Experimentální Analýza Napětí 2007

IDENTIFICATION OF PARAMETERS OF HYPERELASTIC MODELS FROM BIAXIAL TENSION TESTS

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Abstract: *Relationships between stress and strain components are nonlinear isotropic for elastomers such as rubber and nonlinear anisotropic for soft tissues such as artery wall. Isotropic as well as orthotropic hyperelastic constitutive equations are used for describing these properties. The quantitative definition of hyperelasticity is that material behaviour is such that the stress component is the derivative of an elastic potential function (or strain energy density function) with respect to the corresponding strain component. For a credible identification of hyperelastic constitutive equations and their parameters, it is necessary to use appropriate types of experimental tests. Typical tests for isotropic hyperelastic materials are: volumetric test, equibiaxial tension test, uniaxial tension test and pure shear test (or some of their equivalent tests). Combinations of data from multiple tests will enhance the characterization of the hyperelastic behaviour of a material. This paper presents a structural design of the equipment for biaxial testing of soft tissues as well as rubber, an analysis of the test types necessary for a credible identification of constitutive relations and their parameters and a method allowing the identification of parameters of constitutive relations.*

Keywords: *biaxial tension tests, strain energy density function, artery*

1. Introduction

Uniaxial tension tests alone of soft tissues or rubbers are not sufficient for identification of parameters of hyperelastic material models that aim to predict the material behaviour in various types of multi-axial loading states. An credible modelling of hyperelastic materials requires additive test data under conditions of equbiaxial tension and planar tension. A common assumption valid for most of these materials is their incompressibility. Finite Element Analysis (FEA) is used for modelling this material behaviour. FEA software (e.g. ANSYS, ABAQUS) can calculate parameters of hyperelastic constitutive equations from least squares fits to this data.

2. Experimental equipment for biaxial tension tests

The experimental equipment (Fig. 1) consist of bedplate with two servo motors and orthogonal screws, four carriages, equipment for clamping of the specimens, specimen bath with physiological saline solution, support stand with programmable camera and computer with software system for test control. Equipment for clamping of the specimens consist of four carriages with four clips per each one and two sensing heads with force transducer. The specimen is clamped by two or four clips on every edge because the loading must be distributed uniformly. The method of clamping should not damage the specimen. The specimen of arterial wall is immersed in physiological saline solution with specified

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temperature and *pH*. As any contacting strain gauges are unacceptable in soft tissue applications, contactless strain evaluation is required. A programmable CCD camera is used as one of the non-contacting methods that involves the tracking of a finite number of closely spaced markers that are marked or affixed to the specimen. Measurement of strains in loading directions is based on evaluation of the position of markers before and during the loading. The reference markers - four black points are marked on specimen [1] or 1 mm diameter steel balls are glued onto the specimen surface [6], [7] (Fig. 2). Then the specimen is loaded by controlled forces or displacements. Position of reference points or steel balls are on-line monitored by the CCD camera and saved for processing by software for off-line image analysis Tibixus (produced by P. Skácel). The output data consist of the complete deformation gradient tensor (inferred from the black points motions), loading forces and Cauchy (true) stresses.

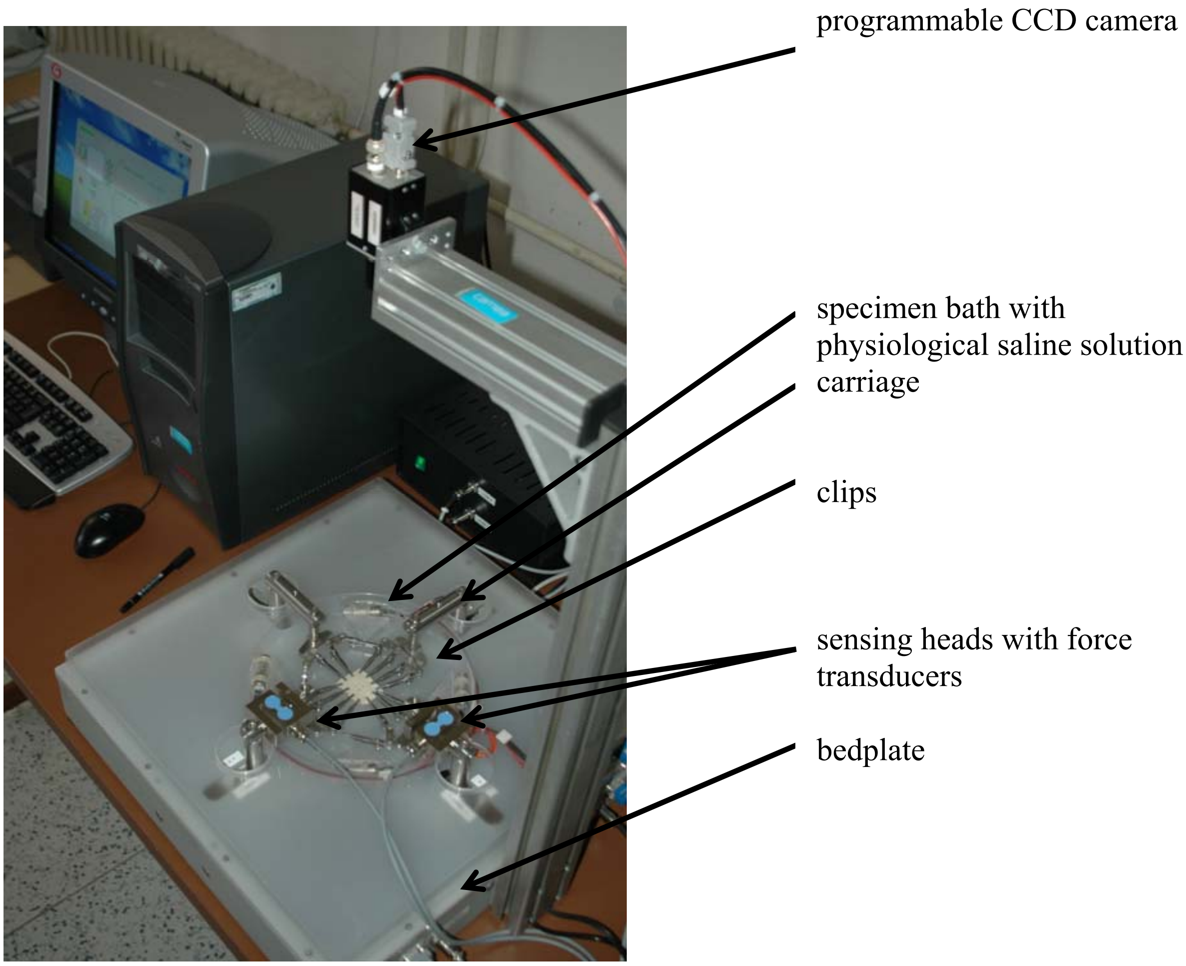

Fig. 1. Experimental equipment

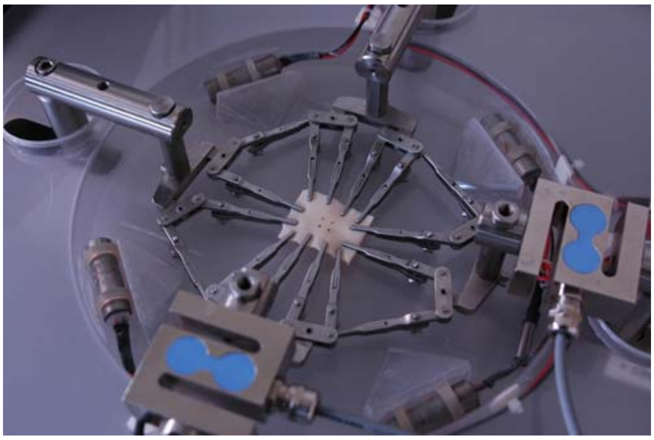

Fig. 2. Gripping of a rectangular specimen with reference points

3. Types of tests

Mechanical tests of soft tissues are realized „in vitro“ using a square or rectangular specimen. The independent control of displacements in both directions enables us to obtain the stress-strain characteristics for various states of biaxial tension. It is possible to obtain stress-strain characteristics in the following types of test [2], [5] (Fig. 3):

a) equibiaxial tension test - with strains equal in both principal directions (curve 1)

b) biaxial tension tests - uniaxial with constraining of transversal contraction (curves 2, 3)

c) biaxial tension test with proportional strain components (curve 4)

d) biaxial tension test with constant strain in either „1“ or „2“ principal direction (curves 5, 6)

e) uniaxial tension tests in either „1“ or „2“ principal direction (curve 7, 8)

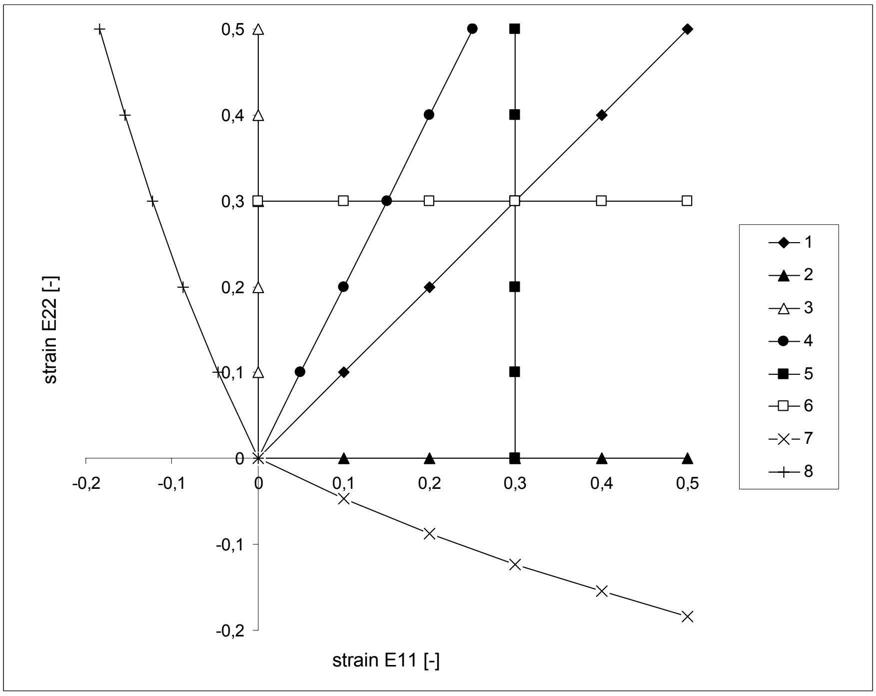

Fig. 3. Strain states in various types of tests

4. Constitutive relations

Hyperelastic materials are described in terms of a strain energy potential W (or strain energy density function) which defines the strain energy stored in the material per unit of reference volume as a function of the strain at that point in the material. Several forms of strain energy potential provided for the simulation of incompressible or nearly incompressible hyperelastic isotropic materials [10], [11] are presented below:

4.1 Isotropic hyperelasticity

a) Neo-Hookean

$$W = \frac{G}{2}\left(\overline{I}_1 - 3\right) + \frac{1}{d}\left(J-1\right)^2 \tag{1}$$

where: G is initial shear modulus of material, d is material incompressibility parameter defined by $K = d/2$ where K is initial bulk modulus, $\overline{I}_1$ is first modified invariant of right Cauchy-Green deformation tensor C_{ij} and J is the total volume change. For biaxial tension testing:

$$C_{ij} = \begin{vmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{vmatrix}$$

where λ_i are principal stretch ratios defined by: $\lambda_i = l_i / l_{i0}$ *for* $i = 1,2,3$ where l_i are deformed dimensions and l_{io} are original dimensions of specimen.

b) Mooney-Rivlin

The form of the strain energy potential can be defined for:

2 *parameter*:

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + \frac{1}{d}\left(J-1\right)^2 \tag{2}$$

3 *parameter*:

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + \frac{1}{d}\left(J-1\right)^2 \tag{3}$$

5 *parameter*:

$$\begin{aligned} W = {} & c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{20}\left(\overline{I}_1 - 3\right)^2 \\ & + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + c_{02}\left(\overline{I}_2 - 3\right)^2 + \frac{1}{d}\left(J-1\right)^2 \end{aligned} \tag{4}$$

9 *parameter*:

$$\begin{aligned} W = {} & c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{20}\left(\overline{I}_1 - 3\right)^2 \\ & + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + c_{02}\left(\overline{I}_2 - 3\right)^2 + c_{30}\left(\overline{I}_1 - 3\right)^3 \\ & + c_{21}\left(\overline{I}_1 - 3\right)^2\left(\overline{I}_2 - 3\right) + c_{12}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right)^2 + c_{03}\left(\overline{I}_2 - 3\right)^3 + \frac{1}{d}\left(J-1\right)^2 \end{aligned} \tag{5}$$

where c_{10}, c_{01}, c_{20}, c_{11}, c_{02}, c_{30}, c_{21}, c_{12}, c_{03} are material constants.

c) Polynomial Form

$$W=\sum_{i+j=1}^{N} c_{ij}\left(\overline{I}_1-3\right)^i\left(\overline{I}_2-3\right)^j+\sum_{k=1}^{N}\frac{1}{d_k}\left(J-1\right)^{2k} \tag{6}$$

where c_{ij} are material constants.

d) Ogden Potential

$$W=\sum_{i=1}^{N}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha i}+\lambda_2^{\alpha i}+\lambda_3^{\alpha i}-3\right)^i+\sum_{k=1}^{N}\frac{1}{d_k}\left(J-1\right)^{2k} \tag{7}$$

where μ_i, α_i are material constants.

e) Arruda-Boyce Model

$$W=G\left[\frac{1}{2}\left(\overline{I}_1-3\right)+\frac{1}{20\alpha_L^2}\left(\overline{I}_1^2-9\right)+\frac{11}{1050\alpha_L^4}\left(\overline{I}_1^3-27\right)\right.$$
$$\left.+\frac{19}{7000\alpha_L^6}\left(\overline{I}_1^4-81\right)+\frac{519}{673750\alpha_L^8}\left(\overline{I}_1^5-243\right)\right]+\frac{1}{d}\left(\frac{J^2-1}{2}-\ln J\right) \tag{8}$$

where G, α_L are material constants.

For soft tissues applications, the most frequent two-dimensional models of the strain-energy function describing orthotropic hyperelastic material behavior [1], [8] are:

4.2 Orthotropic hyperelasticity

a) Polynomial model: Patel and Vaishnav (1972):

$$W=AE_{11}^2+BE_{11}E_{22}+CE_{22}^2+DE_{11}^3+EE_{11}^2E_{22}+FE_{11}E_{22}^2+GE_{22}^3 \tag{9}$$

where A, B, C, D, E, F, G are material constants and E_{ij} are components of Green-Langrange strain tensor.

b) Exponential model:

- Fung (1973); Fung, Fronek, Patitucci (1979):

$$W=\frac{C}{2}\exp\left[c_1E_{11}^2+c_2E_{22}^2+2c_3E_{11}E_{22}\right] \tag{10}$$

where C, c_1, c_2, c_3 are material constants

- Maltzahn (1984):

$$W=\frac{C}{2}\left[\left(\exp Q\right)-1\right] \quad where \quad Q=c_1E_{11}^2+c_2E_{22}^2+2c_3E_{11}E_{22} \tag{11}$$

where C, c_1, c_2, c_3 are material constants

c) Logarithmic model: Takamizawa and Hayashi (1987) [3]:

$$W=-C\ln\left(1-Q\right) \quad kde \quad Q=\frac{1}{2}c_1E_{11}^2+\frac{1}{2}c_2E_{22}^2+c_3E_{11}E_{22} \tag{12}$$

where C, c_1, c_2, c_3 are material constants

If such a strain-energy function exists, the stress components can be obtained as derivatives of W with respect to strain components. For biaxial tension testing:

$$S_{11} = \frac{\partial W}{\partial E_{11}} \qquad S_{22} = \frac{\partial W}{\partial E_{22}} \tag{13}$$

where S_{ij} is 2. Piola Kirchhoff stress tensor which is conjugated with Green Lagrange strain tensor E_{ij} .

5. Data Fitting

The next step in the analysis is fitting of the nonlinear constitutive model to experimental data. During an equibiaxial tension test, a hyperelastic specimen is loaded equally along two of its axes. For isotropic material, the principal stretch ratios in the directions being loaded are identical. Hence, for equibiaxial tension, the pricipal stretches λ_i are given by [10], [11]:

$$\lambda_1 = \lambda_2 = \lambda_L \qquad \textit{stretch ratio in direction being loaded} \tag{14}$$

Utilizing incompressibility condition:

$$\lambda_3 = \lambda_L^{-2} \qquad \textit{stretch in direction not being loaded} \tag{15}$$

The strain energy potential can then be defined as function of the parameters below:

$$W = W(\overline{I}_1, \overline{I}_2) = W(\lambda_1, \lambda_2, \lambda_3) \tag{16}$$

For equibiaxial tension, the deviatoric strain invariants then become:

$$\overline{I}_1 = 2\lambda_L^2 + \lambda_L^{-4} \qquad \textit{and} \qquad \overline{I}_2 = \lambda_L^4 + 2\lambda_L^{-2} \tag{17}$$

From virtual work:

$$\delta W = 2 S_L \delta \lambda_L \tag{18}$$

where S_L is the component of the second Piola-Kirchhoff stress tensor. And it follows that:

$$\begin{aligned} S_L &= \frac{1}{2}\frac{\partial W}{\partial \lambda_L} = \frac{1}{2}\left(\frac{\partial W}{\partial \overline{I}_1}\frac{\partial \overline{I}_1}{\partial \lambda_L} + \frac{\partial W}{\partial \overline{I}_2}\frac{\partial \overline{I}_2}{\partial \lambda_L} \right) \\ &= 2(\lambda_L - \lambda_L^{-5})\left(\frac{\partial W}{\partial \overline{I}_1} + \lambda_L^2 \frac{\partial W}{\partial \overline{I}_2} \right) \end{aligned} \tag{19}$$

The principal true (Cauchy) stress for equibiaxial tension:

$$\sigma_L = S_L \lambda_L = 2(\lambda_L^2 - \lambda_L^{-4})\left(\frac{\partial W}{\partial \overline{I}_1} + \lambda_L^2 \frac{\partial W}{\partial \overline{I}_2} \right) \tag{20}$$

5.1 Nonlinear least square fit for isotropic hyperelastic materials:

FEA software (e.g. ANSYS, ABAQUS) contains curve fitting toolbox for approximation and determination of material constants for isotropic hyperelastic material models. Experimental data are smoothed to remove the noise from the test data based on the Savitzky-Golay method and the material constants are determined through a nonlinear least-squares-fit procedure based on Marquard-Levenberg algorithm. The least squares fit minimizes the sum of squared errors between experimental and predicted Cauchy stress values. The sum of the squared errors is defined by:

$$E = \sum_{i=1}^{n} \left(S_i^E - S_i\left(c_j\right) \right)^2 \tag{21}$$

where E is least squares residual error, S_i^E is a stress value from the test, and $S_i\left(c_j\right)$ comes from one of the nominal stress expressions derived above and n is the number of experimental data points. Once the strain energy potential is determined, the behaviour of the hyperelastic model is established. However, the quality of this model behaviour should be assessed: the prediction of material behaviour under different deformation modes should be compared with the experimental data.

5.2 Non-linear least square fit for orthotropic hyperelastic materials:

Fitting of the constitutive model to experimental data is achieved by optimizing (minimizing) the stress-based nonlinear function [9]:

$$f_s = \sum_{i=1}^{n} \left[w_1 \left(\frac{\partial W}{\partial E_{11}}\Big|_i - S_{11}^i \right)^2 + w_2 \left(\frac{\partial W}{\partial E_{22}}\Big|_i - S_{22}^i \right)^2 \right] \tag{22}$$

where w_1 and w_2 are weighting factors, $\frac{\partial W}{\partial E_{11}}\Big|_i$ and $\frac{\partial W}{\partial E_{22}}\Big|_i$ are 2. Piola Kirchhoff stresses predicted by the constitutive model for i-th data record and S_{11}^i and S_{22}^i are the experimental 2.Piola Kirchhoff stresses calculated directly from the original experimental data and n is number of experimental data records.

Alternatively to the stress-based approach expressed by eq. (22), an energy-based nonlinear function f_w may also be chosen. Thus:

$$f_w = \sum_{i=1}^{n} \left(\psi_i - W_i \right)^2 \quad n = number\ of\ experimental\ data\,records \tag{23}$$

where ψ_i is the strain energy for i-th data record predicted by the constitutive model and

$$W_i = \int_0^{E_{11}^i} S_{11}^i dE_{11}^i + \int_0^{E_{22}^i} S_{22}^i dE_{22}^i \tag{24}$$

is the strain energy computed from experimental data. From a mathematical point of view both approaches are equivalent.

Transformation of experimental data to a mathematical model is shown in the example described bellow:

5.3 Determination of material constants for orthotropic hyperelastic material models

In this example the energy-based approach and numerical integration of Eq.(24) are used. Thus:

$$W = \sum_{i=1}^{n} \sum_{j=11}^{22} \frac{S_j^i + S_j^{i-1}}{2} \left(E_j^i - E_j^{i-1} \right), \quad n = number\ of\ experimental\ data \tag{25}$$

Strain-Energy function W is computed from experimental data of three types of tests:

- two biaxial tension tests with constrained transversal deformation: (Fig. 3) - (curves 2, 3)

- equibiaxial tension test: (Fig. 3) - (curve 1)

Using an off-line image analysis software Tibixus, Cauchy stresses and principal stretch ratios in two orthogonal directions are obtained. These data are then expressed as Kirchoff stresses (2.P.K.) and Green-Lagrange strain. The parameters C, c_1, c_2, c_3 are then obtained by means of the standard nonlinear Levenberg-Marquardt algorithm for multivariate nonlinear regression by minimizing the strain-energy function. In the physiological range of loading, the best-fit parameters are obtained by using logarithmic model for the strain-energy function (Fig. 4):

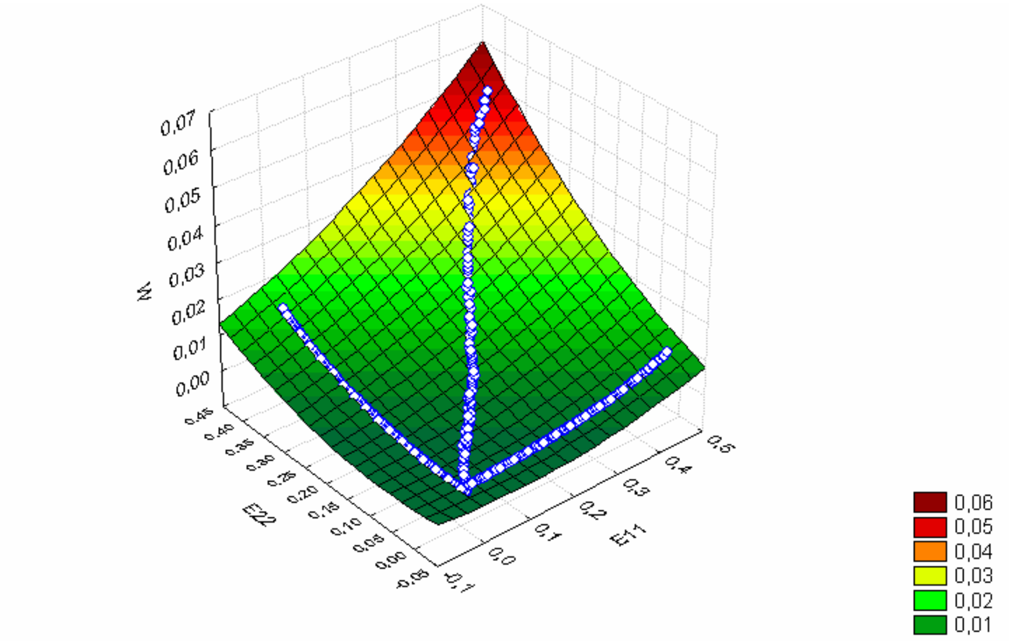

Fig. 4. The regression surface of strain-energy function

When the strain-energy function with the known constants exists, the stress components can be obtained as derivatives of W (Eq. 13) with respect to strain components:

$$S_{11} = \frac{\partial W}{\partial E_{11}} = \frac{c\left(c_1 E_{11} + c_3 E_{22}\right)}{1 - \left(\frac{1}{2} c_1 E_{11}^2 + c_3 E_{11} E_{22} + \frac{1}{2} c_2 E_{22}^2\right)} \tag{26}$$

$$S_{22} = \frac{\partial W}{\partial E_{22}} = \frac{c\left(c_3 E_{11} + c_2 E_{22}\right)}{1 - \left(\frac{1}{2} c_1 E_{11}^2 + c_3 E_{11} E_{22} + \frac{1}{2} c_2 E_{22}^2\right)} \tag{27}$$

Relations between Cauchy σ_{11}, σ_{22} and Kirchoff stresses (2.P.K.) stress S_{11}, S_{22} are:

$$\sigma_{11} = S_{11} \cdot \lambda_1^2 \tag{28}$$

$$\sigma_{22} = S_{22} \cdot \lambda_2^2 \tag{29}$$

Comparison between experimental data and data calculated by Eq. 26, 27, 28, and 29 are on Fig. 5, Fig. 6, Fig. 7 (stress-strain curves):

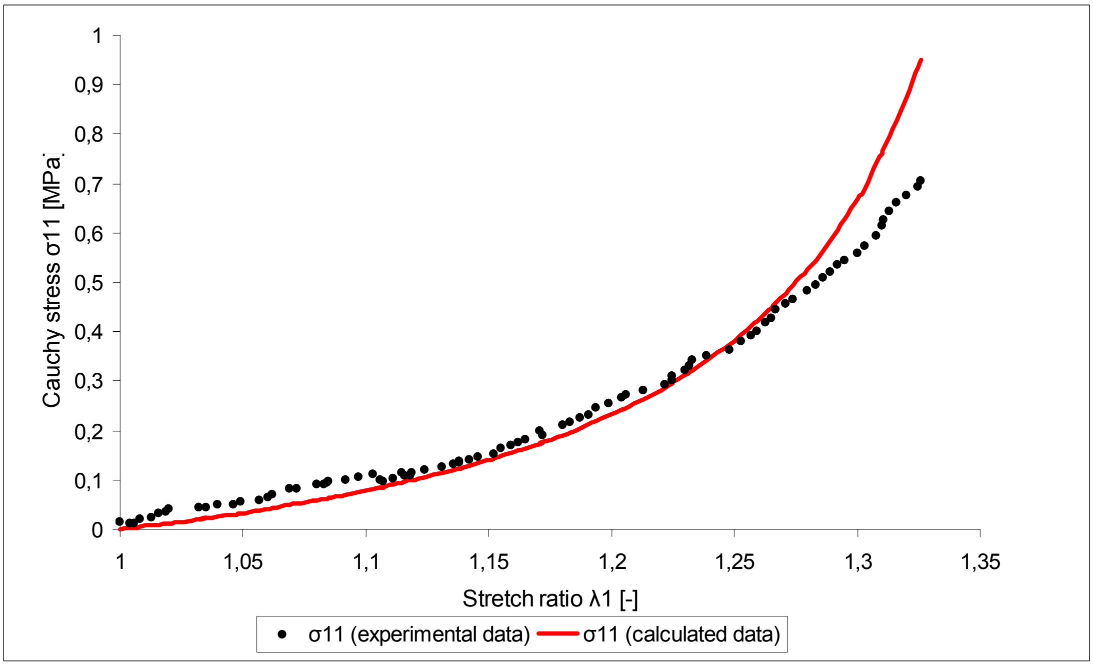

Fig. 5. Biaxial tension test – uniaxial in „1“ direction with constraining of transversal contraction in „2“ direction

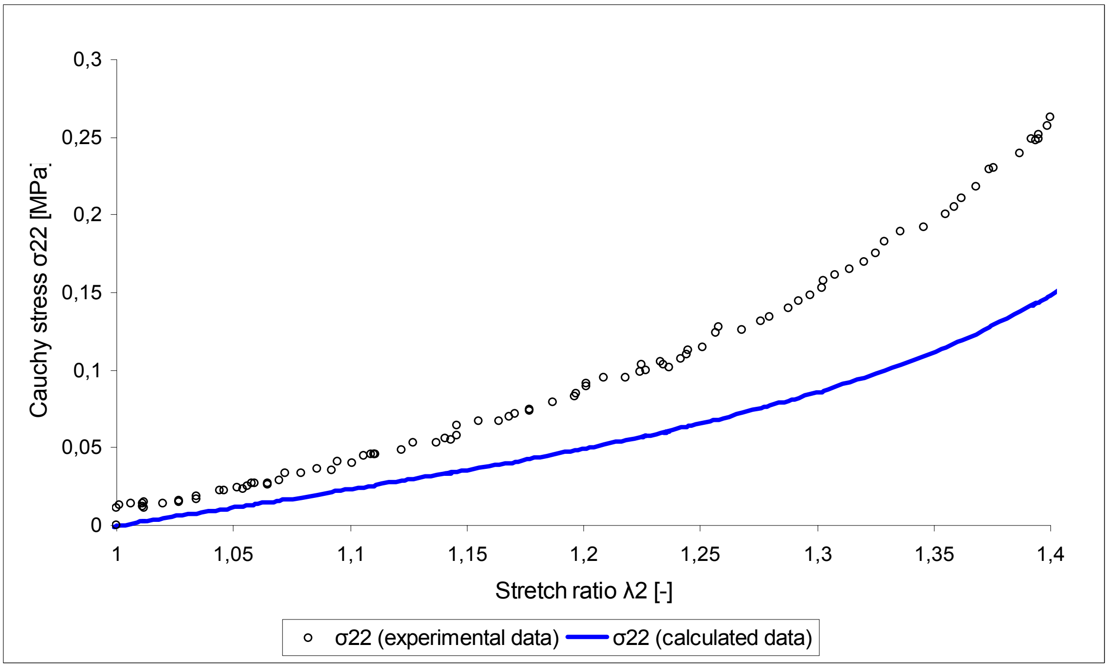

Fig. 6. Biaxial tension test – uniaxial in „2“ direction with constraining of transversal contraction in „1“ direction

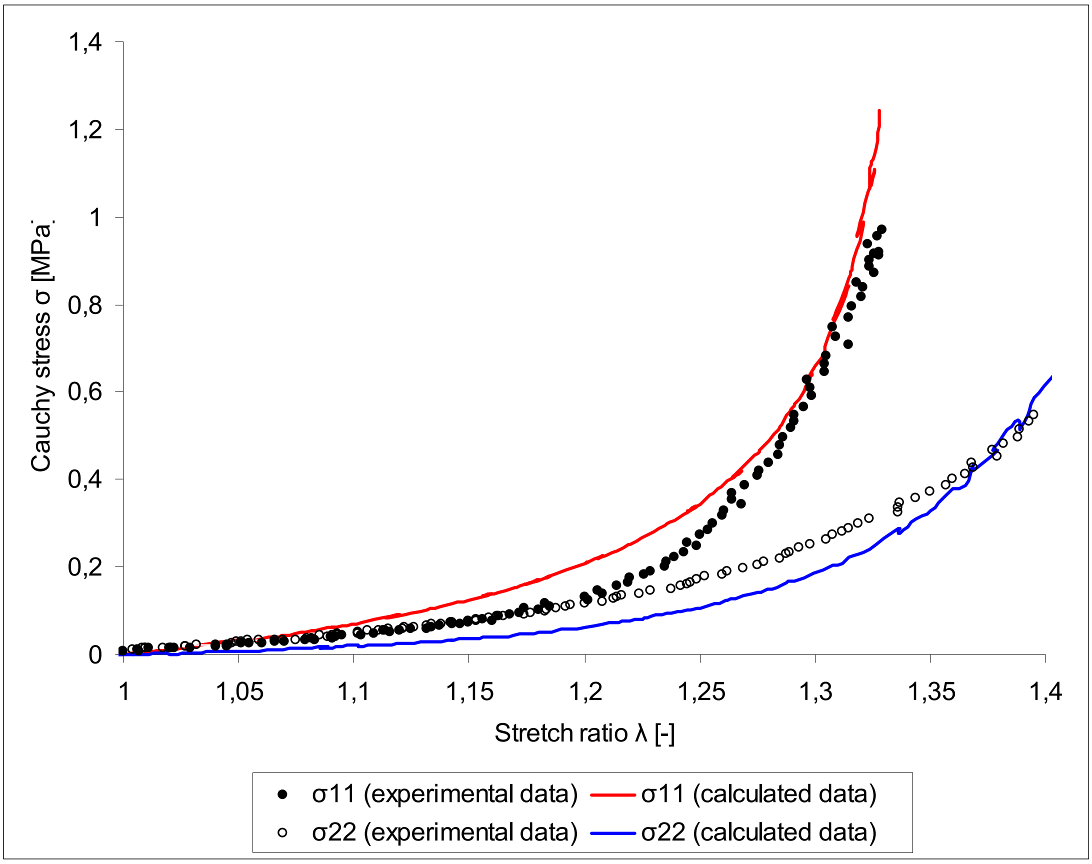

Fig. 7. Equibiaxial tension test

6. Conclusion

This paper presents a structural design of the equipment for biaxial testing of elastomers and soft tissues, such as artery wall and a method allowing the identification of constitutive relations and their parameters for FEA modelling. The advantage of the presented experimental equipment is in the possibility to obtain the stress-strain characteristic for various states of biaxial tension e.g. equibiaxial, biaxial with constant or proportional strain components and standard uniaxial tension test. Non-contacting strain measurement techniques are used to eliminate problems with deformation and local stress concentrations at the contacting points. Simulation of physiological conditions during testing is required for soft tissue applications, therefore specimens should be tested in a temperature-controlled saline (physiological) bath.

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