PŘÍSPĚVEK K URČENÍ DIAGRAMU SKUTEČNÉ NAPĚTÍ – SKUTEČNÁ DEFORMACE MATERIÁLU

CONTRIBUTION TO THE DETERMINATION OF THE TRUE STRESS – TRUE STRAIN DIAGRAM OF MATERIAL

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Abstrakt

Příspěvek se zabývá možností určení diagramu skutečné napětí – skutečná deformace použitím smluvního pracovního diagramu a mechanických vlastností materiálu. Uvedený přístup byl ověřen experimentálně a pomocí Gursonova modifikovaného modelu. Navržená konstrukce diagramu byla potvrzena jako dostatečně přesná.

Klíčová slova: smluvní pracovní diagram, mechanické vlastnosti materiálu, diagram skutečné napětí – skutečná deformace.

Abstract

The article deals with a possibility of determination of true stress – true strain diagram of material using engineering stress – strain diagram and mechanical properties of material. Presented approach was verified experimentally and using Gurson´s modified model. The proposed approach of diagram plot was validated as a sufficient accurate.

Keywords: engineering stress-strain diagram, mechanical properties of material, determination of true stress-true strain diagram.

INTRODUCTION

The true stress-true strain diagram of material is necessary to solve problems in the field of large plastic deformations. To determine that diagram using tensile test, it is necessary to measure continuously the change in the specimen diameter during testing. It is complicated. There are cases of testing when the change in of cross-sectional area can not be measured at all, for example in the case when only tubes are available for testing. The article provides a possibility of approximate determination of the true stress-true strain diagram using engineering stress-strain diagram and basic mechanical property of material.

There was published theoretical principle of approach of using the rule of constant volume for determination of an increment of plastic deformation [1], [2], when only elastic deformation causes change in the cross-sectional area was taking into account and neglected. But the problem is more complicated because of the change in Poisson’s ratio ν from ν_0 in elastic state to $\nu = 0.5$ in the case of developed plastic state. An influence of that process was analysed and a design of an approximate true stress-true strain diagram was proposed.

DESCRIPTION OF THE METHOD AND USED MATERIAL

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As it was mentioned above the elastic-plastic transition of the Poisson's ratio from the value ν_0 corresponding to the elastic state of material to the value $\nu_1 = 0.5$ in the case of developed plastic state will be taken into account. There was serious research in that field (for example [3], [4]) to describe that process. The results published in [3] were taking into account. The change in Poisson's ratio in the range of $\varepsilon \in \langle \varepsilon_y, \varepsilon_1 \rangle$ (Fig. 1) can be then expressed as a function of strain in the form

$$\nu = a_0 + a_1 \varepsilon + a_2 \varepsilon^2 \ . \tag{1}$$

The constants a_0, a_1, a_2 can be determined using boundary conditions (Fig.1).

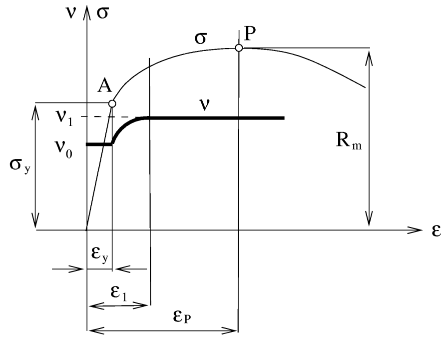

Fig.1 Engineering stress σ - strain ε diagram and plot of elastic-plastic transition of Poisson's ratio ν versus strain ε

Point *A* in the engineering stress-strain diagram represents yield point, point *P* is the point of instability and R_m ultimate strength. Corresponding boundary conditions are

$$\varepsilon = \varepsilon_y, \quad \nu = \nu_0 \ , \tag{2}$$

resulting from the elastic state

$$\varepsilon \in \langle 0, \varepsilon_y \rangle, \quad \nu = \nu_0 \ ,$$

where $\varepsilon_y = \dfrac{\sigma_y}{E}$, where σ_y is well defined or 0.2 percent offset yield stress, *E* is Young''s modulus of elasticity

$$\varepsilon = \varepsilon_1, \quad \nu = \nu_1 = 0.5 \tag{3}$$

and $$\left(\frac{d\nu}{d\varepsilon}\right)_{\varepsilon_1} = 0 \ . \tag{4}$$

Inserting the boundary conditions expressed by Eq. (2), (3), (4) into relation (1) the constants will be

$$a_1 = 2\varepsilon_1 \frac{\nu_1 - \nu_0}{(\varepsilon_1 - \varepsilon_0)^2} \ ,$$

$$a_2 = -\frac{1}{2\varepsilon_1} a_1 \ , \tag{5}$$

$$a_0 = \frac{\nu_0 \varepsilon_1^2 - 2\varepsilon_1 \varepsilon_0 \nu_1 + \nu_1 \varepsilon_0^2}{(\varepsilon_1 - \varepsilon_0)^2} \ .$$

Let us consider the rule of constant volume in the field of the developed plastic deformation when $\nu_1 = 0.5$ and let it is fulfilled for $\varepsilon = \varepsilon_1$. Then the rule of constant volume can be expressed as

$$\overline{A}_0 \, \bar{l}_0 = A\,l \,, \tag{6}$$

where $\overline{A}_0$ is the chosen original cross-sectional area of the specimen in plastic state and $\bar{l}_0 = l_0 + \Delta l_{\varepsilon 1}$ is the corresponding length of the specimen taking into account corresponding deformation of the bar ε_1 . Then $\bar{l}_0 = l_0 (1 + \varepsilon_1)$.

The chosen original cross-sectional area $\overline{A}_0$ can be then expressed as

$$\overline{A}_0 = A_0 - \Delta A \,, \tag{7}$$

where change in cross-sectional are is

$$\Delta A = \int_{\varepsilon = 0}^{\varepsilon = \varepsilon_1} dA \,, \tag{8}$$

in the range of $\varepsilon \in \langle 0, \varepsilon_1 \rangle$ and A_0 is the original cross-sectional area of the specimen. An infinitesimal change dA can be expressed as

$$dA = -2 A_0 \; \nu \; d\varepsilon \tag{9}$$

and change in cross-sectional area with respect to Eq. (1) then will be

$$\Delta A = -2 A_0 \left[\nu_0 \int_0^{\varepsilon_0} d\varepsilon + \int_{\varepsilon_0}^{\varepsilon_1} \left(a_0 + a_1 \, \varepsilon + a_2 \, \varepsilon^2 \right) d\varepsilon \right] . \tag{10}$$

After treating, the last expression can be rearranged as

$$\overline{A}_0 = K \, A_0 \; . \tag{11}$$

After integration of Eq. (10) with respect to Eq. (5) for $\nu_0 = 0.3$ in the case of steel and $\nu_1 = 0.5$ at $\varepsilon_1 = 0.02$ the constant $K = 1 - 0.0159 = 0.984$. Equation (6) can be then expressed as

$$A_0 \, l_0 \; K (1 + \varepsilon_1) = A\,l \; . \tag{12}$$

Taking into account the value of the constant K and assumed value of $\varepsilon_1 = 0.2$ it is obvious that the influence not only of the elastic change in cross-sectional area, but also of the change in cross-sectional area because of plastic deformations with respect transition in Poisson's ratio can be ignored and Eq. (6) can be rearranged as

$$A_0 \, l_0 \; = A\,l \,, \tag{13}$$

as it is was used previously without mentioned above demonstration.

To express an approximate true stress $\tilde{\sigma} = F / A$ - true strain $\tilde{\varepsilon}$ diagram, the known procedure of determination of tangent at point P' corresponding to point of instability P of engineering stress $\sigma = F / A_0$ - strain $\varepsilon = (l - l_0)/l$ diagram, was used. Then the following relation can be obtained (Fig. 1)

$$\left(\frac{d\tilde{\sigma}}{d\varepsilon} \right)_P = \frac{\tilde{\sigma}_P}{1 + \varepsilon_P} = tg\alpha \,, \tag{14}$$

for tangent of the true stress – true strain diagram at $\varepsilon = \varepsilon_P$, point P' in Fig. 2. The true stress $\tilde{\sigma}$ - true strain $\tilde{\varepsilon}$ diagram in the range $\varepsilon \geq \varepsilon_P$ is then replaced by line identical with that tangent. Then it will be

$$\tilde{\sigma} = R_m + \frac{\tilde{\sigma}_P - R_m}{\varepsilon_P} \, \varepsilon \quad \text{for} \quad \varepsilon \geq \varepsilon_P \, . \tag{15}$$

Using Eq. (13) corresponding strain after treating is given by

$$\varepsilon = \frac{l}{l_0} - 1 = \frac{A_0}{A} - 1 \ . \tag{16}$$

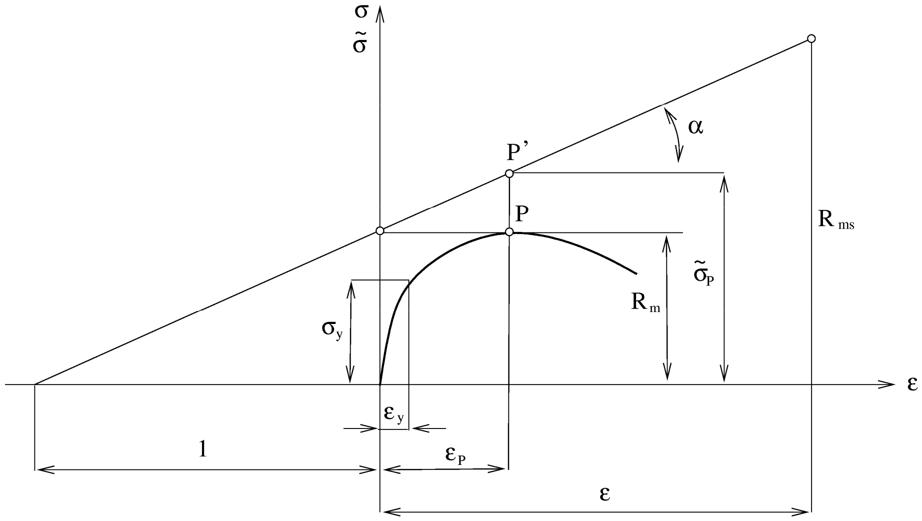

Fig.2 Engineering stress σ - strain ε diagram with the structure of the tangent to the true stress $\tilde{\sigma}$ - true strain $\tilde{\varepsilon}$ curve at point P'

Then the following relationship can be expressed as

$$\tilde{\sigma} = \sigma(1+\varepsilon) \ . \tag{17}$$

The last relationship is used to determine the true stress $\tilde{\sigma}$ using engineering stress σ in the range $\varepsilon \in \langle \varepsilon_1, \varepsilon_p \rangle$. It is supposed that $\tilde{\varepsilon} = \varepsilon$, because of homogenous deformation in the whole volume of the testing part of the specimen in that case. The presented here approach was compared with experimental results and using Gurson's model [5] in the case of mild carbon steel and aluminium alloy.

RESULTS AND DISCUSSION

The presented approach was applied to two materials, as it is presented in Fig. 3 for mild carbon steel and in Fig. 4 for aluminium alloy.

In the case of mild steel the true stress $\tilde{\sigma}$ – true strain $\tilde{\varepsilon}$ diagram plotted by mentioned above approach was validate experimentally by measurement of change in diameter of the specimen of the original diameter of 5 mm and calculating the true stress using instantaneous cross-sectional area in the range $\varepsilon > \varepsilon_P$. In the range of $\varepsilon \in \langle \varepsilon_1 = 0.02, \varepsilon_P = 0.15 \rangle$ was obtained results compared with ones obtained by Gurson's modified model as it is presented in Fig. 3.

In the case of aluminium alloy the validation of presented approach was done in the instant of rupture, when corresponding true strain and true stress were calculated using the measured change in the diameter of the specimen of original diameter of 6 mm after rupture. That is presented in Fig. 4.

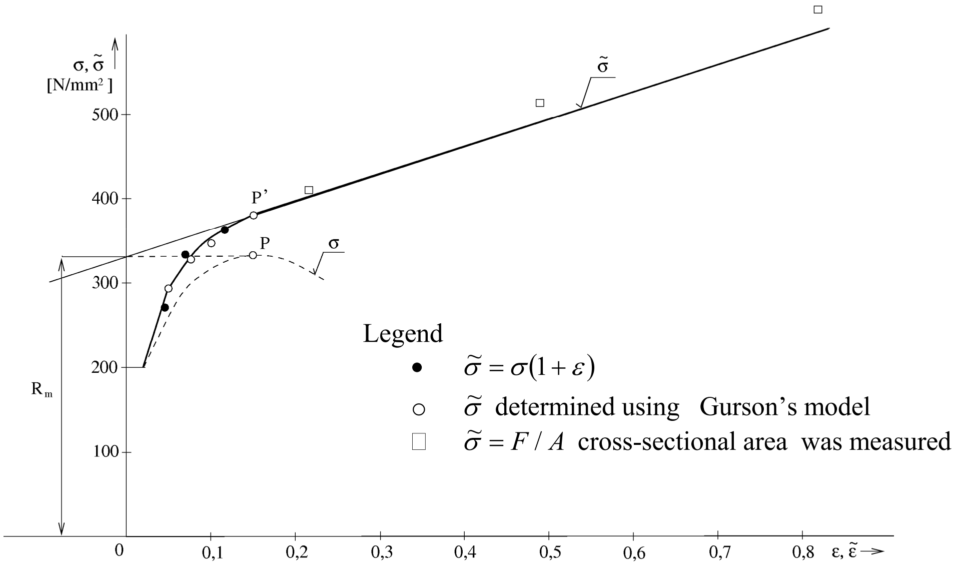

Fig.3 True stress $\tilde{\sigma}$ - true strain $\tilde{\varepsilon}$ and engineering stress σ - strain ε diagrams for mild steel with experimental and modified Gurson's model results

Taking into account obtained results the proposed method was validated as the accurate enough.

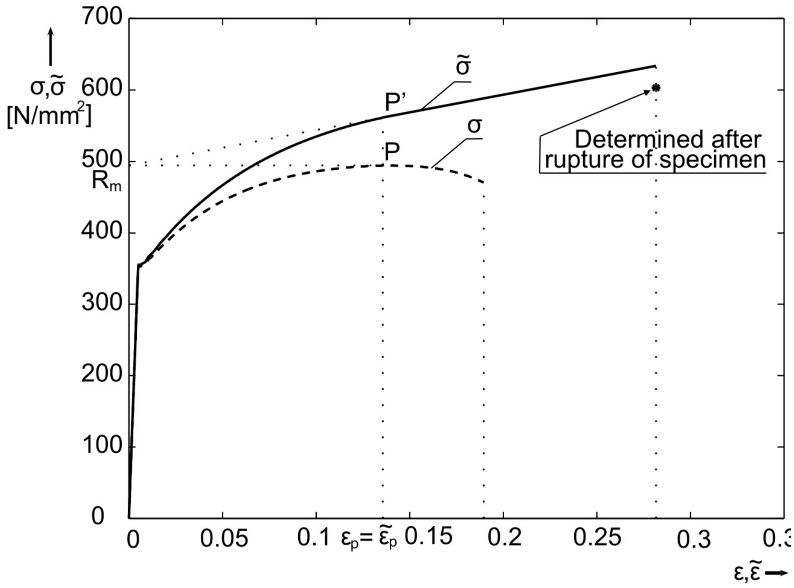

Fig.4 True stress $\tilde{\sigma}$ - true strain $\tilde{\varepsilon}$ and engineering stress σ - strain ε diagrams of aluminium alloy

CONCLUSIONS

The presented approach gives possibility of approximate plot of true stress $\tilde{\sigma}$ - true strain $\tilde{\varepsilon}$ diagram using engineering one. The method was verified experimentally and using modified Gurson's model for the mild steel specimen (Fig.3). In the case of aluminium alloy experimentally was verified only state at the rupture, see Fig.4. The presented approach of the diagram plot was validated. It was proved that difference in comparison with the exact diagrams does not exceed 20% in the range near the rupture of the specimens.

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