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**NONLINEAR NUMERICAL ANALYSIS OF BEAD STIFFENED COMPOSITE
RECTANGULAR PLATES SUBJECTED TO SHEARING IN THE LIGHT OF EXPERIMENTAL
RESULTS**

**NELINEÁRNÍ NUMERICKÁ ANALÝZA KOMPOZITNÍCH OBDÉLNÍKOVÝCH DESEK,
VYZTUŽENÝCH LEMY, KTERÉ JSOU NAMÁHÁNY SMYKEM, VE SVĚTLE
EXPERIMENTÁLNÍCH VÝSLEDKŮ.**

Abstract

The paper presents the results of numerical analysis of the bead stiffened composite rectangular plates subjected to shearing, considered as a geometric non-linear problem. The solution comes down to receive the representative equilibrium path of the structure. On the basis of the test series numerical parameters were selected. The reliability of the results was evaluated on the base of the comparison between an experimental and numerical conclusions.

Abstrakt

V příspěvku jsou prezentovány výsledky numerické analýzy kompozitních obdélníkových desek, vyztužených lemy, které jsou namáhány smykem. Jedná se o problém geometrických nelinearit. Na základě zkoušky byly vybrány skupiny numerických parametrů. Spolehlivost výsledků byla ověřena porovnáním numerického a experimentálního řešení.

1 INTRODUCTION

Composite structures belong to bearing systems applied more often and more often in many disciplines of the contemporary technology. The suitable selection of components and characteristic parameters is enabling the design to shape rational solutions, ensuring the suitable rigidity, the load capacity, time of life and reliability.

Assigning of that life and reliability requires the specification of the stress state operating in the full field of the loading resulting from exploitation. Fulfilling of this condition requires achievements of numerical analyses of the system taking into consideration of geometrically non-linearity. It refers to thin-wall structures in particular.

Methodology and some effects of the numerical analysis of thin-wall plate stiffened with bead elements were presented in the work. Pure shearing constituted the load of the structure.

The numerical model in the formulation of the FEM method taking into consideration geometrically non-linearity were elaborated on the basis of the observation of experimental studies. Compatibility of the character of deformation and paths of balance were being accepted as the criterion of correctness of the numeric model.

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2 SUBJECT OF EXAMINATION

The measurements of the glass - epoxy composite rectangular plate (400mm x 360mm, thickness 0.7mm) are subject of investigation. In area of the plate three bead stiffeners were shaped uniformly, parallel to one of the borders. The plate was subjected to the pure shearing in the special station.

Fig.1 presents the plate prepared for fixing in the station, whereas on the fig. 2 post buckling state during examinations is shown. The character of deformation was underlying to work out of the FEM numerical model. Credible stress distribution received in results of calculation making grounds for the estimation of the structure.

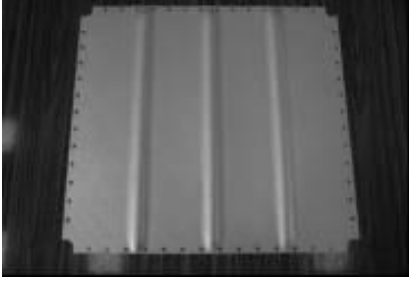


Fig. 1 The plate prepared to tests

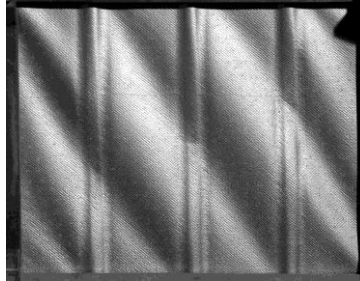


Fig. 2 Post buckling state of the plate

3 NON-LINEAR FEM ANALYSIS

Numerical simulation of post buckling deformation phenomena requires the non-linear procedure application. Large deformations and the change of the structure rigidity have to be taken into consideration.

Non-linear formulation of the problem is managing to the discrete equilibrium equations encountered in non-linear static structural analysis, formulated by the displacement method presented in the compact force residual form [8]

$$\mathbf{r}(\mathbf{u}, \Lambda) = \mathbf{0}.$$

Here \mathbf{u} is the state vector, containing the degrees of freedom, that characterize the configuration of the structure, Λ is an array of control parameters, containing the components of external loading, whereas \mathbf{r} is the residual vector containing out-of-balance forces conjugate to \mathbf{u} . Varying the vector \mathbf{r} with respect to the components of \mathbf{u} in assumption - $\Lambda = \text{constant}$, tangent stiffness matrix \mathbf{K} in structural mechanics application can be written:

$$\mathbf{K} = \frac{\partial \mathbf{r}}{\partial \mathbf{u}}.$$

An alternative version of equation (2) is the force-balance form

$$\mathbf{p}(\mathbf{u}) = \mathbf{f}(\mathbf{u}, \Lambda).$$

The \mathbf{p} vector contains components of internal forces, resulting from deformation of the structure; however \mathbf{f} are the control-dependent external forces, posing the set introduced respectively during the next stages of the analysis, which are also able to be dependent on current geometry of the structure.

Philosophy of the non-linear analysis in FEM is based on the gradual application of control parameters, realized in the next stages. It is corresponding to the stage for every reliable state of the structure, which a static balance is specific for corresponding to the solution of equation (2). Control parameters connected to external forces components are expressed, in general, as functions of reliable quantity λ , called the stage control parameter. The result of the non-linear analysis poses the set of

solutions, corresponding to each values of the λ parameter. They create the equilibrium path of the system. The unambiguous graphical interpretation of the equilibrium path is possible for at most two degrees of freedom. However on the basis of the external loading knowledge, value of stage control parameters and related geometric configurations of the structure, it is possible to obtain approximated dependence between selected values describing deformation of the structure versus external loading. In cases of considered numerical models equilibrium paths were determined into the system: torsional moment versus total torsional angle.

Algorithms of non-linear analysis are based, mainly, on iterative and incremental - iterative procedures. The stiffness matrix \mathbf{K} is treated in every solution stage as a constant and it is experiencing the increase as far, as λ stage control parameter grows. Newton - Raphson algorithm constitutes the basic iterative method. Lack of the chance to obtain the solution convergence is the method defect. It is bound up with the appearance of the limit of bifurcation points on the equilibrium path. In such situations the arc length method is applied, which makes possible to determine the balance of the system.

In considered problem, applying MSC MARC 2005 programme, non-linear numerical analyses were made. This programme is creating the chance of the user's meaning intervention in the issue of the iteration parameters selection.

On the fig. 3 the results of computation are shown. Fig. 3 presents form of deformation in post buckling state of plates in the real scale. Effective stress distribution according to Von Misses criterion in external side of plate for the same range of deformation like in fig. 3 presents fig. 4.

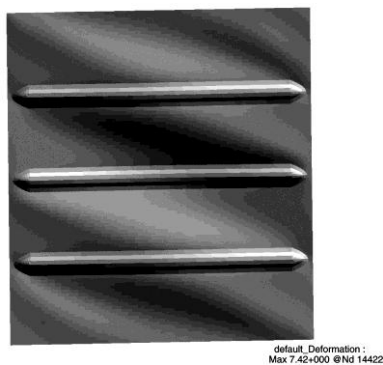


Fig.3 Computer model presenting post buckling state of the plate.

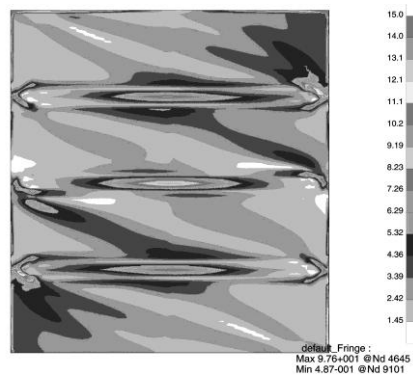


Fig. 4 Effective stress distribution in external side of plate (post buckling deformation).

3 CONCLUSIONS

On the basis of numerical and experimental results some statements could be formulated, essential for engineering practice.

Comparing effects of numerical calculations to results of experimental studies easily to notice satisfying convergence of the form of deformation in post buckling state of plate. It is proving to correctness of the elaborated numerical model and the accepted strategy of the solution to the non-linear problem.

Presented investigations denoted experimental revision, an information source about structure behaviour under loading indeed, performs verification function for numerical FEM model in particular, when the solution of the problem requires the non-linear formulation.

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