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MODELLING AND DATA EVALUATION OF FORCE TRANSDUCERS
MODELOVÁNÍ A VYHODNOCENÍ DAT SILOVÝCH SNÍMAČŮ

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A multiaxial transducer that enables measuring whole load force vector is recently not aviable on a commercial market. Point is to introduce construction of a force transduces, that would provide measurement of full force and momentum load. To fullfill this requirement a special testing beam shouldered by a number of strain ganges is used. Despite a common distribution of the strain gages over the measuring device it is possible to count a full vector of force and momentum load. The measuring procedure consist of two independent steps that we are presenting a following article. In a final data processing total least squares method is being used.

Keywords

Multiaxial, ForceTransducer, Evaluation, Identification

Introduction

Multiaxial transducers of mechanical force quantities are not included in the usual product range of com-panies manufacturing force sensors. Only uniaxial transducers of forces and moments are commonly available (see the catalogues of top world companies, which are selling their products on our market, i.e. HBM-HBP, Devetron, Vishay).

Demands on more complex sensors are usually mat-ter of custom design and manufacturing. Thus their price is generally substantially higher. Multiaxial transducers are much more complex

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as regards de-mands for their basic mechanical construction. The measuring of a particular deformation field on the construction is the basic principle of their use. The design of a transducer sets broad range of require-ments, which have to be fulfilled. These are: compli-ance of bodies of respective sensors with strength criteria, optimization of range of measuring circuit signal sensitivity and stability guarantee of sensor parameters. Complexity of demands concerning the

design of multiaxial transducers leads towards opti-mization techniques, because of the necessary crite-ria, which have to be fulfilled, are of different char-acter. This is the prime reason, why the multiaxial sensors are not commonly sold.

High-strength steel with low material hysteresis and closely fitting the Hooke's law are used for bodies of sensors at most. Modern materials based on a basis of laminated composites also show similar qualities, moreover with another important feature, which is stiffness controllability in different directions. Elastic response of sensor's body to a load is measured by resistance strain gauges, alternatively by more sensi-tive semiconductor ones. Any measurement set-up requires to have implemented a compensation of temperature effect on force response of the sensor.

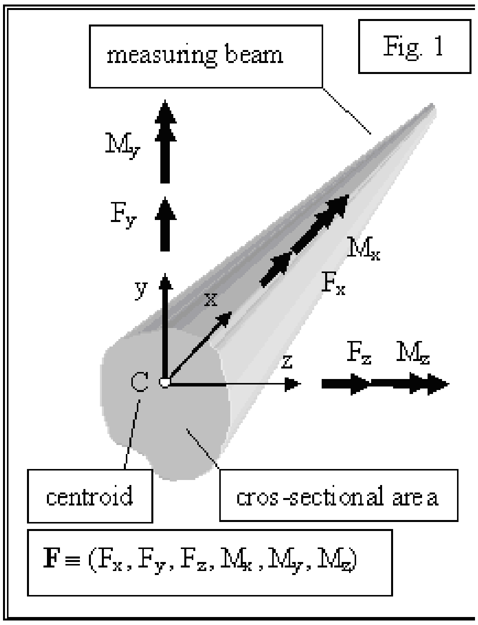

Fig. 1. Forces and moments marks

Design of transducers

A multiaxial transducer should be produced from a material that fulfills the Hooke's Law as good as possible so that the principle of superposition can be exploited. Further it is supposed that a identification of a force vector that loads a reference place (Fig. 1) is realized through measuring of a strain on the transducer using strain gages. A force vector is composed of three force Fx, Fy, Fz and three momentum Mx, My, Mz components who are mutually independent. In case of a beam transducer there are several simple computing theorems that enable us to a state of stress, respectively a strain distribution in case of single force or momentum loads. These theo-ries are valid also for a superposition of such simple loads and by their application we may produce an equation system of physical constrains that can im-prove the accuracy of measured data (data reconcilia-tion).

Ordering of directions of the strain gages that are used for measuring on the beam is analyzed in [2] and for a typical example of a shaft with circle cross-sectional area it is possible to place strain gages, which identify dominant components of shifting force

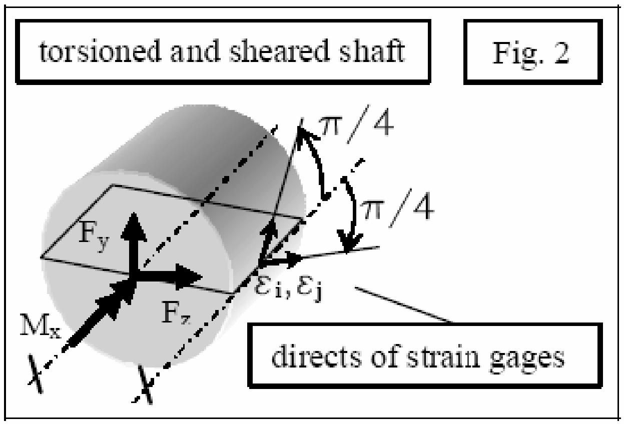

Fig. 2. Position of strain gages used for torsion and shaft measuring

and torsion moment according to scheme (Fig. 2). Components of the bending moments impact the greatest values of strain in the direction of the shaft axes, what is marked on scheme (Fig. 3).

Identification of the measuring component

In following chapter a it is described a identification process of measuring device used or measuring of a force load. Calibration of the device must be made before the measurement, as the calibration of the device is necessary.

On the schemes (Fig. 1, Fig. 2, Fig. 3) there are marked several strain gages (sensors) whose data are further preceded. Each of the sensors reacts reflects every kind of load that is measuring device exposed to. As device can be practically exposed to six independent kinds of load, we assume that the final value of the sensor ε_i is a linear combination of individual contributions that are done by the individual loads

$$\varepsilon_i = c_{i1}F_x + c_{i2}F_y + c_{i3}F_z + c_{i4}M_x + c_{i5}M_x + c_{i5}M_x, \qquad (1)$$

where F_k denotes a force in the directions of the coordinates, M_k denotes a momentum to axis x, y and z and finally c_{ik} marks the sensitivity of i-th sensor to the k -th variable. Supposing m measuring sensors there are m linear equations like (1) that contain $m\times 6$ unknown coefficients c_{ik}. We formulate (1) in matrix form like

$$\begin{bmatrix} c_{i1} & c_{i2} & c_{i3} & c_{i4} & c_{i5} & c_{i6} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = [\varepsilon_i] \qquad (2)$$

and for all measuring places it form an equation system

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ \vdots & & & & & \vdots \\ c_{m1} & c_{m2} & c_{m3} & c_{m4} & c_{m5} & c_{m6} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix} \qquad (3)$$

The sensitivities c_{ik} are constant time invariant values that need to be identified by an experiment. To identify these values we need to load a measuring device by well-known forces and moments independently one by one. Suppose we have an independent loading force $\overline{F}_y$ and all other influencing powers and moments are zero. In that case we measure a value on all the sensors denoted like $\boldsymbol{\varepsilon}_{\overline{F}_y}$. Putting this case into equation (3) yields

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ \vdots & & & & & \vdots \\ c_{m1} & c_{m2} & c_{m3} & c_{m4} & c_{m5} & c_{m6} \end{bmatrix} \begin{bmatrix} 0 \\ \overline{F}_y \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \boldsymbol{\varepsilon}_{\overline{F}_y} \qquad (4)$$

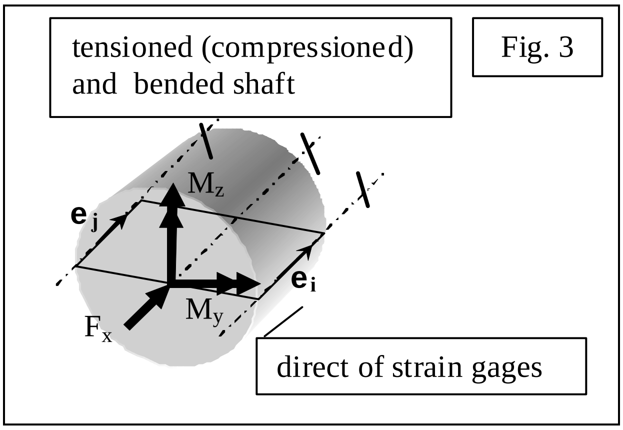

Fig. 3. Position of strain gages used for measuring of bend and tension

Multiplication (4) selects second column of the matrix $\mathbf{C} = [c_{ik}]$ so that we can rewrite equation as

$$\begin{bmatrix} c_{12} \\ \vdots \\ c_{m2} \end{bmatrix} \overline{F}_y = \boldsymbol{\varepsilon}_{\overline{F}_y} .$$

The magnitude of load $\overline{F}_y$ is well known with its relative error $\delta_{\overline{F}_y}$. Sensors values $\boldsymbol{\varepsilon}_{\overline{F}_y}$ evoked by load $\overline{F}_y$ are known with their relative error $\boldsymbol{\delta}_{\boldsymbol{\varepsilon}_{\overline{F}_y}}$. Sensitivities to $\overline{F}_y$ can be reached together with their relative like

$$\begin{bmatrix} c_{12} \\ \vdots \\ c_{m2} \end{bmatrix} = \frac{1}{\overline{F}_y} \boldsymbol{\varepsilon}_{\overline{F}_y} . \qquad (5)$$

When dividing we have to find relative error δ_{i2} of c_{i2} from relative errors of the dividend and divider like

$$\delta_{i2} = \sqrt{(\delta_{\varepsilon_i})^2 + (\delta_{\overline{F}})^2} . \qquad (6)$$

To get all the coefficients of matrix $\mathbf{C}$, we have to make six measurements caused each time just by one isolated force or momentum. Product of this is a sensitivity matrix together with matrix $\boldsymbol{\delta}$

of the relative uncertainty of individual matrix components. True component of matrix $\mathbf{C}$ belongs than certainly to interval

$$\hat{c}_{ij} \in \left(c_{ij}\left(1-\delta_{ij}\right), c_{ij}\left(1+\delta_{ij}\right)\right). \qquad (7)$$

Measuring the unknown force

If we impose the specimen to the unknown combination of forces and applied moments it evokes a sensor reaction denoted by $\boldsymbol{\varepsilon}_u$ with relative uncertainty $\boldsymbol{\delta}_\varepsilon$. Unknown vector of forces and moments $\mathbf{x}$ equal to

$$\mathbf{x} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix}, \qquad (8)$$

that has n components must according to (3) fulfill the equation

$$\mathbf{Cx} = \boldsymbol{\varepsilon}_u .$$

To provide all the unknown forces we have to have at least n identified sensors. To test the existence of solution rank of matrix $\mathbf{C}$ must be equal

$$rank(\mathbf{C}) = n .$$

On the other hand number of sensors m can be larger than n. In that case we have more equations than variables. As all the sensitivities are produced by the identification its clear that the Frobenius law [1]

$$rank(\mathbf{C}) = rank([\mathbf{C}, \boldsymbol{\varepsilon}_u]) \qquad (9)$$

is not fulfilled as the vector $\boldsymbol{\varepsilon}_u$ is linearly independent on columns of matrix $\mathbf{C}$. As the components of matrix $\mathbf{C}$ and components of vector $\boldsymbol{\varepsilon}_u$ are all known imprecisely, we should solve the problem using the Total Least Squares algorithm (TLS) [1]. This algorithm solves the problem formulated like

$$\min_{\forall[\hat{\mathbf{C}},\hat{\boldsymbol{\varepsilon}}]\in\Re^{m\times(n+1)}} \left\{ \left\| [\mathbf{C}, \boldsymbol{\varepsilon}_u] - [\hat{\mathbf{C}}, \hat{\boldsymbol{\varepsilon}}] \right\|_F : \hat{\boldsymbol{\varepsilon}} \in R(\hat{\mathbf{C}}) \right\} \qquad (10)$$

where $\hat{\mathbf{C}}$ is more concrete matrix $\mathbf{C}$ and $\hat{\boldsymbol{\varepsilon}}$ is more appropriate measured value, that fulfills a Frobenius law (9). If we find a matrices $\hat{\mathbf{C}}$ and $\hat{\boldsymbol{\varepsilon}}$ than unknown vector $\mathbf{x}^*$ can be found by solving a equation system

$$\hat{\mathbf{C}}\mathbf{x}^* = \hat{\boldsymbol{\varepsilon}} . \qquad (11)$$

Solution of the problem can be found in [1, page 39]. The minimization of the Frobenius norm of matrix difference (10) can be performed using singular value decomposition (SVD) [1]. We perform a SVD of original matrices

$$\mathrm{svd}\left[\mathbf{C},\boldsymbol{\varepsilon}_u\right]=\mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V},\ \sigma_1 \geq \sigma_2 \geq \cdots \sigma_n \geq \sigma_{n+1}\ .$$

The minimum difference matrix

$$\left[\Delta\mathbf{C},\Delta\boldsymbol{\varepsilon}\right]=\left[\mathbf{C},\boldsymbol{\varepsilon}_u\right]-\left[\widehat{\mathbf{C}},\widehat{\boldsymbol{\varepsilon}}\right]$$

can be according to [1] found like

$$\left[\Delta\mathbf{C},\Delta\boldsymbol{\varepsilon}\right]=\sigma_{n+1}\mathbf{v}_{n+1}\mathbf{u}_{n+1}^{\mathrm{T}},$$

where $\mathbf{v}_{n+1}$ and $\mathbf{u}_{n+1}$ are $n+1$-th columns of the matrices $\mathbf{V}$ and $\mathbf{U}$. Final solution of the equation system (11) can be found from the original system like

$$\mathbf{x}^{*}=\left(\mathbf{C}^{\mathrm{T}}\mathbf{C}+\mathbf{I}\sigma_{n+1}\right)\mathbf{C}^{\mathrm{T}}\boldsymbol{\varepsilon}_u\ . \tag{12}$$

Performing this solution according to (12) satisfies uncertainty requirements, as the possible mistake is distributed into matrix $\mathbf{C}$ as well as into vector $\boldsymbol{\varepsilon}_u$.

Our future goal is to find a statistical procedure, that would find a gross error in the set of measurements and so that to acquire which of the sensors has a possible malfunction.

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