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Coefficient Functions of Hole-Drilling Method

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Abstract: This paper presents the hole-drilling measurement method corresponding to the E 837 standard method, but, at the same time, it is more universal. This method transforms the full stress tensor of the drilled hole position by the regression coefficients and describes the state of strains released in the hole surrounding, based on the hole center distance and its depth. The regress coefficients are not defined in the method concretely for the rosette but they are universal both for the isotropic Hooke's materials and for the other measuring elements. The method defines the way for the processing of the released strains measured with a defined measuring element and involves naturally the influence of the drilled hole eccentricity and so it is possible, in the hole-drilling method, to apply measuring elements more simply, without determining their specified regression coefficients.

Keywords: hole-drilling method, residual stress, regression coefficients

1. Decomposition theory of hole drilling method

The theory of this experimental principle take advantage of the analytical Kirsch's stress-state solution of a thin plate with a hole drilled through perpendicularly and uniaxially loaded by principal stress [1]. The thin plate in Cartesian coordinates x, y, z under the loading by principal stress σ_x is depicted in Fig. 1. On the surface of this plate are defined polar coordinates R, α stresses $\sigma_r, \sigma_\theta, \tau$ and strains ε_r, ε_θ, γ, ε_z .

$$\sigma'(\alpha) = \begin{Bmatrix} \sigma'_r \\ \sigma'_\theta \\ \tau' \end{Bmatrix} = \begin{Bmatrix} \dfrac{\sigma_x}{2}(1+\cos 2\alpha) \\ \dfrac{\sigma_x}{2}(1-\cos 2\alpha) \\ \dfrac{\sigma_x}{2}\sin 2\alpha \end{Bmatrix} \tag{1}$$

We define the relative radius $r = R/R_0 \geq 1$, where R_0 is the hole radius and R is the arbitrary radius from hole center according to [2]. If a thin plate is (without a drilled hole) loaded by the principal stress σ_x then stress state components $\sigma'_r, \sigma'_\theta, \tau'$ are described in Eqs. 1 in the polar coordinates R, α. The Kirsch's equations (Eqs. 2) describe the state of plane strain in the vicinity of the through hole of radius R_0 (Fig. 1).

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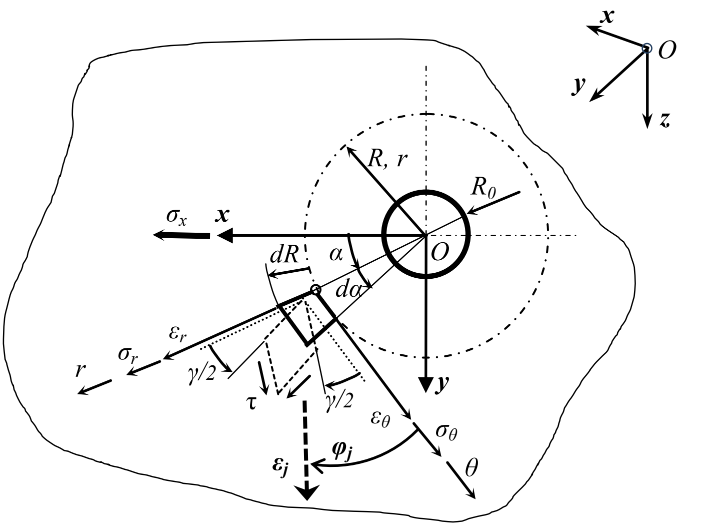

Fig. 1. Components of the stress tensor and strain tensor in the drilled hole vicinity.

$$\sigma''(r,\alpha) = \begin{Bmatrix} \sigma''_r \\ \sigma''_\theta \\ \tau'' \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{Bmatrix} \tag{2}$$

$$\sigma''(r,\alpha) - \sigma'(\alpha) = \begin{Bmatrix} \frac{\sigma_x}{2}(-\frac{1}{r^2}) + \frac{\sigma_x}{2}(\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \frac{\sigma_x}{2}(\frac{1}{r^2}) - \frac{\sigma_x}{2}(\frac{3}{r^4})\cos 2\alpha \\ \frac{\sigma_x}{2}(-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{Bmatrix} \tag{3}$$

The change of straining induced by the hole drilling in comparison to the original state is defined by the difference of corresponding components of Eqs. (1) and (2) in Eqs. (3). In comparison with Eqs. (1), the Eqs. (2) include terms dependent on the drilled hole, which are left in the Eqs. (3) that are otherwise of a character similar to Eqs. (1) and (2).

The Hole drilling strain-gage method used for the residual stress state identification is currently standardized by the E 837 international standard [3]. This hole drilling method theory is based on two parameters adjusted for particular designs of drilling rosettes and requires very accurate the experimental hole drilling. It is valid for isotropic Hooke's materials with a known strain response to the drilling of the hole. The response is measured by strain gauges assembled to a drilling rosette. The response function is similar to strains identified in the Kirsch's solution of the thin plate with a hole as described in Eq. (3). The measurement properties of the rosettes during the hole drilling according to E 837 standard are considerably dependent on the accuracy of compliance with standardized conditions of the experiment. The drilled experimental holes are often eccentric with respect to the ideal position, which the standard theory assumes to be situated in the drilling rosette center.

We expect, that the components of strain in the surroundings of the blind drilled hole as written in Eqs. (4) are analogous to Eqs. (3) of the straight-through hole. Let for the blind hole we also modify [4] all the seven polytropic terms of the complete Kirsch's theory by seven parameters $c_k\,(r,\,z)$, which are dependent on the distance from the center of the drilled hole. The distance is described by the relative radius r and the depth z of the drilled hole. Regression coefficients $c_1,\ldots c_7$ can be determined via regression of the results yield by FEM by the experiment test flat plate as depicted in Fig.1. The specimen is loaded with a unidirectional principal stress σ_x collinear with the plate axis of symmetry. Further, the same specimen with a drilled hole produced by a drilling process is modelled by FEM as well, where the hole is normal to the test surface at the plate axis of symmetry.

$$\sigma(r,\alpha,z)=\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \tau\end{Bmatrix}=\begin{Bmatrix}\dfrac{\sigma_x}{2}(-\dfrac{1\cdot c_1(r,z)}{r^2})+\dfrac{\sigma_x}{2}(\dfrac{3\cdot c_2(r,z)}{r^4}-\dfrac{4\cdot c_3(r,z)}{r^2})\cos 2\alpha\\ \dfrac{\sigma_x}{2}(\dfrac{1\cdot c_4(r,z)}{r^2})-\dfrac{\sigma_x}{2}(\dfrac{3\cdot c_5(r,z)}{r^4})\cos 2\alpha\\ \dfrac{\sigma_x}{2}(-\dfrac{3\cdot c_6(r,z)}{r^4}+\dfrac{2\cdot c_7(r,z)}{r^2})\sin 2\alpha\end{Bmatrix} \tag{4}$$

$$H(E,\nu)=\frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\\ -\nu & -\nu & 0\end{bmatrix} \tag{5}$$

By the way, a similar approach is also used by E 837 standard primarily for radial strain. If E stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_θ, τ can be used for any isotropic material for a calculation of changes related to strains ε_r, ε_θ, γ and ε_z (see Fig. 1) in a point on the plate using the Hooke's law matrix (5) for transformation the stress to the strain Eq. (6). A strain state on planes perpendicular to the surface can be set by an angular transformation, where the use of the first three components ε_r, ε_θ, γ in Eqs. (6) is sufficient, because of the principal strain ε_z does not have any effect on it.

The strain ε_j tangential to the winding direction (see Fig. 1) of the measuring strain gage is derived from ε_r, ε_θ, γ strains according to the transformation Eq. (7) for an acute angle φ_j. Subsequently it is expressed using goniometric functions of a double angle $2\varphi_j$. The latter statement is a consequence of the fact that strain gages primarily measure along the winding tangent. We expect the direction of the principal stress σ_x given by the angular parameter $\overline{\alpha}$ and principal stress σ_y angular parameter $\overline{\alpha}+\pi/2$, both measured from axis x. The bonded strain gauge reads the strain field of the contact surface. Therefore, the deformation under the strain-gauge, at a specified section of its winding, is proportional to the contribution of this winding section into the total signal measured with the strain-gauge. We set a unit vector in the direction of the principal stress σ_x under the $\overline{\alpha}$ in the first case and in the direction of the stress σ_y under angle $\overline{\alpha}+\pi/2$. Relieved strain $\overline{\varepsilon}_j$ is multiplied with a unit dummy load vector introduced in the direction of principal stress and transformed to the winding direction using the strains $\overline{\varepsilon}_r, \overline{\varepsilon}_\theta, \overline{\tau}_g$ of the winding point j. The both considered sensitivities t_i of the i-th strain gauge to the strains relieved during the drilling can be formulated by average strain in the direction of the strain gauge winding according to Eqs. (8). The curvilinear integral of strain along the winding length w_i has an argument including strain $\overline{\varepsilon}_j$.

$$
\left\{
\begin{aligned}
&\varepsilon(r,\alpha,z,E,\nu) = H\cdot\sigma = \begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma\\ \varepsilon_z\end{Bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\\ -\nu & -\nu & 0\end{bmatrix}\cdot\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \tau\end{Bmatrix} = \\
&= \begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma\\ \varepsilon_z\end{Bmatrix} = \frac{\sigma_x}{2E}\left\{
\begin{matrix}
\left\{\left[-\frac{c_1}{r^2}-\frac{c_4}{r^2}\nu\right]+\left[\frac{c_2}{r^4}3+\frac{c_5}{r^4}3\nu-\frac{c_3}{r^2}4\right]\cdot\cos 2\alpha\right\}\\
\left\{\left[+\frac{c_4}{r^2}+\frac{c_1}{r^2}\nu\right]-\left[\frac{c_5}{r^4}3+\frac{c_2}{r^4}3\nu-\frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\\
\left\{\left[-\frac{c_6}{r^4}3+\frac{c_7}{r^2}2-\frac{c_6}{r^4}3\nu+\frac{c_7}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\}\\
\left\{\left[\frac{c_1}{r^2}\nu-\frac{c_4}{r^2}\nu\right]+\left[-\frac{c_2}{r^4}3\nu+\frac{c_5}{r^4}3\nu+\frac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}
\end{matrix}
\right\}
\end{aligned}
\right\}
\tag{6}
$$

$$
\begin{aligned}
&\varepsilon_j(\varphi_j) = \varepsilon_\theta\cos^2(\varphi_j)+\varepsilon_r\cos^2(\pi/2-\varphi_j)+\gamma\sin(\varphi_j)\cdot\cos(\varphi_j) = \varepsilon_\theta\cos^2(\varphi_j)+\\
&+\varepsilon_r\sin^2(\varphi_j)+\gamma\sin(\varphi_j)\cdot\cos(\varphi_j) = \frac{\varepsilon_\theta+\varepsilon_r}{2}+\frac{\varepsilon_\theta-\varepsilon_r}{2}\cos(2\varphi_j)+\frac{\gamma}{2}\sin(2\varphi_j)
\end{aligned}
\tag{7}
$$

$$
t_i(\alpha) = \frac{\oint_{w_i}\overline{\varepsilon}_j(\overline{\alpha})\cdot dw_i}{\oint_{w_i} dw_i}, \quad \text{or} \quad t_i(\alpha+\pi/2) = \frac{\oint_{w_i}\overline{\varepsilon}_j(\overline{\alpha}+\pi/2)\cdot dw_i}{\oint_{w_i} dw_i}
\tag{8}
$$

The angle φ_j (see Fig. 1 and Eq. (7)) is function of the particular position of the winding point and is not a function of the parameter $\overline{\alpha}$. The strains $\overline{\varepsilon}_j, \overline{\varepsilon}_r, \overline{\varepsilon}_\theta, \overline{\tau}_g$ normed by a unit vector are goniometric functions of the particular position of the winding point *j* and parameter $\overline{\alpha}$ defining the position of the unit vector introduced to the direction of the principal stress. A system of at least three independent Eqs. (9) of *i*-th strain gauge signals ε_i read in the vicinity of the drilled hole for unknown principal stresses σ_x, σ_y and the angle of their position $\overline{\alpha}$. A superposition including effects of both principal stresses is done.

$$\varepsilon_i = \sigma_x \cdot t_i(\overline{\alpha}) + \sigma_y \cdot t_i(\overline{\alpha} + \pi/2) \tag{9}$$

2. Functions of regression coefficients

The regress coefficients $c_k(r,z)$ defined in Eqs. (4) are dependent only on the radial distance $r=R/R_0$ from the center *O* of the drilled hole and on the hole depth *z.* They can be therefore identified [5] by substituting Eqs. (4) into Eqs. (10) with the data $\overline{\sigma}_r, \overline{\sigma}_\theta, \overline{\tau}$ employed in the FEM analysis of drilling in *i-th* points of the surface layer of the first hole quadrant mapped with the coordinates *x,y* as depicted in Fig.1.

$$\left\{\begin{array}{l} \sigma_r(\alpha,r,c_1,c_2,c_3) = \dfrac{\sigma_x}{2}\left(-\dfrac{1\cdot c_1(r,z)}{r^2}\right) + \dfrac{\sigma_x}{2}\left(\dfrac{3\cdot c_2(r,z)}{r^4} - \dfrac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha = \overline{\sigma}_r(\alpha,r,z) \\ \sigma_\theta(\alpha,r,c_4,c_5) = \dfrac{\sigma_x}{2}\left(\dfrac{1\cdot c_4(r,z)}{r^2}\right) - \dfrac{\sigma_x}{2}\left(\dfrac{3\cdot c_5(r,z)}{r^4}\right)\cos 2\alpha = \overline{\sigma}_\theta(\alpha,r,z) \\ \tau(\alpha,r,c_6,c_7) = \dfrac{\sigma_x}{2}\left(-\dfrac{3\cdot c_6(r,z)}{r^4} + \dfrac{2\cdot c_7(r,z)}{r^2}\right)\sin 2\alpha = \overline{\tau}(\alpha,r,z) \end{array}\right\} \tag{10}$$

$$\left\{\begin{array}{l} \min\sum_i(\sigma_r - \overline{\sigma}_r)_i^2 = \min\sum_i\left[\dfrac{\sigma_x}{2}\left(-\dfrac{1\cdot c_1(r,z)}{r^2}\right) + \dfrac{\sigma_x}{2}\left(\dfrac{3\cdot c_2(r,z)}{r^4} - \dfrac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha - \overline{\sigma}_r\right]_i^2 = \min F1(c_{1,2,3}) \\ \min\sum_i(\sigma_\theta - \overline{\sigma}_\theta)_i^2 = \min\sum_i\left[\dfrac{\sigma_x}{2}\left(\dfrac{1\cdot c_4(r,z)}{r^2}\right) - \dfrac{\sigma_x}{2}\left(\dfrac{3\cdot c_5(r,z)}{r^4}\right)\cos 2\alpha - \overline{\sigma}_\theta\right]_i^2 = \min F2(c_{4,5}) \\ \min\sum_i(\tau - \overline{\tau})_i^2 = \min\sum_i\left[\dfrac{\sigma_x}{2}\left(-\dfrac{3\cdot c_6(r,z)}{r^4} + \dfrac{2\cdot c_7(r,z)}{r^2}\right)\sin 2\alpha - \overline{\tau}\right]_i^2 = \min F3(c_{6,7}) \end{array}\right\} \tag{11}$$

The region of radii used for identification is assumed according the possible drilling experiment sensitivity in the relative radius interval $1 \leq r \leq 5$ and in the angle coordinate interval $0 \leq \alpha \leq \pi/2$. The possible degrees of freedom, i.e., the unknown coefficients $c_1,\ldots c_7$, can be determined by using the least squares method in Eqs. (11), which minimizes the residual errors between the analytical and numerical methods in the comparative points *i* of the numerical model from Fig. 1. This task can be transformed by the minimization of three independent functionals *F1, F2* and *F3*, yielding the seven linear equation system in the form of Eqs. (12). The conditions for the minimization of the functionals *F1, F2* and *F3* can be separated into the three independent linear equation systems as stated in Eqs. (13a,b,c). The first three equations follow from *F1*, other two from *F2* and, finally, the sixth and seventh equations from *F3*. The unknown coefficients $c_1,\ldots c_7$ can be determined from many possible initial point combinations, which can be selected from many various angle coordinates α in the interval $0<\alpha_k<\pi/2$ for concrete depth z.

$$\left\{\begin{aligned}
&\frac{\partial}{\partial c_1}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot c_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\left(-\frac{\sigma_x}{2r^2}\right)\right]_i=0\\
&\frac{\partial}{\partial c_2}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot c_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\left(\frac{3\sigma_x}{2r^4}\cos 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial c_3}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot c_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3\cdot c_2(r,z)}{r^4}-\frac{4\cdot c_3(r,z)}{r^2}\right)\cos 2\alpha-\bar{\sigma}_r\right\}\cdot\left(-\frac{2\sigma_x}{r^2}\cos 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial c_4}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot c_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot c_5(r,z)}{r^4}\right)\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(\frac{\sigma_x}{2r^2}\right)\right]_i=0\\
&\frac{\partial}{\partial c_5}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot c_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot c_5(r,z)}{r^4}\right)\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(-\frac{3\sigma_x}{2r^4}\cos 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial c_6}F3=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{3\cdot c_6(r,z)}{r^4}+\frac{2\cdot c_7(r,z)}{r^2}\right)\sin 2\alpha-\bar{\tau}\right\}\cdot\left(-\frac{3\sigma_x}{2r^4}\sin 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial c_7}F3=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{3\cdot c_6(r,z)}{r^4}+\frac{2\cdot c_7(r,z)}{r^2}\right)\sin 2\alpha-\bar{\tau}\right\}\cdot\left(\frac{\sigma_x}{r^2}\sin 2\alpha\right)\right]_i=0
\end{aligned}\right\} \tag{12}$$

$$\begin{bmatrix}
\sum_i -\frac{1}{r^4} & \sum_i \frac{3\cos 2\alpha}{r^6} & -\sum_i \frac{4\cos 2\alpha}{r^4}\\
\sum_i -\frac{\cos 2\alpha}{r^6} & \sum_i \frac{3\cos^2 2\alpha}{r^8} & -\sum_i \frac{4\cos^2 2\alpha}{r^6}\\
\sum_i -\frac{\cos 2\alpha}{r^4} & \sum_i \frac{3\cos^2 2\alpha}{r^6} & -\sum_i \frac{4\cos^2 2\alpha}{r^4}
\end{bmatrix}\cdot\begin{Bmatrix}c_1\\c_2\\c_3\end{Bmatrix}=\frac{2}{\sigma_x}\cdot\begin{Bmatrix}\sum_i \frac{\bar{\sigma}_r}{r^2}\\ \sum_i \frac{\bar{\sigma}_r}{r^4}\cos 2\alpha)\\ \sum_i \frac{\bar{\sigma}_r}{r^2}\cos 2\alpha)\end{Bmatrix} \tag{13a}$$

$$\begin{bmatrix}
\sum_i \frac{1}{r^4} & \sum_i -\frac{3\cos 2\alpha}{r^6}\\
\sum_i \frac{\cos 2\alpha}{r^6} & \sum_i -\frac{3\cos^2 2\alpha}{r^8}
\end{bmatrix}\cdot\begin{Bmatrix}c_4\\c_5\end{Bmatrix}=\frac{2}{\sigma_x}\cdot\begin{Bmatrix}\sum_i \frac{\bar{\sigma}_\theta}{r^2}\\ \sum_i \frac{\bar{\sigma}_\theta}{r^4}\cos 2\alpha\end{Bmatrix} \tag{13b}$$

$$\begin{bmatrix}
\sum_i -\frac{3\sin^2 2\alpha}{r^8} & \sum_i \frac{2\sin^2 2\alpha}{r^6}\\
\sum_i -\frac{3\sin^2 2\alpha}{r^6} & \sum_i \frac{2\sin^2 2\alpha}{r^4}
\end{bmatrix}\cdot\begin{Bmatrix}c_6\\c_7\end{Bmatrix}=\frac{2}{\sigma_x}\cdot\begin{Bmatrix}\sum_i \frac{\bar{\tau}}{r^4}\sin 2\alpha\\ \sum_i \frac{\bar{\tau}}{r^2}\sin 2\alpha\end{Bmatrix} \tag{13c}$$

The regression model, introduced in Eqs. (10), contains in the functionals F1 and F3 the pairs of coefficients c2, c3 and c6, c7, respectively, which virtually introduce a linear dependence into the equation system, when using data measured on one particular radius. In order to obtain a non-singular equation system, it is required to use equations assembled at several radii. The number of equations increases linearly with the number of different radii used which smoothes the coefficient functions. For a certain drill hole depth ***z***, we approximate the coefficients values calculated from individual stripes by the coefficient regression functions, as illustrated in Fig. 2. These are further used in the mathematical description of the method. The regression coefficients $c_{1\text{-}7}$ cannot be calculated when using solely data obtained at a single radius because the model structure would generate a linearly dependant equation system. But it also implies that the model can be simplified by the replacement of c_2, c_3 and c_6, c_7 coefficient couples by two new coefficients $\bar{c}_2, \bar{c}_6$. The equation system using the $\bar{c}_1, \bar{c}_2, \bar{c}_4, \bar{c}_5, \bar{c}_6$ is introduced in Eq. (14). This equation system can be assembled by using data from a single or multiple radii.

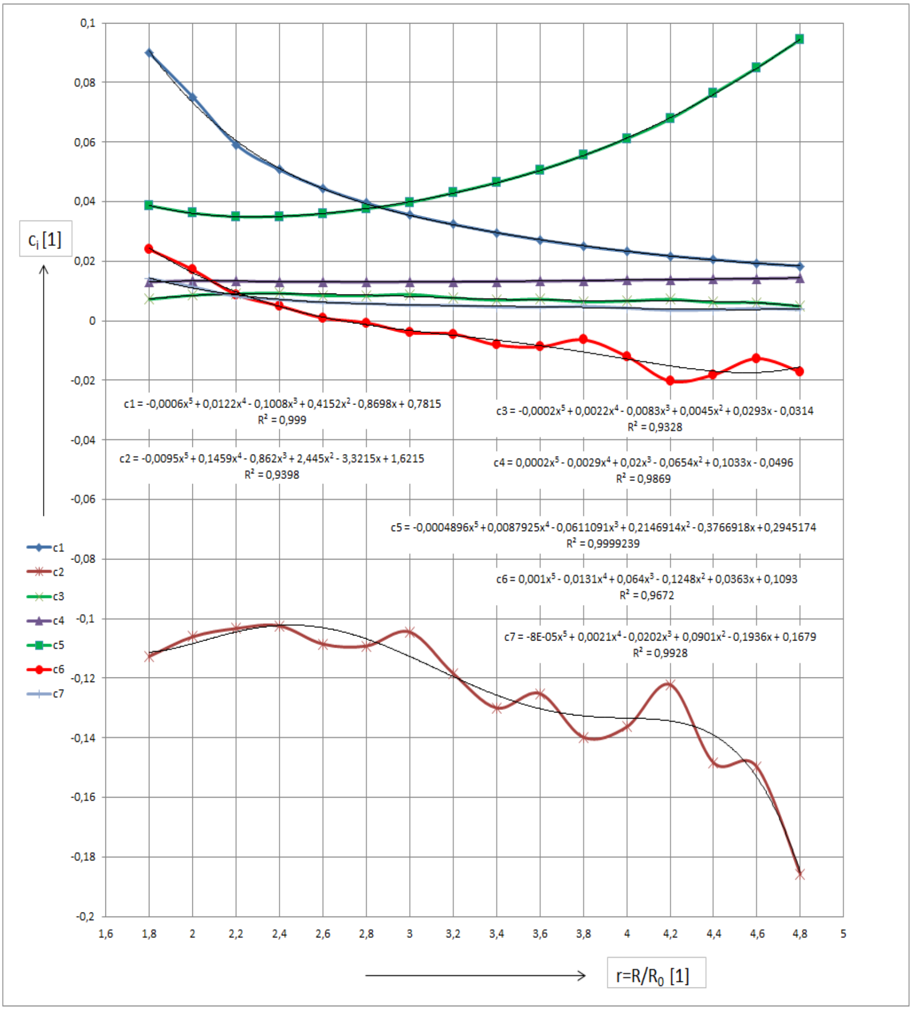

Fig. 2. Regression coefficients functions c_1, c_2, c_3, c_4, c_5, c_6, c_7 for hole depth z/R_0=0,05.

$$\left\{\begin{aligned}
&\sigma_r(\alpha,r,\bar{c}_1,\bar{c}_2)=\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha=\bar{\sigma}_r(\alpha,r,z)\\
&\sigma_\theta(\alpha,r,\bar{c}_4,\bar{c}_5)=\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha=\bar{\sigma}_\theta(\alpha,r,z)\\
&\tau(\alpha,r,\bar{c}_6)=\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha=\bar{\tau}(\alpha,r,z)
\end{aligned}\right\} \tag{14}$$

The regression coefficients $\bar{c}_1,\bar{c}_2,\bar{c}_4,\bar{c}_5,\bar{c}_6$ of the reduced model can be obtained analogously by searching for coefficients of the full model. By using the least squares method in Eqs. (15), which minimizes the residual errors between the analytical and numerical methods in the comparative points *i* of the numerical model from Fig. 1. This task can be transformed by the minimization of three independent functionals *F1, F2* and *F3*, yielding the seven linear equation system in the form of Eqs. (16). The conditions for the minimization of the functionals *F1, F2* and *F3* can be separated into the three independent linear equation systems as stated in Eqs. (17a,b,c). The approximated regression coefficients $\bar{c}_1,\bar{c}_2,\bar{c}_4,\bar{c}_5,\bar{c}_6$ by polynomial regression functions, as calculated from the individual radius data, are presented in Fig. 3.

$$\left\{\begin{aligned}
&\min\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right]_i^2=\min F1(\bar{c}_1,\bar{c}_2)\\
&\min\sum_i(\sigma_\theta-\bar{\sigma}_\theta)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta(\alpha,r,z)\right]_i^2=\min F2(c_4,c_5)\\
&\min\sum_i(\tau-\bar{\tau})_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha-\bar{\tau}(\alpha,r,z)\right]_i^2=\min F3(\bar{c}_6)
\end{aligned}\right\} \tag{15}$$

$$\left\{\begin{aligned}
&\frac{\partial}{\partial\bar{c}_1}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right\}\cdot\left(-\frac{\sigma_x}{2r^2}\right)\right]_i=0\\
&\frac{\partial}{\partial\bar{c}_2}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right\}\cdot\left(\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cos 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial\bar{c}_4}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(\frac{\sigma_x}{2r^2}\right)\right]_i=0\\
&\frac{\partial}{\partial\bar{c}_5}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(-\frac{3\sigma_x}{2r^4}\cos 2\alpha\right)\right]_i=0\\
&\frac{\partial}{\partial\bar{c}_6}F3=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha-\bar{\tau}(\alpha,r,z)\right\}\cdot\left(\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\sin 2\alpha\right)\right]_i=0
\end{aligned}\right\} \tag{16}$$

$$\begin{bmatrix}
\sum\limits_i-\frac{1}{r^4} & \sum\limits_i\left(\frac{3}{r^6}-\frac{4}{r^4}\right)\cos 2\alpha\\
\sum\limits_i-\left(\frac{3}{r^6}-\frac{4}{r^4}\right)\cos 2\alpha & \sum\limits_i\left(\frac{9}{r^8}-\frac{24}{r^6}+\frac{16}{r^4}\right)\cos^2 2\alpha
\end{bmatrix}\cdot\begin{Bmatrix}\bar{c}_1\\ \bar{c}_2\end{Bmatrix}=\frac{2}{\sigma_x}\cdot\begin{Bmatrix}
\sum\limits_i\frac{\bar{\sigma}_r(\alpha,r,z)}{r^2}\\
\sum\limits_i\bar{\sigma}_r(\alpha,r,z)\cdot\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cos 2\alpha
\end{Bmatrix} \tag{17a}$$

$$\begin{bmatrix} \sum\limits_i \frac{1}{r^4} & \sum\limits_i -\frac{3\cos 2\alpha}{r^6} \\ \sum\limits_i \frac{\cos 2\alpha}{r^6} & \sum\limits_i -\frac{3\cos^2 2\alpha}{r^8} \end{bmatrix} \cdot \begin{Bmatrix} \bar{c}_4 \\ \bar{c}_5 \end{Bmatrix} = \frac{2}{\sigma_x} \cdot \begin{Bmatrix} \sum\limits_i \frac{\bar{\sigma}_\theta(\alpha, r, z)}{r^2} \\ \sum\limits_i \frac{\bar{\sigma}_\theta(\alpha, r, z)}{r^4} \cos 2\alpha \end{Bmatrix} \tag{17b}$$

$$\bar{c}_6 \cdot \sum_i \left(\frac{9}{r^8} - \frac{12}{r^6} + \frac{4}{r^4}\right) \sin^2 2\alpha = \frac{2}{\sigma_x} \sum_i \bar{\tau}(\alpha, r, z) \cdot \left(-\frac{3}{r^4} + \frac{2}{r^2}\right) \sin 2\alpha \tag{17c}$$

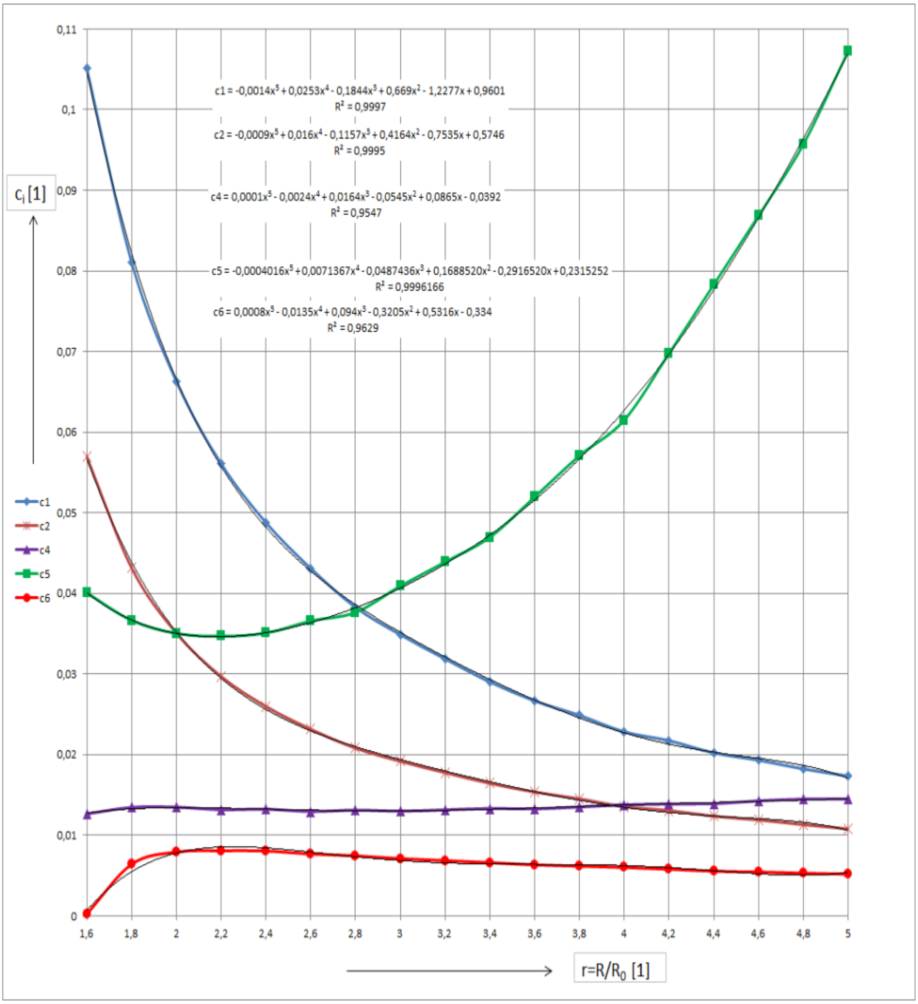

Fig. 3. Regression coefficients functions $\bar{c}_1, \bar{c}_2, \bar{c}_4, \bar{c}_5, \bar{c}_6$ for hole depth *z/R₀=0,05*.

3. Conclusions

The coefficient values $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ obtained from various radii data combinations move in a restricted range and do not change significantly. The reduced coefficients $\bar{c}_1, \bar{c}_2, \bar{c}_4, \bar{c}_5, \bar{c}_6$ are sufficient for the formulation of the stress tensor and for the formulation of this method. The results for a single radius can be also alternatively obtained by solving system of the further simplified equations, see Eqs. (18a,b,c).

$$\begin{bmatrix} \sum\limits_i -\frac{1}{r^2} & \sum\limits_i (\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sum\limits_i -(\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha & \sum\limits_i (\frac{9}{r^6}-\frac{24}{r^4}+\frac{16}{r^2})\cos^2 2\alpha \end{bmatrix} \cdot \begin{Bmatrix} \bar{c}_1 \\ \bar{c}_2 \end{Bmatrix} = \frac{2}{\sigma_x} \cdot \begin{Bmatrix} \sum\limits_i \bar{\sigma}_r(\alpha, r, z \\ \sum\limits_i \bar{\sigma}_r(\alpha, r, z) \cdot (\frac{3}{r^2} - 4)\cos 2\alpha \end{Bmatrix} \quad (18a)$$

$$\begin{bmatrix} \sum\limits_i \frac{1}{r^2} & \sum\limits_i -\frac{3\cos 2\alpha}{r^4} \\ \sum\limits_i \frac{\cos 2\alpha}{r^2} & \sum\limits_i -\frac{3\cos^2 2\alpha}{r^4} \end{bmatrix} \cdot \begin{Bmatrix} \bar{c}_4 \\ \bar{c}_5 \end{Bmatrix} = \frac{2}{\sigma_x} \cdot \begin{Bmatrix} \sum\limits_i \bar{\sigma}_\theta(\alpha, r, z) \\ \sum\limits_i \bar{\sigma}_\theta(\alpha, r, z) \cdot \cos 2\alpha \end{Bmatrix} \quad (18b)$$

$$\bar{c}_6 \cdot \sum_i (\frac{9}{r^6} - \frac{12}{r^4} + \frac{4}{r^2}) \sin^2 2\alpha = \frac{2}{\sigma_x} \sum_i \bar{\tau}(\alpha, r, z) \cdot (-\frac{3}{r^2} + 2) \sin 2\alpha \quad (18c)$$

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