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Once More about Stress Concentration in the Case of a Thin-Walled Tube with a Circular Hole under Torsion

Ještě jednou o koncentraci napětí v případě tenkostěnné trubky s kruhovým otvorem namáhané krutem

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The paper deals with the stress concentration caused by a circular hole in the wall of a thin-walled tube under torsion. The contribution to the same problem was published at 40th conference EAN in Prague. We return to this problem because the results of the experiment and the numerical simulation were nearly the same for the shear stress concentration factor ($\alpha_\tau \doteq 1{,}44$ *) but the value of this factor determined analytically using diagram in [1] (* $\alpha_\tau = 3{,}5$ *) was more than twice higher. Because the question of so high difference has not been explained we made afford to answer it. For this reason further numerical simulations have been performed and a new search in literature has been done. The results complement the previous ones.*

Příspěvek se zabývá koncentrací napětí vlivem kruhového otvoru ve stěně tenkostěnné trubky namáhané krutem a byl publikován na 40. konferenci EAN v Praze. K uvedené otázce se vracíme proto, protože výsledky experimentu a numerické simulace byly pro součinitel koncentrace smykového napětí od krutu téměř stejné ($\alpha_\tau \doteq 1{,}44$ *), ale hodnota tohoto součinitele určená analyticky pomocí grafu v [1] (* $\alpha_\tau = 3{,}5$ *) byla o více jak o 100% větší. Protože otázka tak velkého rozdílu zůstala nezodpovězenou, byla snaha ji objasnit. Za tímto účelem byla provedena další počítačová simulace a hledáno v literatuře. Výsledky doplňují původní.*

Keywords *Thin-walled tube, stress riser, experimental results, numerical modelling, analytical solution, stress concentration.*

Klíčová slova *Tenkostěnná trubka, koncentrátor napětí, experimentální výsledky, numerické modelování, analytické řešení, koncentrace napětí.*

1. Introduction

The paper deals with the stress concentration caused by a circular hole in a thin walled tube subjected to torsion. The shape and dimensions of the specimen made from the annealed mild steel ČSN 411523.1 are shown in fig. 1.

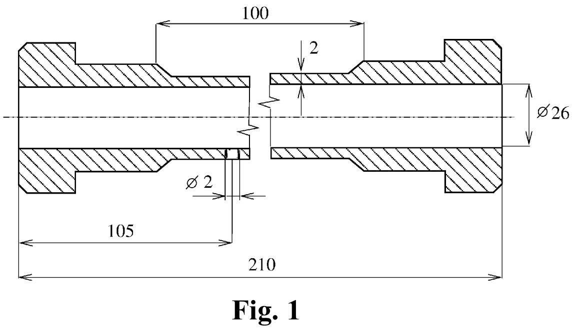

Fig. 1

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The chemical composition is presented in table 1.

C [%]	Mn [%]	Si [%]	P [%]	S [%]	N [%]
0.20	1.35	0.38	0.020	0.018	0.008

Tab. 1

The thin walled tube of the thickness of 2 *mm* was chosen to satisfy the assumption of constant stresses distribution along the tube thickness. But the numerical analysis has shown, that this assumption is valid only approximately, as it is possible to see later.

Static mechanical properties of the material are summarised in table 2. Young modulus of elasticity in tension $E = 2{,}185 \cdot 10^5\, MPa$, Poisson ratio $\mu = 0{,}3$.

Yield stress σ_Y [MPa]	Ultimate stress σ_U [MPa]	Ductility A [%]	Area reduction Z [%]
365,1	550,97	32	73

Tab. 2

The attention was paid to the explanation of the high difference of the stress concentration factors α_τ under torsion loading obtained by the different ways. That is why further numerical simulation was done. The new computation model differed only in the mesh density in the region of the stress riser. Further search in literature to find other analytical relations for the stress concentration factor determination [5] was done, too.

2. Numerical analysis

The numerical stress analysis was performed by FEM for the 3D – model with the stress riser of diameter $d = 3\,mm$. The 3D model and its loading by the torque as well as the coordinate system are obvious from fig. 2. The computational model was created by the MENTAT code and the numerical solution itself was performed by the MARC system.

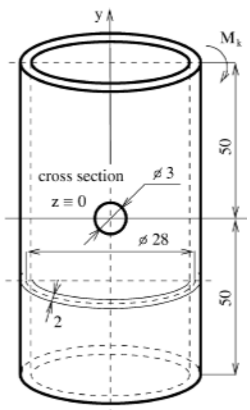


Fig.2

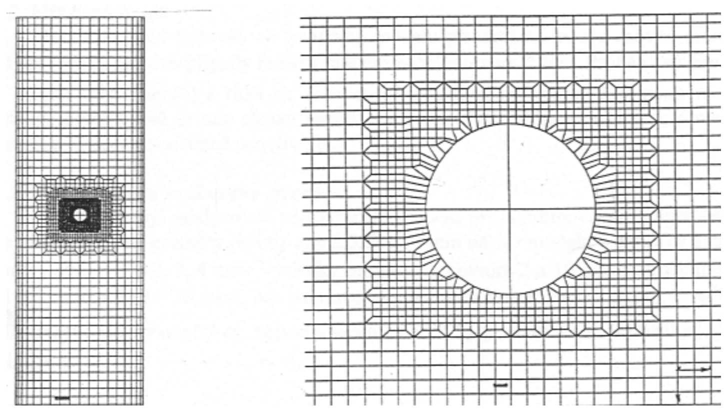


Fig. 3

The model meshing with the detail of the stress riser region zone is shown in fig. 3, number of layers along the tube wall thickness was chosen 4. The model was subjected to the torque causing shear stress $\tau = 100\ MPa$ in the part of the body without the riser.

3. Experimental analysis

For the verification of the shear stress under torque two electric-resistance rosettes $0^\circ/45^\circ/90^\circ$ RY91-1,5/120 (T_2 and T_3) were used. Gauges are products of HBM Ltd: The gauge T_3 is used as a check one, because of the possibility to assume that its location is uninfluenced by the riser. The locations of strain gauges are obvious from fig. 4. The mentioned above assumption was acknowledged by the numerical PC solution. The location of the gauge T_2 was found as a compromise between the point of maximum value of component τ_{yz} determined by PC simulation and the gauge dimensions. The strain gauge T_1 was used for the stress concentration under loading of the tube by the axial force. The coincidence of the stress concentration factors was very good in this case, see [4]. The measurement was conducted by the measuring system UGR60 of HBM, Ltd under loading in the electro-hydraulic testing machine INOVA YUY 200-1 with computer controlled function.

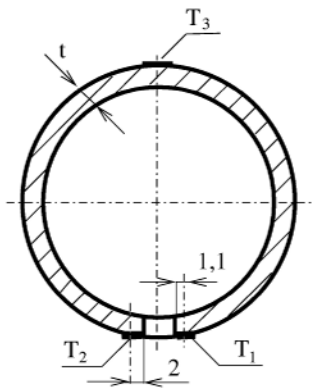
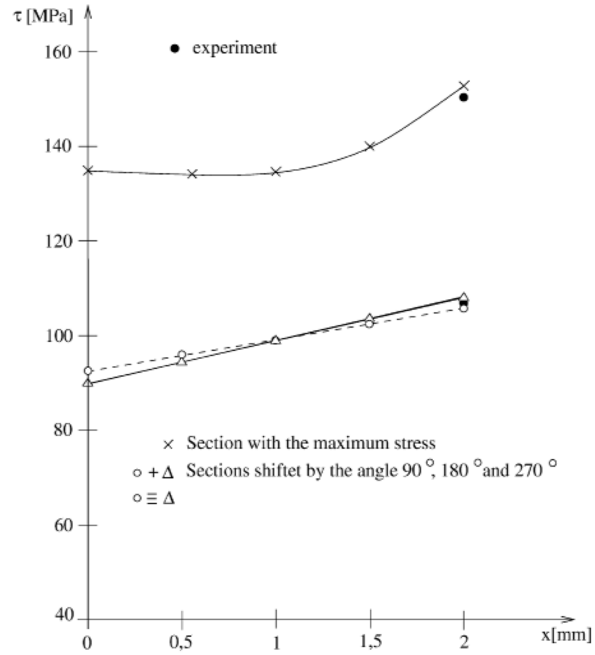


Fig. 4

4. Stress analysis

Numerical solution using finite element method showed distribution of resultant shear stress along the tube wall thickness. The results are presented in fig. 5 for the place of maximum shear stress and in sections turned about 90°, 180° and 270° from the vertical axis coming through the centre of riser under loading corresponding to the shear stress of 100 *MPa*. It is obvious from the diagram, that even in the case of a very thin tube wall the shear stress is changing along the thickness of the wall. In the locus of its maximum values the change is non-linear and the difference between the values at the inner and outer surfaces is about 13%. In the sections, which are not influenced by the riser, the difference is nearly the same but the stress distribution is quite different. It is linear and it co-insides with the theory of torsion. The maximum value of the shear stress $\tau = 155.3\ MPa$ corresponds very well to the experimentally determined value 149.5 *MPa*, fig. 5. The courses of the shear stress components τ_{yz} and τ_{yx} along the medium cylindrical surface and the maximum value of τ_{yz} obtained by another numerical simulation and maximum value of shear stress τ obtained experimentally are presented in fig. 6. It gives possibility to compare all obtained results by mentioned above ways.



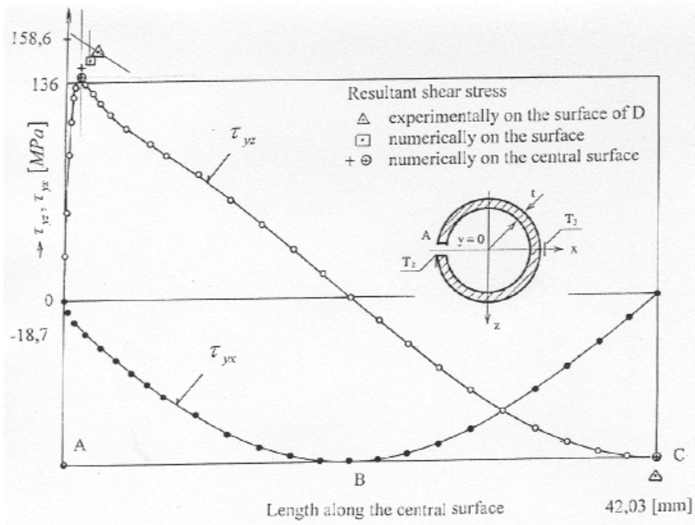


Fig. 5 **Fig. 6**

It is obvious, that experimentally and numerically obtained results co-inside again. Original discrepancy was in analytical solution. Therefore a new research was made in that field. A solution was found in [5], fig. 7, for transverse circular hole.

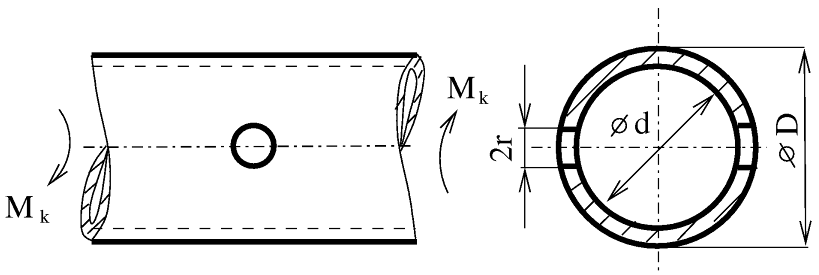


Fig. 7

Taking into account assumption that the mutual influence of holes on shearing stress distribution under torque can be neglected, that results can be used for taking into account case too. The validity of the theory is limited by $\eta = d / D \leq 0.8$ and $\xi = 2r / D \leq 0.4$. Then nominal stress is taken into account as

$$\tau_{nom} = \frac{16 D M_k}{\pi \left(D^4 - d^4\right)} , \tag{1}$$

$$K_t = K_1 + K_2 \xi + K_3 \xi^2 + K_4 \xi^3 , \tag{2}$$

where corresponding constants are

$$K_1 = 4.000 , \qquad K_2 = -6.055 + 3.184\eta - 3.461\eta^2$$

$$K_3 = 32.764 - 30.121\eta + 39.887\eta^2, \quad K_4 = -38.330 + 51.542\sqrt{\eta} - 27.483\eta^2 \tag{3}$$

Then stress concentration factor is

$$\alpha_\tau = \frac{\tau_{\max}}{\tau_{nom}} = \frac{K_t}{2} \ . \tag{4}$$

Inserting in corresponding eqn. one obtains:

$K_1 = 4.00$, $K_2 = -5.9$, $K_3 = 36.75$, $K_1 = -14.17$, $\eta = 0.866$, this necessary condition was slightly exceed, $\xi = 0.115 \le 0.4$ was satisfied. Inserting into eqn. (2) and (4) $K_t = 3.8$ and $\alpha_\tau = 1{,}9$.

To compare all obtained results, stress concentration factors were calculated using maximum values obtained by: numerical solution $\alpha_\tau = 1{,}6$, by experiment $\alpha_\tau = 1{,}55$. The point of shearing stress measurement was not (because of gauge dimensions) identical with the point maximum shear stress. If the determined value is extrapolated with the same trend in the point of the same distance of maximum shear stress determined by PC simulation (see thin solid straight line in Fig. 6) the stress concentration factor will be $\alpha_\tau = 1{,}64$. By the assumption mentioned above the nominal stress was expressed by eqn. (1) and it was $\tau_{nom} = 96{,}59\,MPa$ for original nominal stress in minimum cross section equal to 100 *MPa*.

5. Conclusion

Stress and strain fields in bodies of different shapes and under different loadings are determined numerically using FEM at present. The presented case showed, that the values of stress concentration factor in followed case using mentioned above three methods lay between $1{,}55 \le \alpha_\tau \le 1{,}9$. The difference did not exceed 19%. It is necessary to take into account that the presented case is not too convenient for PC simulation because of the change of shear stress along very small distance in the range of it maximum value and in the case of electrical resistance strain gauge method when the result is influenced also by the gauge dimension and its position.

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