6\ kg.m^{-1}s^{-2}$

Written above dependence was obtained by approximation from initial datas, which are in Table 1, where last squares method was used. The form of the dependence is:

$$\pi_o = A.\pi_1^m \Rightarrow \frac{R_{c,cu}\, t_i}{\overline{R}_b} = 2{,}795.\left(\frac{t}{a}\sqrt{\frac{\overline{R}_b}{\rho_q}}\right)^{0{,}23517} \qquad (13)$$

The dependence (13) is valid for $0 \le t \le 365$ days. Correlation index IK = 0,9905 that is dimensionally correct verifies propriety of obtained fitting curve.

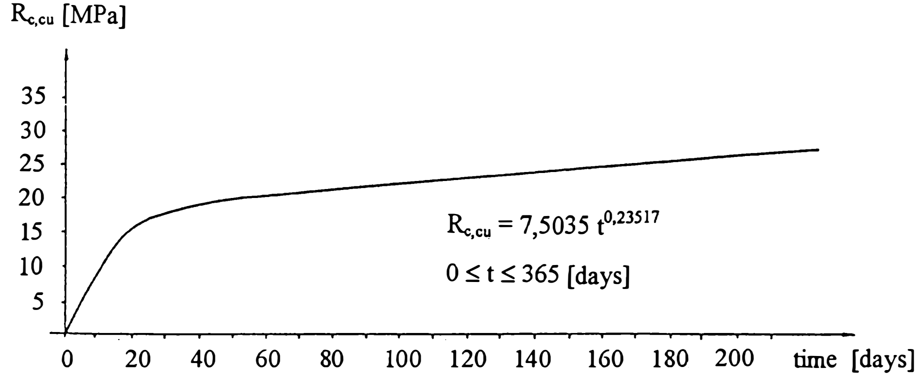

Fig. 1

The dependence (13) can be simplified by substituting values from the bottom part of a table for $\overline{R}_b$, ρ_q. Modified dependence is then:

$$R_{c,cu} = 7{,}5035\, t^{0{,}235171} \qquad (14)$$

Dependences (13), resp. (14) give the possibility to obtain the values $R_{c,cu}$ at time $t \in (0; 365)$ in the days when measurements were not carried out.

3. Conclusion

Numerical application solutions shown that the written above mathematical process is advisable for expressing of influence time concrete compressive strength $R_{c,cu}$ relationships.

Mathematical processing procedure is summarized in this steps:

- processing of quantities equation (2)
- selecting of quantities, which are tabling in valuation (see (3) and (5))
- processing of non-dimensional arguments π_1 , π_0 (see (10))
- criterial equation as mathematical model (see (9), (11))
- processing of initial datas (see Table 1)
- approximation by least squares method for obtaining A, m (see (13))
- verifying of the propriety replace - fitting curve by correlation index.

Obtained dependence is dimensionally correct. It is the base for next more detailed theoretical analysis.

4. References

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