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ALGORITHM FOR AUTOMATIC MANUFACTURING CONTROL OF GENERAL HYDRAULIC SURFACE

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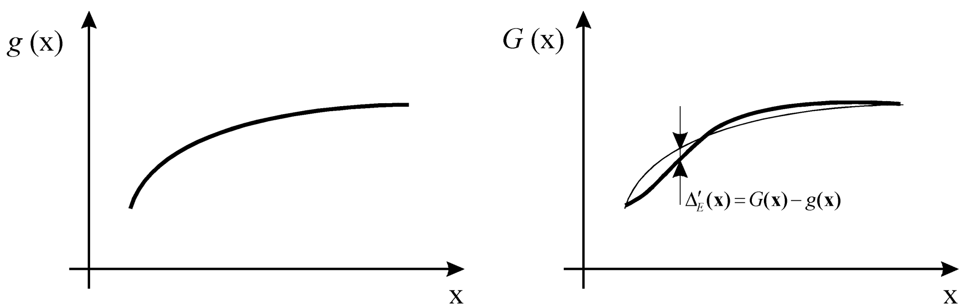
ABSTRACT: *The article is dedicated to the derivation of the computing algorithm which will serve for the automatization of the control of manufacturing of a hydraulic surface. However the algorithm can be applied for many problems of the type of the comparison of two surfaces. One of them is called the reference surface and it is the requirement according to which the object surface is made with some inaccuracy, of course. The second surface called the real one is obtained as the result of the measurement of the given object. The so called error function is introduced as the difference of both surfaces in corresponding points. The direct determination of its is unable under the requirement of the automatic control by virtue of the fact that surfaces are translated and rotated to each other. The result of the comparison is the so called total deviation function. It consists of two parts: the mentioned error function and the other function called the trend one. The form of its depends on the transformation and the work contains its derivation. The concrete computation proceeds using the well-known method of least squares.*

KEY WORDS: automatic control, error function, trend function, total deviation function

1. Analysis of situation, deviations

The reference surface is the standard according to which the object surface is made. It can be given by various ways e.g. by the technical drawing. The object surface is manufactured with some inaccuracy which have to be found out by the measurement. The measurement is often called the topography.

Let's mark values of the topographic depth of the real surface in the general point $(\mathbf{x},\mathbf{y})$ by the symbol $\underline{\mathbf{z}} = G(\mathbf{x},\mathbf{y})$ and requisite values of the topographic depth in the general point $(\mathbf{x},\mathbf{y})$ by the symbol $\mathbf{z_R} = g(\mathbf{x},\mathbf{y})$. We get two surfaces in Cartesian co-ordinate systems $(\mathrm{O},\mathbf{x},\mathbf{y},\mathbf{z})$. The real and reference surface are collections of points $(\mathbf{x},\mathbf{y},\underline{\mathbf{z}})$ and $(\mathbf{x},\mathbf{y},\mathbf{z_R})$ respectively. The example of both surfaces is on Fig.1 for the illustration (as functions of only the one independent variation **x** for the simplicity).



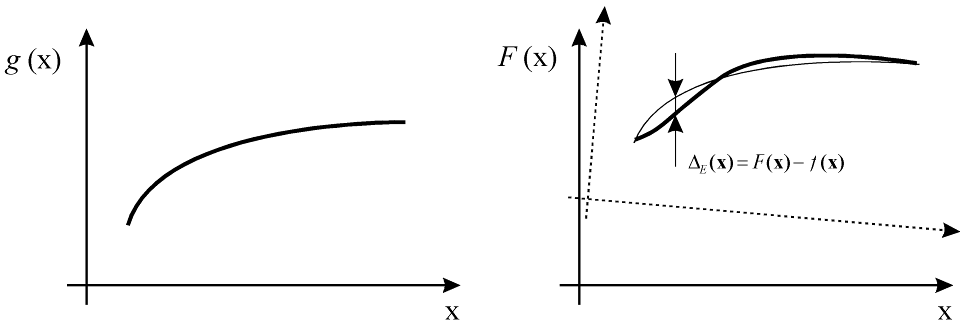
(a) (b)

Figure 1: The reference (a) and real (b) surface in independent coordinate systems.

The reference and real surface are similar ones evidently (see Fig.1(b)) the real surface is described by the same function as the reference one (the thin curve) modulated by the error function expressing the accuracy of manufacturing of the object. Then the next relation is valid

$$\begin{aligned} G(\mathbf{x},\mathbf{y}) &= g(\mathbf{x},\mathbf{y}) + \varDelta'_E(\mathbf{x},\mathbf{y}) \\ \varDelta'_E(\mathbf{x},\mathbf{y}) &= G(\mathbf{x},\mathbf{y}) - g(\mathbf{x},\mathbf{y}) \end{aligned} . \tag{1}$$

The comparison of surfaces can be comprehended as placing of independent co-ordinate systems of both surfaces into the one main co-ordinate system. It can be chosen to be the same as the co-ordinate system of the reference surface. The co-ordinate system of the real surface and its location compared with the main co-ordinate system respectively depend on the experimental set-up of the topography. Generally this co-ordinate system is translated and rotated compared with the main one. These transformations are mathematically described in the next chapter. The following figure shows for the illustration surfaces from Fig.1 placed in the main co-ordinate system. Axes of the co-ordinate system of the real surface (dotted lines) are also shown on Fig.2(b) in the main co-ordinate system. The transformation is evident from their reciprocal location.



(a) (b)

Figure 2: The location of the reference (a) and real (b) surface in the main coordinate system.

The function $g(\mathbf{x},\mathbf{y})$ describing the reference surface in the main co-ordinate system stays unchanged. On the other hand the function $G(\mathbf{x},\mathbf{y})$ describing the real surface is transformed by the translation and the rotation of the co-ordinate system to the function $F(\mathbf{x},\mathbf{y})$. The relation (1) is modified into the form

$$\begin{aligned} F(\mathbf{x},\mathbf{y}) &= f(\mathbf{x},\mathbf{y}) + \varDelta_E(\mathbf{x},\mathbf{y}) \\ \varDelta_E(\mathbf{x},\mathbf{y}) &= F(\mathbf{x},\mathbf{y}) - f(\mathbf{x},\mathbf{y}) \end{aligned} . \tag{2}$$

The error function $\varDelta_E(\mathbf{x},\mathbf{y})$ is the aim we are interested in. It is evidently valid the function $f(\mathbf{x},\mathbf{y})$ is the transformation of the function $g(\mathbf{x},\mathbf{y})$. But the transformation and of course also the function $f(\mathbf{x},\mathbf{y})$ are unknown. Then it can be directly determined not the error function $\varDelta_E(\mathbf{x},\mathbf{y})$ but only the so called total deviation function $\varDelta(\mathbf{x},\mathbf{y})$ according to Fig.3 as

$$\varDelta(\mathbf{x},\mathbf{y}) = F(\mathbf{x},\mathbf{y}) - g(\mathbf{x},\mathbf{y}) = f(\mathbf{x},\mathbf{y}) - g(\mathbf{x},\mathbf{y}) + \varDelta_E(\mathbf{x},\mathbf{y}) = \varDelta_{TR}(\mathbf{x},\mathbf{y}) + \varDelta_E(\mathbf{x},\mathbf{y}), \tag{3}$$

where the so called transformation deviation function is marked as

$$\varDelta_{TR}(\mathbf{x},\mathbf{y}) = f(\mathbf{x},\mathbf{y}) - g(\mathbf{x},\mathbf{y}). \tag{4}$$

The total difference of both surfaces – the total deviation function $\varDelta(\mathbf{x},\mathbf{y})$ – consists of two parts: the difference caused by the inaccuracy of manufacturing – the error function

$\Delta_E(\mathbf{x},\mathbf{y})$ – and the difference caused by the translation and the rotation of the co-ordinate system during the comparison – the transformation deviation function $\Delta_{TR}(\mathbf{x},\mathbf{y})$. It is better called the trend function from reasons introduced in the following text.

Under the condition the co-ordinate system of the real surface is identical with the main co-ordinate system (that means they are not transformed to each other) the function $g(\mathbf{x},\mathbf{y})$ is identical with the function $f(\mathbf{x},\mathbf{y})$ and according to relations (3) and (4) the total deviation function $\Delta(\mathbf{x},\mathbf{y})$ equals the error function $\Delta_E(\mathbf{x},\mathbf{y})$. The trend function is the zero one.

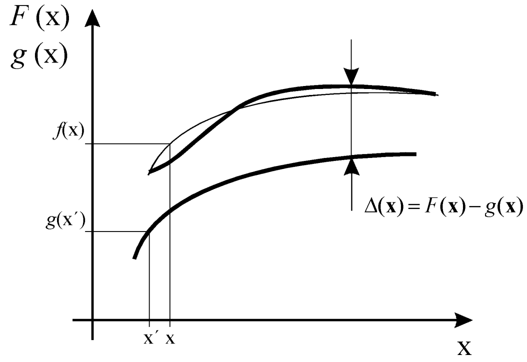


Figure 3: To the derivation of the total deviation, error and trend function.

2. Transformation of co-ordinate systems

The transformation mentioned above can be described with the help of matrixes of rotation **R** and translation **T** by the system of linear equations expressing the relation between two corresponding points $P[\mathbf{x},\mathbf{y},\mathbf{z}]$ in the main co-ordinate system and $P'[\mathbf{x}',\mathbf{y}',\mathbf{z}']$ in the transformed one (see Fig.3). This system can be written in the form of the matrix relation

$$\begin{pmatrix}\mathbf{x}'\\ \mathbf{y}'\\ \mathbf{z}'\end{pmatrix} = \mathbf{R}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\\ \mathbf{z}\end{pmatrix} + \mathbf{T} \tag{5}$$

and for coorditates of corresponding points of functions f (x,y) and g (x,y) in the form

$$\begin{aligned} \mathbf{x}' &= r_{11}\mathbf{x} + r_{12}\mathbf{y} + r_{13} f(\mathbf{x},\mathbf{y}) + t_1 \\ \mathbf{y}' &= r_{21}\mathbf{x} + r_{22}\mathbf{y} + r_{23} f(\mathbf{x},\mathbf{y}) + t_2 \; . \\ g(\mathbf{x}',\mathbf{y}') &= r_{31}\mathbf{x} + r_{32}\mathbf{y} + r_{33} f(\mathbf{x},\mathbf{y}) + t_3 \end{aligned} \tag{6}$$

Inverse transformation equations can be written similarly in the form

$$\begin{aligned} \mathbf{x} &= p_{11}\mathbf{x}' + p_{12}\mathbf{y}' + p_{13} g(\mathbf{x}',\mathbf{y}') + s_1 \\ \mathbf{y} &= p_{21}\mathbf{x}' + p_{22}\mathbf{y}' + p_{23} g(\mathbf{x}',\mathbf{y}') + s_2 \; . \\ f(\mathbf{x},\mathbf{y}) &= p_{31}\mathbf{x}' + p_{32}\mathbf{y}' + p_{33} g(\mathbf{x}',\mathbf{y}') + s_3 \end{aligned} \tag{7}$$

Let’s restrict in next to little angles of rotations and little values of translations. Only the translation in the z-axis direction will be arbitrary. Matrixes have the approximation form

$$\mathbf{R} \approx \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}; \; \mathbf{P} \approx \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}; \; \mathbf{T} \approx \begin{pmatrix}0\\ 0\\ t_3\end{pmatrix}; \; \mathbf{S} \approx \begin{pmatrix}0\\ 0\\ s_3\end{pmatrix}. \tag{8}$$

3. Modification of equations, simplification

Let's start with the equation (3) where $\Delta(\mathbf{x},\mathbf{y})$ is measured and $\Delta_E(\mathbf{x},\mathbf{y})$ is wanted. Only the trend function $\Delta_{TR}(\mathbf{x},\mathbf{y})$ reminds to determine. The last relation of the system (7) is modified by the substitution of values $\mathbf{x}'$ a $\mathbf{y}'$ from transformation relations (6)

$$\begin{aligned} f(\mathbf{x},\mathbf{y}) = & \frac{(\mathrm{p}_{31}\mathrm{r}_{11}+\mathrm{p}_{32}\mathrm{r}_{21})}{[1-\mathrm{p}_{31}\mathrm{r}_{13}-\mathrm{p}_{32}\mathrm{r}_{23}]}\mathbf{x}+\frac{(\mathrm{p}_{31}\mathrm{r}_{12}+\mathrm{p}_{32}\mathrm{r}_{22})}{[1-\mathrm{p}_{31}\mathrm{r}_{13}-\mathrm{p}_{32}\mathrm{r}_{23}]}\mathbf{y}+ \\ & +\frac{\mathrm{p}_{33}}{[1-\mathrm{p}_{31}\mathrm{r}_{13}-\mathrm{p}_{32}\mathrm{r}_{23}]}g(\mathbf{x}',\mathbf{y}')+\frac{(\mathrm{p}_{31}\mathrm{t}_1+\mathrm{p}_{32}\mathrm{t}_2+\mathrm{s}_3)}{[1-\mathrm{p}_{31}\mathrm{r}_{13}-\mathrm{p}_{32}\mathrm{r}_{23}]}. \end{aligned} \tag{9}$$

Although the function $g(\mathbf{x}',\mathbf{y}')$ is known co-ordinates $\mathbf{x}'$ and $\mathbf{y}'$ of the point which the point of our interest of co-ordinates $\mathbf{x}$ and $\mathbf{y}$ is transformed to are not known. It is not possible to substitute the third equation of the system (6) for $g(\mathbf{x}',\mathbf{y}')$. We would obtain the identity. According to simplifications (8) the function $g(\mathbf{x}',\mathbf{y}')$ can be written with the help of the first two members of the Taylor analysis of a function in the point $(\mathbf{x},\mathbf{y})$ in the form

$$g(\mathbf{x}',\mathbf{y}') = g(\mathbf{x},\mathbf{y}) + \left.\frac{\partial g(\mathbf{x},\mathbf{y})}{\partial \mathbf{x}}\right|_{(\mathbf{x},\mathbf{y})}(\mathbf{x}'-\mathbf{x}) + \left.\frac{\partial g(\mathbf{x},\mathbf{y})}{\partial \mathbf{y}}\right|_{(\mathbf{x},\mathbf{y})}(\mathbf{y}'-\mathbf{y}). \tag{10}$$

Let's mark partial derivations of the function $g(\mathbf{x},\mathbf{y})$ by symbols $D_x(\mathbf{x},\mathbf{y})$ and $D_y(\mathbf{x},\mathbf{y})$. Differences $(\mathbf{x}'-\mathbf{x})$, $(\mathbf{y}'-\mathbf{y})$ can be determined from relations of the system (6)

$$\begin{aligned} \mathbf{x}'-\mathbf{x} &= (\mathrm{r}_{11}-1)\mathbf{x}+\mathrm{r}_{12}\mathbf{y}+\mathrm{r}_{13}f(\mathbf{x},\mathbf{y})+\mathrm{t}_1 \\ \mathbf{y}'-\mathbf{y} &= \mathrm{r}_{21}\mathbf{x}+(\mathrm{r}_{22}-1)\mathbf{y}+\mathrm{r}_{23}f(\mathbf{x},\mathbf{y})+\mathrm{t}_2 \end{aligned} \tag{11}$$

and then the modified relation (10) can be substitute into (9) by which we obtain

$$\begin{aligned} f(\mathbf{x},\mathbf{y}) = & \mathrm{K}_1\mathbf{x}+\mathrm{K}_2\mathbf{y}+\mathrm{K}_3[(\mathrm{r}_{11}-1)\mathbf{x}+\mathrm{r}_{12}\mathbf{y}+\mathrm{r}_{13}f(\mathbf{x},\mathbf{y})+\mathrm{t}_1]D_x(\mathbf{x},\mathbf{y})+ \\ & +\mathrm{K}_3[\mathrm{r}_{21}\mathbf{x}+(\mathrm{r}_{22}-1)\mathbf{y}+\mathrm{r}_{23}f(\mathbf{x},\mathbf{y})+\mathrm{t}_2]D_y(\mathbf{x},\mathbf{y})+\mathrm{K}_3\,g(\mathbf{x},\mathbf{y})+\mathrm{K}_4 \end{aligned}. \tag{12}$$

Algebraic terms composed by the element of transformation matrixes according to (9) are marked K_1, K_2, K_3 and K_4. The next step is the determination of $f(\mathbf{x},\mathbf{y})$ and the substitution into (4). The expression of the trend function $\Delta_{TR}(\mathbf{x},\mathbf{y})$ is obtained

$$\begin{aligned} \Delta_{TR}(\mathbf{x},\mathbf{y}) = & -g(\mathbf{x},\mathbf{y})+[\mathrm{C}_1\mathbf{x}+\mathrm{C}_2\mathbf{y}+\mathrm{C}_3\,g(\mathbf{x},\mathbf{y})+\mathrm{C}_4\,D_x(\mathbf{x},\mathbf{y})+\mathrm{C}_5\,D_y(\mathbf{x},\mathbf{y})+\mathrm{C}_6\mathbf{x}D_x(\mathbf{x},\mathbf{y})+ \\ & +\mathrm{C}_7\mathbf{x}D_y(\mathbf{x},\mathbf{y})+\mathrm{C}_8\mathbf{y}D_y(\mathbf{x},\mathbf{y})+\mathrm{C}_9\mathbf{y}D_x(\mathbf{x},\mathbf{y})+\mathrm{C}_{10}](1-\mathrm{C}_{11}\,D_x(\mathbf{x},\mathbf{y})-\mathrm{C}_{12}\,D_y(\mathbf{x},\mathbf{y}))^{-1} \end{aligned}, \tag{13}$$

where constants $\mathrm{C}_1,\ldots,\mathrm{C}_{12}$ are algebraic terms again. Transformation matrixes are unknown or their determination is very complicated respectively and practically impossible in the condition of the automatic analysis. Values of constants $\mathrm{C}_1,\ldots,\mathrm{C}_{12}$ have to be determined by the other way. This is the contain of the rest of the article. But it is necessary to transform the relation (13) into a linear form according to constants for next purposes.

3.1. Approximation No.1

One of the possibilities of the linearization of the equation (13) is introduced now. In accordance with approximations (8) we can write $\mathrm{K}_3\mathrm{r}_{13}\approx 0$; $\mathrm{K}_3\mathrm{r}_{23}\approx 0$. Let's restrict derivations by some suitable value for example $D_x(\mathbf{x},\mathbf{y})<3,\ D_y(\mathbf{x},\mathbf{y})<3$ and then this relation is valid

$$\left(1-\mathrm{K}_3\mathrm{r}_{13}D_x(\mathbf{x},\mathbf{y})-\mathrm{K}_3\mathrm{r}_{23}D_y(\mathbf{x},\mathbf{y})\right)^{-1}\approx\left(1+\mathrm{K}_3\mathrm{r}_{13}D_x(\mathbf{x},\mathbf{y})+\mathrm{K}_3\mathrm{r}_{23}D_y(\mathbf{x},\mathbf{y})\right). \tag{14}$$

Substituting (14) into (13) the result relation can be obtained

$$\begin{aligned}\varDelta_{TR}(\mathbf{x},\mathbf{y}) &= \mathrm{A}_1+\mathrm{A}_2\mathbf{x}+\mathrm{A}_3\mathbf{y}+\mathrm{A}_4\, g(\mathbf{x},\mathbf{y})+\mathrm{A}_5\, D_x(\mathbf{x},\mathbf{y})+\mathrm{A}_6\, D_y(\mathbf{x},\mathbf{y})+\\ &+\mathrm{A}_7\, D_x(\mathbf{x},\mathbf{y})D_y(\mathbf{x},\mathbf{y})+\mathrm{A}_8\, D_x^2(\mathbf{x},\mathbf{y})+\mathrm{A}_9\, D_y^2(\mathbf{x},\mathbf{y})+\mathrm{A}_{10}\mathbf{x}D_x(\mathbf{x},\mathbf{y})+\\ &+\mathrm{A}_{11}\mathbf{x}D_x^2(\mathbf{x},\mathbf{y})+\mathrm{A}_{12}\mathbf{x}D_y(\mathbf{x},\mathbf{y})+\mathrm{A}_{13}\mathbf{x}D_y^2(\mathbf{x},\mathbf{y})+\mathrm{A}_{14}\mathbf{x}D_x(\mathbf{x},\mathbf{y})D_y(\mathbf{x},\mathbf{y})+.\\ &+\mathrm{A}_{15}\mathbf{y}\,D_x(\mathbf{x},\mathbf{y})+\mathrm{A}_{16}\mathbf{y}\,D_x^2(\mathbf{x},\mathbf{y})+\mathrm{A}_{17}\mathbf{y}\,D_y(\mathbf{x},\mathbf{y})+\mathrm{A}_{18}\mathbf{y}\,D_y^2(\mathbf{x},\mathbf{y})+\\ &+\mathrm{A}_{19}\mathbf{y}\,D_x(\mathbf{x},\mathbf{y})D_y(\mathbf{x},\mathbf{y})+\mathrm{A}_{20}\, g(\mathbf{x},\mathbf{y})D_x(\mathbf{x},\mathbf{y})+\mathrm{A}_{21}\, g(\mathbf{x},\mathbf{y})D_y(\mathbf{x},\mathbf{y})\end{aligned} \tag{15}$$

3.2. Approximation No.2

The second possibility of the linearization is to substitute the equation (2) into the right side of (12). The modified equation can be used for the expression of the trend function $\varDelta_{TR}(\mathbf{x},\mathbf{y})$ according to (4). Members of the sum containing the error function $\varDelta_E(\mathbf{x},\mathbf{y})$ are eliminated under the presumption of little values of this function. The result is in the form

$$\begin{aligned}\varDelta_{TR}(\mathbf{x},\mathbf{y}) &= \mathrm{B}_1+\mathrm{B}_2\mathbf{x}+\mathrm{B}_3\mathbf{y}+\mathrm{B}_4\, g(\mathbf{x},\mathbf{y})+\mathrm{B}_5\, D_x(\mathbf{x},\mathbf{y})+\mathrm{B}_6\, D_y(\mathbf{x},\mathbf{y})+\\ &+\mathrm{B}_7\mathbf{x}D_x(\mathbf{x},\mathbf{y})+\mathrm{B}_8\mathbf{x}D_y(\mathbf{x},\mathbf{y})+\mathrm{B}_9\mathbf{y}\,D_x(\mathbf{x},\mathbf{y})+\mathrm{B}_{10}\mathbf{y}\,D_y(\mathbf{x},\mathbf{y})+\quad .\\ &+\mathrm{B}_{11}\, F(\mathbf{x},\mathbf{y})D_x(\mathbf{x},\mathbf{y})+\mathrm{B}_{12}\, F(\mathbf{x},\mathbf{y})D_y(\mathbf{x},\mathbf{y})\end{aligned} \tag{16}$$

4. Method for determination of error function

The infinite number of transformations of the co-ordinate system is necessary to take into account evidently. For every of them the reference surface $g(\mathrm{x,y})$ is transformed to some function $f(\mathrm{x,y})$ by the help of which we can find out the wanted error function $\varDelta_E(\mathbf{x},\mathbf{y})$ after subtraction from the real surface $F(\mathrm{x,y})$ in accordance with (2). We obtain the infinite number of error functions generally. Some reducing condition is necessary to introduce to choose one the most suitable error function. It shows the condition can be the minimisation of the sum of squares of values of the error function in measured points $(\mathbf{x}_\mathrm{i},\mathbf{y}_\mathrm{i})$ which number is N generally. The form of this condition is

$$S=\sum_{\mathrm{i}=1}^{\mathrm{N}}\left[\varDelta_E(\mathbf{x}_\mathrm{i},\mathbf{y}_\mathrm{i})\right]^2=\sum_{\mathrm{i}=1}^{\mathrm{N}}\left[\varDelta(\mathbf{x}_\mathrm{i},\mathbf{y}_\mathrm{i})-\varDelta_{TR}(\mathbf{x}_\mathrm{i},\mathbf{y}_\mathrm{i})\right]^2\to min. \tag{17}$$

The method is well-known in the mathematical statistics as the method of least squares. The deviation $\varDelta_{TR}(\mathbf{x},\mathbf{y})$ caused by the transformation of the co-ordinate system is some carrier function (a skeleton) on which the error function $\varDelta_E(\mathbf{x},\mathbf{y})$ is modulated. The presumption is the fluctuation character of its. The carrier function is often called the trend. The sum of the trend and the error function is the total deviation function $\varDelta(\mathbf{x},\mathbf{y})$ which has then the random character.

Solving of problems of this type expects the knowledge of the general form of the trend function. The article contains the two possible form of it – the Approximation No.1, relation

(15) and the Approximation No.2, relation (16). These relations are linearized according to constants. For our purposes we can write generally

$$\begin{aligned}\Delta_{TR}(\mathbf{x}_i,\mathbf{y}_i) &= R(\mathbf{x}_i;\mathbf{y}_i;A_1;A_2;\ldots;A_{v-1};A_v) = D_1\varphi_1(\mathbf{x}_i,\mathbf{y}_i) + \\ &+ D_2\varphi_2(\mathbf{x}_i,\mathbf{y}_i) + \ldots + D_{v-1}\varphi_{v-1}(\mathbf{x}_i,\mathbf{y}_i) + A_v\varphi_v(\mathbf{x}_i,\mathbf{y}_i)\end{aligned}, \tag{18}$$

where $\varphi_j(\mathbf{x}_i,\mathbf{y}_i)$ express functions of independent variables **x** and **y** like e.g. the product of the derivation and the variable etc. according to the relation (15) or (16). If the Approximation No.1 is taking into account the equation $D_j=A_j$ (j=1,…,21) is valid. Or the equation $D_j=B_j$ (j=1,…,12) is valid in case of the Approximation No.2. The number of members of the sum (18) is v=21 or v=12 but it can be much less because some members are insignificantly small in the concrete control of the given surface.

5. Discussion, conclusion

Several simplifications were carried out in previous chapters during the derivation of equations. The discussion of the possibility to achieve these simplifications will follow in the rest of the article. The first of them are approximate forms of transformation matrixes (8). Elements of matrixes **R** and **P** are cosines of angles of rotations in diagonals and sines of these angles out of diagonals respectively. Equations (8) are valid for rotations less then 5° due to *sin* $5°\approx 0{,}087$ and *cos* $5°\approx 0{,}996$. The limitation of the magnitude of translations is necessary to study in the context to dimensions of the measured area.

The other condition is the limitation of derivations by the value 3. It is very simply realized when meaning of derivations is considered. They are tangents of angles between the tangent of the surface in the direction of the axis x or y and this axis. The condition can be interpreted the angle between the tangent and the plane xy in an arbitrary point will be less than 70° due to *tg* 70°≈3. The most of real surfaces are smooth functions without extremes under the plane xy. Such functions realize the mentioned condition.

It looks like useful to discuss using of above mentioned approximations. The Approximation No.1 is made under previous conditions which are well-realized. But it contains the great number of unknown variables – 21. Opposite to this the Approximation No.2 contains the less number of constants – 12. But moreover the requirement of little values of the error function $\Delta_E(\mathbf{x},\mathbf{y})$ have to be solved. It is possible to say generally that several members of sums (15) and (16) can be ignored. However it depends on the concrete case.

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