Mathematical Method of Shooting Capability Evaluation

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Abstract: This article introduces a brand new method of shooting accuracy assessment; this method may properly supplement the traditional methods. When using the new method the shooting accuracy is assessed as one number; this number contains the information of the capability of a shooter to hit the assigned part of a target from the short-term (immediate) perspective as well as from the long-term perspective.

Keywords: Series Shooting Capability Index (SSCI), Process Shooting Capability Index (PSCI), Time Shooting Capability Index (TSCI).

1. Introduction

Shooting a firearm is a test of a shooter's sensorimotor performance; the shooter is required to surely hit an assigned target. Trained shooters are able to fulfil such a task due to their ability to attain an appropriate shooting pose, to fix necessary concentration, to aim the firearm accurately and to master the finger movement while pressing and during the shot itself. Hit accuracy is a suitable criterion for the assessment of the shooter's success.

A number of methods for hit accuracy assessment have been described in available literature; such methods use, for example, average radius, R50 circle, R100 circle, 2R100 circle, plain sum of values, standard deviation and eventually estimated universe standard deviation, the ellipsis of dispersion, probability of the hit, etc.

The introduced method is a suitable supplement for the above mentioned traditional methods of assessment for the needs of the Firearm Licence applicants training. It is a method of shooting accuracy assessment where one number rates the ability to hit the assigned part of a target (for the purpose of the competence test training in particular) and to hit the scoring rings; all of it from the short-term (immediate) as well as from the long-term perspective. The method it based on specification of the Series Shooting Capability Index (SSCI) and the Process Shooting Capability Index (PSCI).

2. Shooting Capability

While sitting for the competence test, the applicant is obliged to hit the assigned target with the assigned number of hits. The hit must be aimed and carried out within the previously specified part of the target. For example, the international pistol target 50/20 is being used while handguns are being fired; shooting is carried out from the distances of 10 or 15 meters.

A great number of forces influence the bullet trajectory; this shows, in fact, that the hit points are variable and do not exactly correspond with the point being aimed at. Thus, every process of shooting shows a certain degree of consistency (or inconsistency).

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The variability in the process of firearms shooting (aimed at the acquisition of the Firearms Licence) originates not only in the shooter, but also in the type of firearm and ammunition used. A considerable source of inconsistency is the seriousness of the moment; that is, the social environment at the shooting range at the moment of the competence test. Another source of variability in consistency is the environment at the shooting range where the shooting itself takes place.

Capability is the proposed criteria for the assessment of the quality of the shooting process. The capability is judged on the basis of the analysis of the overall consistency in the result of shooting.

The term shooting process capability represents the shooters' ability to consistently hit the assigned target.

Three capability indices are proposed for the use of the shooting capability evaluation:

- The Series Shooting Capability Index (SSCI) $^d C_s$, which evaluates the capability in the process of shooting one series; the conditions of shooting do not change during the course of the series;

- The Process Shooting Capability Index (PSCI) $^d C_p$, which assesses the shooters' ability to hit the assigned target from the long-term perspective. The Process Shooting Capability Index must be set on the basis of the results of several series (minimum of 25 series). The conditions of shooting change in an anticipated way (for example the time devoted to shooting a single series, temperature and other characteristics of the environment, etc.) within the time when the minimal number of series will be carried out.

- The Process Shooting Capability Index (PSCI) $^d C_t$ (assessed from several series) evaluates the shooters' permanent (long-term) ability to fulfil the given shooting performance in a given time (minimal permissible time span of shooting).

3. Series Shooting Capability Index

We suggest using the capability index C_s for evaluation of the shooting process within one series; that is a short-term process shooting capability. The above mentioned index may be expressed as a ratio of requirement for a target and the consistency within the shooting process

$$C_s = \frac{\text{requirement for target}}{\text{consistency within a process}}. \tag{1}$$

Requirement for a target represents the required outcome of the shooting process; that is, usually the type of target and its size.

Consistency of the shooting process is defined by an interval which includes all the shots' scores. In case the consistency is subject to a normal dispersion, such intervals may be defined as a multiple of standard deviation *s*.

If we were to evaluate the consistency of the shooting process using the interval ± 1. *s* (standard deviation), 68.27 % of hits' scores would fall within this interval. If the interval was ± 2. *s* (picture 3.4b), the consistency of the shooting process would be related to 95.45 % of all the hits' scores. If the interval was ± 3. *s*, the consistency of the shooting process would be related to 99.73 % of all the hits' scores.

The interval (± 3. *s)* is suggested to become a standard for the consistency evaluation of the shooting process. Such intervals represent the measure which may be perceived as

accurate enough for the evaluation of the shooting process quality (that means a sure target hit).

The term process shooting capability in one series represents the shooters' ability to hit the required target with all shots within the given series.

A statistical data normality check must be carried out after the adjustment of all hits' scores acquired from the performed shots (concordance of the measured data with the normal dispersion test). In case of non-refusal of the hypothesis that the consistency of the shots is a subject of normal dispersion, we may define the estimate of the Process Shooting Capability Index C_s for the assessed series by the following formula

$$\hat{C}_s = \frac{\text{requirement}}{6s}, \tag{2}$$

where s represents the standard deviation within the shooting process of one series. The requirement for hits may be expected to have the shape of a circular scoring ring with the diameter T.

If the coordinates of the hits (being shot from the distance d are marked with the symbols $^d x$ and $^d y$, the value of the hit radius $^d r$ may be determined according to the formula (3). The hit radius represents the distance of a given hit from the target point (decision point specified in the certain part of a target, in this case, in the center shooting ring).

$$^d r = \sqrt{^d x^2 + ^d y^2} \tag{3}$$

While calculating the Series Shooting Capability Index, the standard deviation of the radius within the given series $^d s$ (shooting from the distance d) may be defined by the formula

$$^d s = \sqrt{\frac{\sum_{j=1}^{n}(^d r_j - \overline{^d r})^2}{n-1}}, \tag{4}$$

where n represents the number of shots (and hits) in the relevant series.

Shooting capability index $^d C_s$ in the relevant series shot from the assessed distance "d" (the only number relevant for the evaluation the process shooting capability) is defined by the formula

$$^d C_s = \frac{0,5.T - \overline{^d r}}{3.^d s_r}, \tag{5}$$

where $\overline{^d r}$ is the median of the radius of the hits which was achieved while shooting from a given distance d; it is defined by the formula (6).

$$\overline{^d r} = \frac{\sum_{j=1}^{n} {}^d r_j}{n}, \tag{6}$$

and $^d s_r$ is the standard deviation of the radius of the hits for the relevant series for shooting from the distance d. In the denominator of the formula (5) calculating the capability

index ^dC_s the value of the three standard deviations is $3.{}^ds_r$; this represents the interval of the estimate of the consistency in the shooting process from the given firearm. The probability is 99.73 % (on condition of normal dispersion).

Basic criteria for evaluation of short-term shooting process capability are introduced in Table 1. These criteria are based on the value of the Series Shooting Capability Index.

Table 1. Criteria for evaluation the Series Shooting Capability Index

Series Shooting Capability Index	Evaluation of the Shooter in the Series	Verbal Evaluation of the Shooter's Capability
$^dC_s < 1.00$	**Incompetent**	The shooter either has not achieved satisfactory consistency in shooting or the median of the radius of the hits is too high
$^dC_s = 1.00$	**Competent with condition**	Terminal value of the short-term capability
$^dC_s > 1.00$	**Competent**	Shooter is very consistent in the shooting process in the given series and the median of the radius of the hits is also very good (low)

4. Process Shooting Capability Index

The Process Shooting Capability Index (established on the basis of several series) evaluates the ability of the shooter to permanently (in the long-term) hit the assigned target and abide the imposed features of the shooting process

$$^dC_p = \frac{0{,}5.T - \overset{d=}{r}}{3.{}^ds_p}$$

(7)

The Process Shooting Capability Index is defined by the formula:

where $\overset{d=}{r}$ is the median of the radius of the hits achieved from a given distance d; it is calculated from "m" series (each of the series consists of an identical number of shots) by the formula:

$$\overset{d=}{r} = \frac{\sum_{i=1}^{m} \overset{d-}{r_i}}{m},$$

(8)

and ^ds_p is the standard deviation of the radius of the hits in the shooting process; the shooting is carried out from the distance d.

To enumerate the standard deviation of the shooting process from the distance d, the formula (9) is used

$$^ds_p = \frac{\overset{d}{\overline{R}}}{k},$$

(9)

where $^{d}\overline{R}$ is the average dispersion of the values in the radius of hits; it is calculated from all evaluated series.

k is the statistical constant number dependent on the number of executed shots n in the given series (number of shots in the series). The values of the k constant are stated in Table 2; these are valid for 2 -15 shots in an individual series

Table 2. The Values of the Statistical Constant k

Number of Shots in a Series	k	Number of Shots in a Series	k	Number of Shots in a Series	k
2	1.128	7	2.704	11	3.173
3	1.693	8	2.847	12	3.258
4	2.059	9	2.970	13	3.336
5	2.326	10	3.078	14	3.407
6	2.534	11	3.173	15	3.472

Table 3 contains basic criteria for evaluation of the shooting process based on the value of the Process Shooting Capability Index (long-term capability).

Table 3. Criteria for Evaluation of the Value of the Process Shooting Capability Index

Process Shooting Capability Index	Evaluation of the Shooter in the Series	Verbal Evaluation of the Shooter's Capability
$^{d}C_{p} < 1.00$	**Incompetent**	The shooter either has not achieved long-term satisfactory consistency in shooting or the median of the radius of the hits is too high
$^{d}C_{p} = 1.00$	**Competent with condition**	Terminal value of the shooting capability
$^{d}C_{p} > 1.00$	**Competent**	In long-term, the shooter is very consistent in the shooting process and the median of the radius of the hits is also very good (low)

5. Time Shooting Capability Index

When sitting for the competence test, the ultimate time span for shooting a particular kind of firearm is specified for each group of Firearm Licence; the lengths of the time spans may be found in Table 4.

May the symbol ^{d}t represent the time of shooting (given in seconds) carried out from the distance d. We may assume that the time span for the shooting performance of a particular shooter, carried out from the distance d, will have normal placement with the median $^{d}\overline{t}$ and the standard deviation $^{d}s_{t}$. The Time Shooting Capability Index is defined by the formula

$$^{d}C_{t} = \frac{t_{\max} - ^{d}\overline{t}}{3.^{d}s_{t}}, \tag{10}$$

where t_{max} is the ultimate time span for shooting as indicated in Table 4.

Table 4. The Ultimate Time Span for Shooting a Particular Kind of Firearm when Sitting for the Competence Test

Firearms Licence Category				
A	**B**	**C**	**D**	**E**
5 minutes (Small-bore rifle, handgun) **3 minutes** (scatter gun)	**5 minutes** (Small-bore rifle, handgun) **3 minutes** (scatter gun)	**5 minutes** (small-bore rifle) **3 minutes** (scatter gun)	**2 minutes** (handgun)	**3 minutes** (handgun)

The so-called moving range for the time spans of individual shootings $^{d}R_{k_i}$ is used to calculate the standard deviation $^{d}s_t$. The moving range $^{d}R_{k_i}$ is defined by the formula

$$^{d}R_{k_i} = \left| t_i - t_{i+1} \right|, \tag{11}$$

where t_i is the time span of shooting i,

t_{i+1} is the time span of shooting $i+1$.

This means that, first of all, after the second shot (within the training) we subtract the time span of the second shot from the time span of the first shot; then we subtract the time span of the third shot from the time span of the second shot, etc. Having used the average moving range, we may estimate the standard deviation of the time span of shooting for all assessed series and the particular shooter from the evaluated m series according to the formula

$$^{d}s_t = \frac{^{d}\overline{R_k}}{d_2} = \frac{^{d}\overline{R_k}}{1,128}, \tag{12}$$

where d_2 is a constant number – 1.128 – this number represents the moving range for two values

$^{d}\overline{R_k}$ represents the average moving range which may be defined by the formula

$$^{d}\overline{R_k} = \frac{\sum_{i=1}^{m-1} {}^{d}R_{k_i}}{m-1}, \tag{13}$$

where m represents number of series. For example, the Time Shooting Capability Index for Firearms Licence Category D is defined by the following formula

$$^{15}C_t = \frac{120 - {}^{15}\overline{t}}{3 \cdot {}^{15}s_t}. \tag{14}$$

Basic criteria for the shooting process evaluation (shooting being carried out from a particular kind of firearm) based on the values of the Time Shooting Capability Index may be found in Table 5.

Table 5. Criteria for the Time Shooting Capability Index Assessment

Time Shooting Capability Index	Shooting Process Evaluation from the Ti Span Perspective	Verbal Evaluation of the Shooters' Capability
$^{d}C_t < 1.00$	**Incompetent**	The shooter either has not achieved long-term satisfactory consistency in time span of shooting or the median of the radius of the hits is too high
$^{d}C_t = 1.00$	**Competent on Condition**	Terminal value of the shooting capability from the time span perspective
$^{d}C_t > 1.00$	**Competent**	In long-term, the shooter has consistent time span in the shooting process and the shooting time median

6. Capability Indices Interpretation

The statistical importance of the capability indices is that every single value of the index represents the probability of a shot being placed within the assigned area (as long as shooting time span being evaluated does not exceed the assigned time).

First of all, the importance of the capability index with the value being equal to 1 will be clarified ($^{d}C_s = 1.00$, or $^{d}C_p = 1.00$). It means that, in the given example, we place exactly 3 standard deviations of the shooting process to the formula $0,5.T - \overline{^{d}r}$ (T represents, for example, the diameter of the largest ring of the pistol target 50/20.

In this case, the probability of the target miss equals to 1-$F(3)$ = 0.00135, where $F(3)$ represents the value of the distribution function of the standardized normal dispersion in three standard deviations.

In other words, the probability that the radius of the hits will be larger than 0.5 T is 0.135 %. This means that only one hit out of one thousand hits (thirteen out of ten thousand, etc.) will not fall within the scoring rings of the 5/20 target.

In general, the procedure of interpretation is as follows: We ask: what multiple of the standard deviation of the radius of the hits may be placed within the space $0,5.T - \overline{^{d}r}$. The symbol u represents such a multiple and it is defined by one of the following formulas:

$$u = 3.{^{d}C_s} \tag{15a}$$

$$u = 3.{^{d}C_p} \tag{15b}$$

The probability of the hit being placed within the assigned part of the target from the distance d equals to the distribution function of the standardized normal dispersion. It is defined by the formula

$$p(^{d}r \leq 0,5T) = F(u) = \int_{-\infty}^{u} f(u)du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{u^2}{2}} du \tag{16}$$

The probability of the target miss will thus be equal to

$$p(^d r > 0{,}5T) = 1 - F(u)$$ (17)

In the case of the interpretation of the Time Shooting Capability Index, we ask: what multiple of the standard deviation of the shooting time span may be placed within the time span $t_{max} - ^d\bar{t}$. The symbol u represents this multiple and it is defined by the following formula:

$$u = 3.^d C_t$$ (15c)

The probability of the shooting carried out from the distance d being finished within the assigned time span t_{max} equals to the distribution function of the standardized normal dispersion:

$$p(^d t \le t_{max}) = F(u)$$ (18)

The probability of exceeding the assigned time span t_{max} (see Table 4) equals:

$$p(^d t > t_{max}) = 1 - F(u)$$ (19)

7. Conclusion

This article presents one way of evaluating of the shooting success. The method may be used while evaluating the competence tests of the Firearm Licence applicants. The method is based on calculation of the Series Shooting Capability Index and Process Shooting Capability Index. These two indices are further complemented by the Time Shooting Capability Index. This method is suitable for the evaluation of individual shooting series as well as for the long-term evaluation of the competence test training. The above described method represents a brand new complex attitude to the evaluation of the shooting process. Such a method has already been used in the industrial field, but it has never been applied to the field of firearms shooting evaluation. The shooting evaluation method (using the three capability indices) widens the range of other, already existing, methods of evaluation. Its viability may only be verified by the actual use of the method.

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