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Experimental Stress Analysis

Sensitivity Gain of the Hole Drilling Method for Stress State Identification

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Abstract: The theory for the subsequently drilled experimental holes allows improvement of the measurement sensitivity by increasing the original diameter of the drilled hole. Simultaneously, the hole-drilling experiment can be repeated with a bigger diameter of the drill, while using the same drilling rosette already installed previously, either centrically or eccentrically to the drilled hole. Thus the theory expands the applicability of the hole-drilling principle for the stress state identification.

Keywords: Stress, Identification, Hole, Drilling, Sensitivity gain

1. Introduction

Semi-destructive hole-drilling method for a stress state identification is based on disturbance of a force equilibrium in a strained body by drilling a hole with a relatively small radius R_0 perpendicularly to the surface (see scheme in Fig. 1). Polar coordinates are defined by the relative radius $r = R/R_0 \geq 1$ and angle α. The thin plate is loaded uniaxially by σ_x principal stress here and thus the stresses $\sigma_r, \sigma_\theta, \tau_{r\theta}$ in defined section planes r, θ describe the stress state at a specific position r, α. In the case of a straight-through hole, the Kirch's theory [1, 2] of the thin plate loaded uniaxially by a constant principal stress describes the change of the stress state from the state before drilling of the hole (Eq. (1)) to the final state – Eq. (2).

$$\left\{\begin{aligned} \sigma'_r &= \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma'_\theta &= \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau'_{r\theta} &= \frac{\sigma_x}{2}\sin 2\alpha \end{aligned}\right\} \quad (1) \qquad \left\{\begin{aligned} \sigma''_r &= \frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma''_\theta &= \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \tau''_{r\theta} &= \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \quad (2)$$

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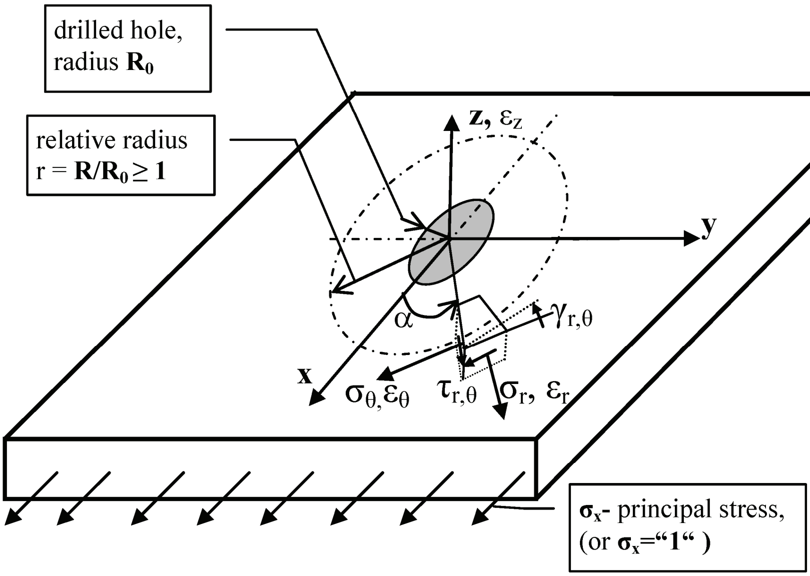

Fig. 1. Description of the stress state in the hole vicinity.

The residual between both stress states is provided in Eq. (3). It manifests itself also in the surface layer around the drilled hole by a measurable strain change, that can be calibrated in advance.

$$\left\{\begin{aligned} \sigma_r &= \sigma''_r - \sigma'_r = \frac{\sigma_x}{2}\left(-\frac{1}{r^2}\right) + \frac{\sigma_x}{2}\left(\frac{3}{r^4} - \frac{4}{r^2}\right)\cos 2\alpha \\ \sigma_\theta &= \sigma''_\theta - \sigma'_\theta = \frac{\sigma_x}{2}\left(\frac{1}{r^2}\right) - \frac{\sigma_x}{2}\left(\frac{3}{r^4}\right)\cos 2\alpha \\ \tau_{r\theta} &= \tau''_{r\theta} - \tau'_{r\theta} = \frac{\sigma_x}{2}\left(-\frac{3}{r^4} + \frac{2}{r^2}\right)\sin 2\alpha \end{aligned}\right\} \tag{3}$$

Thanks to the Hooke's law valid for isotropic material, the changes of related strains $\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}$ a ε_z in the straight-through hole vicinity can be derived from the plane stress state change $\sigma_r, \sigma_\theta, \tau_{r\theta}$, Young's modulus E and Poisson's ratio ν - see Eq. (4).

If the uniaxial stress state with $\sigma_x = 1MPa$ principal stress is expected in the straight-through hole position (see Fig. 1), the graphs in Fig. 2 and Fig. 3 describe courses of tangential ε_θ and radial ε_r strains in dependency on relative radius *r* and deviation angle α from σ_x principal stress direction. The strains are computed with

$\nu = 0{,}3$ Poisson's ratio and are multiplied by the size of Young's modulus for better lucidity. The $\varepsilon_r, \varepsilon_\theta$ strains reach the highest values in $r = 1 \div 2$ relative radius range. Therefore, this is the most sensitive area for placement of drilling rosette strain gauges used for measuring of surface strains.

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\frac{1}{r^2} - \frac{1}{r^2}\nu\right] + \left[\frac{1}{r^4}3 + \frac{1}{r^4}3\nu - \frac{1}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{1}{r^2} + \frac{1}{r^2}\nu\right] - \left[\frac{1}{r^2}3 + \frac{1}{r^2}3\nu - \frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{1}{r^4}3 + \frac{1}{r^2}2 - \frac{1}{r^4}3\nu + \frac{1}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \quad (4)$$

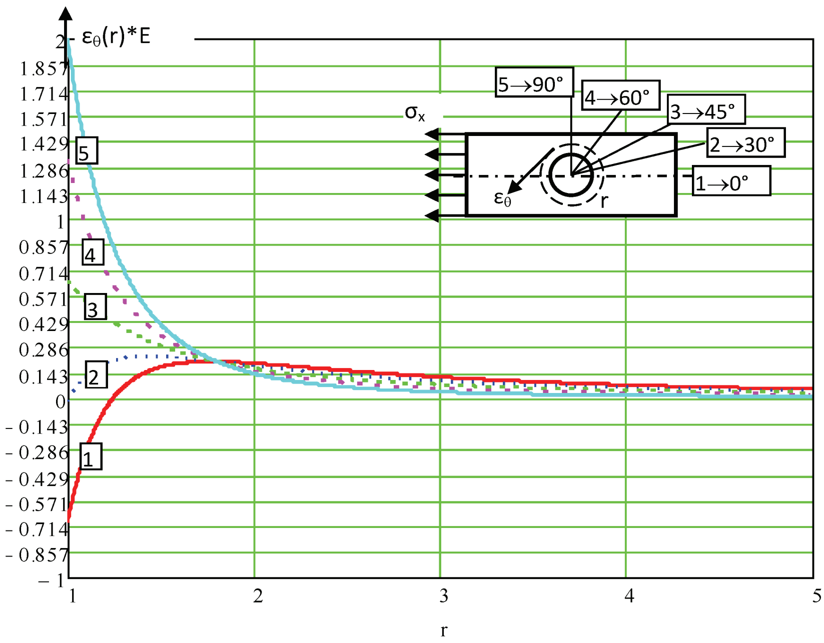

Fig. 2. Strain in the tangential direction, $\varepsilon_\theta(r)$.

Presume the drill radius $2R_0 = 1{,}6mm$. Then the radial dimension of the rosette strain gauge $1{,}7mm$ corresponds to relative radius $r \cong 2 \cong 1{,}7/R_0 = 1{,}7/0{,}8$ in Fig. 4. The simulation of the radial strain gauge measurement in dependency on its relative distance r from the hole center is depicted in Fig. 5 for highest radial strain observed by curve 1 in Fig. 3.

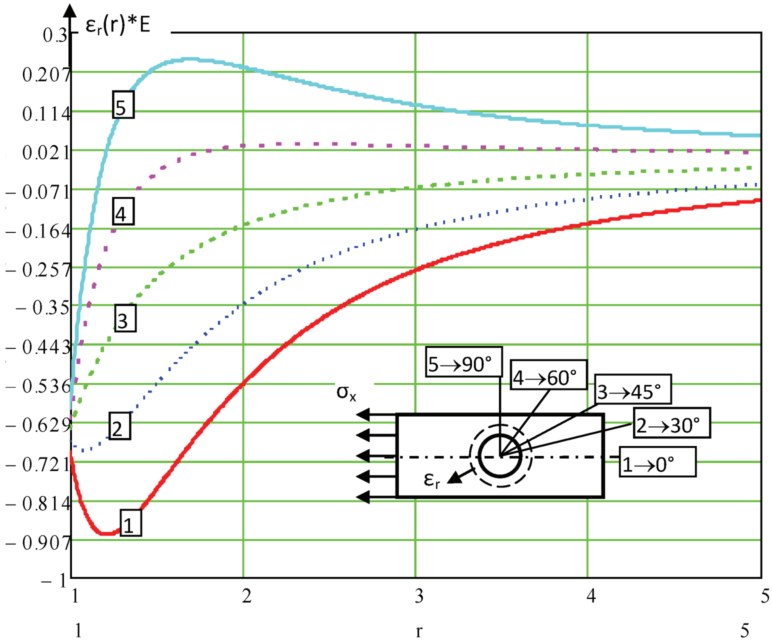

Fig. 3. Radial strain, $\varepsilon_r(r)$.

The quality of strain gauge placement (the strain gauge with $r = 2$, its relative length is expected here) is documented by potential of the signal measured (given by strain along the strain gauge winding) related to the area total potential. The area is presumed to be of $r = 1 \div 5$ relative length. The strain gauge placed closest to the drilled hole is able to indicate 75% of strain, which is released for measuring thanks to the hole drilling. The measurement sensitivity during the hole drilling thus can be increased by putting the strain gauges closer to the hole edge or by relative augmenting of the drill diameter to the diameter, at which are the strain gauges placed in the rosette. The experiment also can be run repeatedly with a gradual increase of the drilled hole diameter. If a minor drill diameter is chosen in the experiment first phase and the rosette strain gauges are installed in a relative distance $r = 2 \div 4$, the measurement of relaxed strain depletes 40% of its potential approximately. The potential of relaxed strains thus can be better exploited by increasing the drill diameter, which results in a relative shift of the strain gauges to the edge of the hole or by a second measurement using the same drilling rosette and the drill of a bigger diameter. In the case of drawing the strain gauge nearer to the hole in the range $r = 1 \div 3$, the repeated measurement allows measurement of approx. 35% of total relaxed strains.

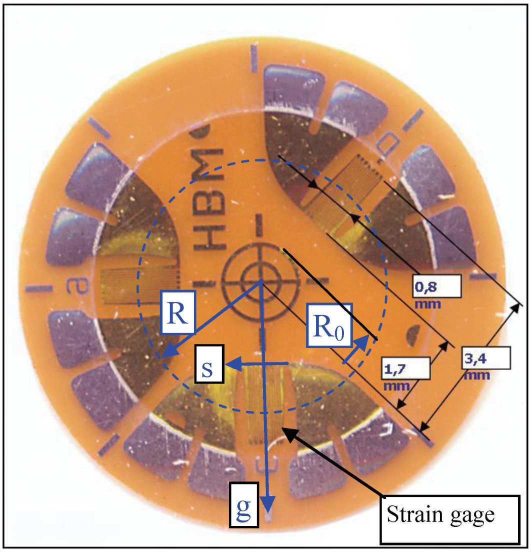

Fig. 4. Hole-drilling rosette.

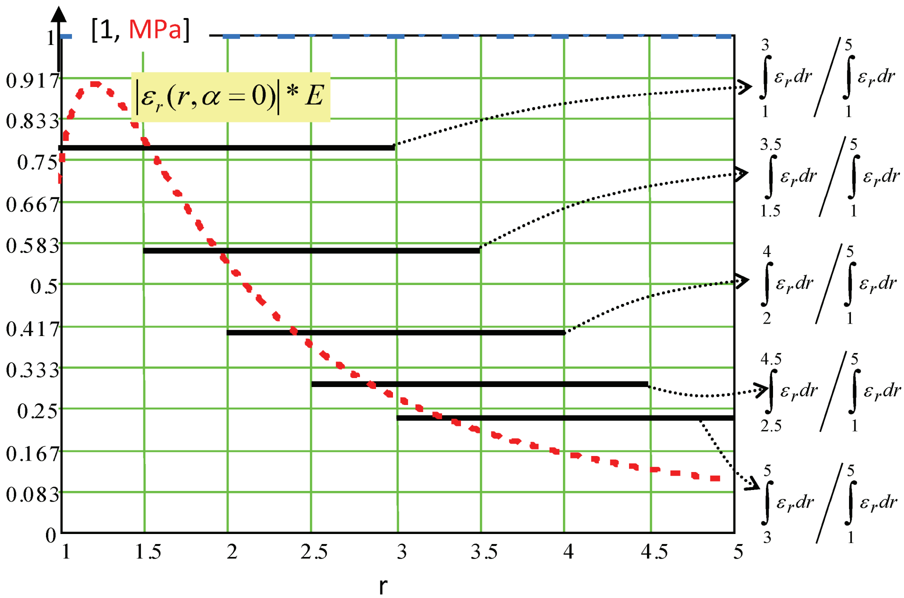

Fig. 5. The signal potential of the radial strain gauge of $2R_0 \cong 1.7$ mm length.

2. The stress state identification theory for repeatedly drilled holes

Let us suppose that the hole (see Fig. 6) of R_h radius and h depth is drilled in the first phase of the hole-drilling experiment. A concentrical hole can be expected in any next drilling of a bigger R_0 radius, because of the guiding provided by the original hole. The first case (two centrical holes in the center of the drilling rosette), where the second hole with the bigger diameter is drilled in one cycle to the same depth $h_0 = h$ and a normalized drilling rosette (standard E 837 [3]) is used for the stress state identification, allows using of the standard theory. Here, the values measured on strain gauges during both measurements are summed and evaluated for the final hole diameter $D_0 = 2R_0$ and depth h. In the second case, where both concentrical holes are drilled to the same depth $h_0 = h$ in one measuring cycle either centrically or eccentrically to the drilling rosette, the generalized hole-drilling principle theory [5, 6] can be used for the stress state identification analogically, this time covering also the potential hole eccentricity. Similarly to E 837 standard, this theory is based on the similitude of the plane stress state on the body surface to the Kirch's plate with a straigh-through hole. In the third case, the situation is further generalized, and we assume the same eccentricity of both holes to the rosette center and different depths of both holes. Also here the situation is close to the Kirch's plate, and thus the hole-drilling principle theory [5, 6] can be further expanded by modifying the individual stress components in Eq. (3) according to Eq. (5) through multiplying by constants related to the stress state in the vicinity of a bottom hole running perpendicularly to the surface of the body.

$$\left\{\begin{aligned} \sigma_r(\sigma_x) &= \frac{\sigma_x}{2}\left(-\frac{1\cdot c_{11}(r,z)}{r^2}\right) + \frac{\sigma_x}{2}\left(\frac{3\cdot c_{12}(r,z)}{r^4} - \frac{4\cdot c_{13}(r,z)}{r^2}\right)\cos 2\alpha \\ \sigma_\theta(\sigma_x) &= \frac{\sigma_x}{2}\left(\frac{1\cdot c_{14}(r,z)}{r^2}\right) - \frac{\sigma_x}{2}\left(\frac{3\cdot c_{15}(r,z)}{r^4}\right)\cos 2\alpha \\ \tau_{r\theta}(\sigma_x) &= \frac{\sigma_x}{2}\left(-\frac{3\cdot c_{16}(r,z)}{r^4} + \frac{2\cdot c_{17}(r,z)}{r^2}\right)\sin 2\alpha \end{aligned}\right\} \tag{5}$$

$$\begin{bmatrix} \varepsilon_r(\sigma_x) \\ \varepsilon_\theta(\sigma_x) \\ \gamma_{r\theta}(\sigma_x) \\ \varepsilon_z(\sigma_x) \end{bmatrix} = \frac{\sigma_x}{2E} \begin{bmatrix} \left\{\left[-\frac{c_{11}}{r^2} - \frac{c_{14}}{r^2}\nu\right] + \left[\frac{c_{12}}{r^4}3 + \frac{c_{15}}{r^4}3\nu - \frac{c_{13}}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{c_{14}}{r^2} + \frac{c_{11}}{r^2}\nu\right] - \left[\frac{c_{15}}{r^4}3 + \frac{c_{12}}{r^4}3\nu - \frac{c_{13}}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{c_{16}}{r^4}3 + \frac{c_{17}}{r^2}2 - \frac{c_{16}}{r^4}3\nu + \frac{c_{17}}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{c_{11}}{r^2}\nu - \frac{c_{14}}{r^2}\nu\right] + \left[-\frac{c_{12}}{r^4}3\nu + \frac{c_{15}}{r^4}3\nu + \frac{c_{13}}{r^4}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \tag{6}$$

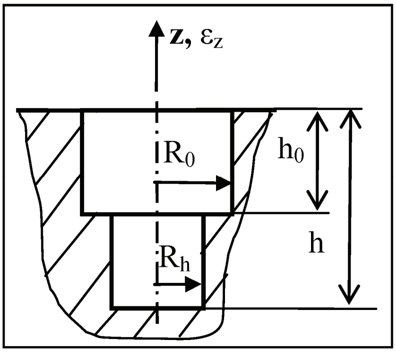

Fig. 6. Repeatedly drilled hole.

We assume, supporting such an extension of a generally accepted and used theory by input of new constants makes sense. Let us suppose the first hole is drilled by a drill of R_h radius to the h depth (see Fig. 6) and the hole is than redrilled to a concentrical hole with a higher radius $R_0 \geq R_h$ to a depth $h_0 \leq h$. The stress state calibration around the hole is realized by applying seven constants $c_{11}, c_{12}, ..., c_{17}$. The constants $c_k(r, h, h_0, R_h, R_0)$ are dependant on relative distance $r = R / R_0$ from the drilled hole, on relation between depths h and h_0 and between radiuses R_0, R_h. A relations of these suitable for the hole-drilling experiment can be further concretized. The application of Hooke's law (5) to Eq. (4) leads to the formulation of strains in Eq. (6).

Let the strain gauge winding be oriented along the g direction (depicted in Fig. 7) with acute φ angle from θ axis at the point described by coordinates r, θ (also see Fig. 1). The strain along the strain-gauge winding g which is identified by the strain-gauge, results from $\varepsilon_\theta, \varepsilon_r$ and $\gamma_{r\theta}$ strains and is derived by the use of 2φ angle through the Mohr's transformation (7).

$$\varepsilon_g = \frac{\varepsilon_\theta + \varepsilon_r}{2} + \frac{\varepsilon_\theta - \varepsilon_r}{2}\cos 2\varphi + \frac{\gamma_{\theta,r}}{2}\sin 2\varphi \tag{7}$$

Figure 1 shows the partially unit vector defined in the direction of σ_x principal stress in the angle α to the evaluated point, above which the $i-th$ strain gauge winding is positioned in the g direction (see Fig. 7). The curvilinear integral of the normalized strain transformed by Eq. (7) along the winding with total length u defines the $t_i(\alpha)$ strain-gauge sensitivity for σ_x principal stress in Eq. (8).

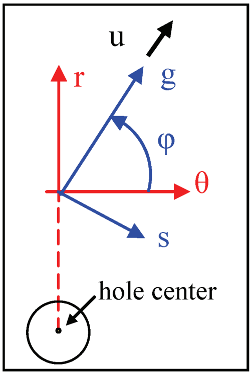

Fig. 7.

A definition of the second sensitivity of this strain-gauge $t_i(\alpha+\pi/2)$ for σ_y principal stress rotated along the surface by $\pi/2$ from the σ_x stress direction follows a similar way. Both sensitivities of the $i-th$ strain gauge of the drilling rosette are functions of the theory constants $c_{11}, c_{12}, \ldots, c_{17}$ and particular positions and orientations r, α, g of points along the winding. The orientation and position of individual strain gauges is defined in accordance with E 837 standard [3], which postulates the hole drilled in the ideal center of the drilling rosette, defines the angle α parameter of a particular strain gauge to σ_x principal stress and derives the placing of the other strain gauges of the rosette from it. Here in this more general theory, the symbol $\overline{\alpha}$ marks the position of a defined base point of the *i-th* strain gauge to σ_x principal stress. This $\overline{\alpha}$ angle of the *i-th* strain gauge thus results from its relative position to the main strain gauge, which is inclined by by α from σ_x principal stress.

$$t_i(\alpha)=\frac{\oint_u \overline{\varepsilon}_{g,j}(\alpha)\cdot du}{\oint_u du} \text{ nebo } \cdots t_i(\alpha+\pi/2)=\frac{\oint_u \overline{\varepsilon}_{g,j}(\alpha+\pi/2)\cdot du}{\oint_u du} \tag{8}$$

For unknown principal stresses σ_x, σ_y and angle α a set of at least three non-linear and independent equations can be established from the computed sensitivities of individual strain gauges in analogy to Eq. (9). The equations encompass the influence of both principal stresses on ε_i strains measured by strain gauges. A convenient way of their solution is reported in [5].

$$\varepsilon_i=\sigma_x\cdot t_i(\overline{\alpha})+\sigma_y\cdot t_i(\overline{\alpha}+\pi/2)\equiv\sigma_x\cdot t_{i,x}(\overline{\alpha})+\sigma_y\cdot t_{i,y}(\overline{\alpha}) \tag{9}$$

The regression model proposed here is an analogy to the hole-drilling method standardized in E 837 and will be likely valid in relative distances $r = R/R_0 \geq 2$, which are also recommended for the standard method. The exploitation of the more sensitive area near the drilled hole changes the regression model characteristics, which is now closer to Eq. (2) than to Eq. (3) and which can require a modification by further five constants. It is also possible on the other hand, that the regression model with seven constants will be satisfactory in the whole range of hole-drilling method application and the extension by five further constants will not be necessary.

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