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Influence of imperfections on the stress state identification accuracy assessment using the "Hole-drilling strain-gauge method"

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Abstract. The hole drilling method used for the stress state identification is currently standardized by the E 837 international standard. It is based on relaxation of the residual stress in vicinity of the drilled hole. Relaxation of the residual stress relieves deformations which are measured with strain gage rosette. However, using the theory E 837 does not correct the influence of the experimental hole imperfections (eccentricity, roundness, perpendicularity) on this experimental method accuracy. This article analyzes the extent of errors that affect these imperfections.

Introduction

The theory of this experimental principle take advantage of the analytical Kirsch's stress-state solution of a thin plate uniaxially loaded by principal stress [2] with a through perpendicularly drilled hole. This thin plate is drawn in Cartesian coordinates *x, y ,z* under the loading with principal stress $\sigma_{x,}$ see Fig. 1. There are defined the polar coordinates *R, α,* the stresses $\sigma_r, \sigma_\theta, \tau$ and the strains ε_r, ε_θ, γ, $\varepsilon_{z,}$ on the surface of this plate.

$$\sigma'(\alpha) = \begin{Bmatrix} \sigma'_r \\ \sigma'_\theta \\ \tau' \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \frac{\sigma_x}{2}\sin 2\alpha \end{Bmatrix} \quad (1) \qquad \sigma''(r,\alpha) = \begin{Bmatrix} \sigma''_r \\ \sigma''_\theta \\ \tau'' \end{Bmatrix} = \begin{Bmatrix} \frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{Bmatrix} \quad (2)$$

A relative radius $r = R/R_0 \geq 1$ is defined, where R_0 is the hole radius and *R* is the arbitrary radius from hole center according to [3]. If a thin plate is (without a drilled hole) loaded by the principal stress $\sigma_{x,}$ then stress state components $\sigma'_r, \sigma'_\theta, \tau'$ are described in Eqs. 1 in the polar coordinates *R, α*. The Kirsch's equations (Eqs. 2) describe the state of plane strain in the vicinity of the through drilled hole of the radius R_0 (Fig. 1).

The change of straining induced by the hole drilling in comparison to the original state is defined by the difference of corresponding components of Eqs. (1) and (2) in Eqs. (3). In comparison with Eqs. (1), the Eqs. (2) include the terms dependent on the drilled hole, which are situated left in the Eqs. (3), that are otherwise of a character similar to Eqs. (1) and (2).

If *E* stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_θ, τ can be used for any isotropic material to calculate the changes related to strains ε_r, ε_θ, γ and ε_z (see Fig. 1) in a point on the plate by using the Hooke's law for transforming the stress to the strain Eq. (4). The strain state in the planes perpendicular to the surface can be assessed by an angular transformation. The radial strain is formulated by cosine functions in Eg. (5). The radial strain is formulated by cosine functions in Eg. (5). Using superposition principal stresses by Eg. (5) is generated of radial strain gauge signal.

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{Bmatrix} = \sigma''(r,\alpha) - \sigma'(\alpha) = \begin{Bmatrix} \frac{\sigma_x}{2}(-\frac{1}{r^2}) + \frac{\sigma_x}{2}(\frac{3}{r^4} - \frac{4}{r^2})\cos 2\alpha \\ \frac{\sigma_x}{2}(\frac{1}{r^2}) - \frac{\sigma_x}{2}(\frac{3}{r^4})\cos 2\alpha \\ \frac{\sigma_x}{2}(-\frac{3}{r^4} + \frac{2}{r^2})\sin 2\alpha \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \\ \varepsilon_z \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{Bmatrix} \quad (4)$$

$$\varepsilon_r = \frac{\sigma_x \cdot (1+\nu)}{2E} \cdot \left[-\frac{1}{r^2} + \left(\frac{3}{r^4} - \frac{4}{(1+\nu) \cdot r^2} \right) \cos 2\alpha \right] \cong A + B \cdot \cos 2\alpha \quad (5)$$

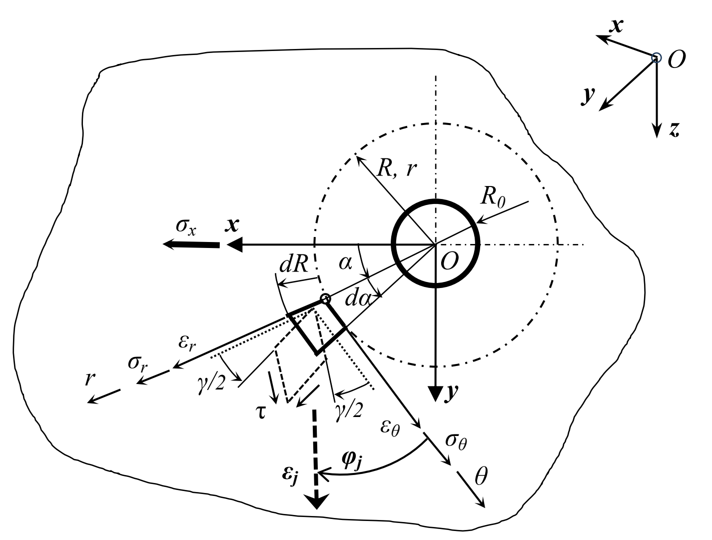

Fig. 1. Components of the stress and strain tensors in the drilled hole vicinity.

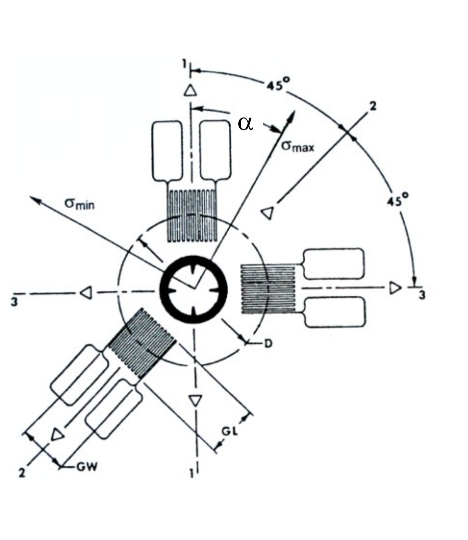

Fig. 2. Position of the radial strain gages in the rosette

Influence of imperfections

The unknown principal stresses σ_x, σ_y and their position given with angle α are assessed using the system of at least three independent Eqs. (6), containing the i-th radial strain gauge signals ε_{ri}, read in the vicinity of the drilled hole. The superposition, including the effects of both principal stresses, is done. The system of Eqs. (6) solution, for the radial hole drilling rosette, is indicated at Eqs. (7), using [3].

$$\left. \begin{aligned} \varepsilon_{r1}(e,o,p) &= A \cdot (\sigma_x + \sigma_y) + B \cdot (\sigma_x - \sigma_y) \cdot \cos 2\alpha \\ \varepsilon_{r2}(e,o,p) &= A \cdot (\sigma_x + \sigma_y) + B \cdot (\sigma_x - \sigma_y) \cdot \cos 2(\alpha + \pi/4) \\ \varepsilon_{r3}(e,o,p) &= A \cdot (\sigma_x + \sigma_y) + B \cdot (\sigma_x - \sigma_y) \cdot \cos 2(\alpha + \pi/2) \end{aligned} \right\} \quad (6)$$

The hole drilling strain-gage method, used for the residual stress state identification, is currently standardized by the E 837 international standard [1]. This hole drilling method theory is based on two parameters adjusted for particular designs of drilling rosettes and requires a very accurate experimental hole drilling. It is valid for isotropic Hooke's materials with a known strain response to the drilling of the hole. The response is measured by strain gauges assembled to a drilling rosette. The response function is similar to the strains identified in the Kirsch's solution of the thin plate with a hole as described in Eqs. (3, 4) or Eq. (5), when using measuring drilling rosette of Fig. 2 with radially oriented axes of strain gauges numbered 1, 2, 3. The measurement properties of the rosettes during the hole drilling according to E 837 standard are considerably dependent on the accuracy of compliance with standardized conditions of the experiment. The standard theory assumes a centric circular hole drilled perpendicularly to the surface. Experimental holes, however, are also eccentric (e), with ovality (o), or being inaccurately perpendicular (p) to the plate, so these imperfections cause errors in the identification of the stress state.

$$\left.\begin{aligned} \sigma_{\max}(e,o,p) &= \frac{\varepsilon_{r1}+\varepsilon_{r3}}{4A} - \frac{1}{4B}\sqrt{(\varepsilon_{r3}-\varepsilon_{r1})^2+(\varepsilon_{r3}+\varepsilon_{r1}-2\varepsilon_{r2})^2} \\ \sigma_{\min}(e,o,p) &= \frac{\varepsilon_{r1}+\varepsilon_{r3}}{4A} + \frac{1}{4B}\sqrt{(\varepsilon_{r3}-\varepsilon_{r1})^2+(\varepsilon_{r3}+\varepsilon_{r1}-2\varepsilon_{r2})^2} \\ \tan 2\alpha(e,o,p) &= \frac{\varepsilon_{r1}-2\varepsilon_{r2}+\varepsilon_{r3}}{\varepsilon_{r1}-\varepsilon_{r3}} \end{aligned}\right\} \qquad (7)$$

$$\Delta\sigma_{\max}[\varepsilon_{r1}(e,o,p),\varepsilon_{r2}(e,o,p),\varepsilon_{r3}(e,o,p)] = \frac{\partial(\sigma_{\max}(e,o,p))}{\partial e}\cdot de + \frac{\partial(\sigma_{\max}(e,o,p))}{\partial o}\cdot do + \frac{\partial(\sigma_{\max}(e,o,p))}{\partial p}\cdot dp \cong$$
$$\cong [\sigma_{\max}(e)-\sigma_{\max}] + [\sigma_{\max}(o)-\sigma_{\max}] + [\sigma_{\max}(p)-\sigma_{\max}] = \sigma_{\max}(e)+\sigma_{\max}(o)+\sigma_{\max}(p)-3\sigma_{\max} \qquad (8)$$

$$\Delta\sigma_{\min}[\varepsilon_{r1}(e,o,p),\varepsilon_{r2}(e,o,p),\varepsilon_{r3}(e,o,p)] \cong \sigma_{\min}(e)+\sigma_{\min}(o)+\sigma_{\min}(p)-3\sigma_{\min} \rightarrow \text{analogously - Eg. (8)} \qquad (9)$$

$$r\Delta\sigma_{\max}(e,o,p) = \frac{\Delta\sigma_{\max}(e,o,p)}{\sigma_{\max}}\cdot 100\,\% \qquad r\Delta\sigma_{\min}(e,o,p) = \frac{\Delta\sigma_{\min}(e,o,p)}{\sigma_{\min}}\cdot 100\,\% \qquad (10)$$

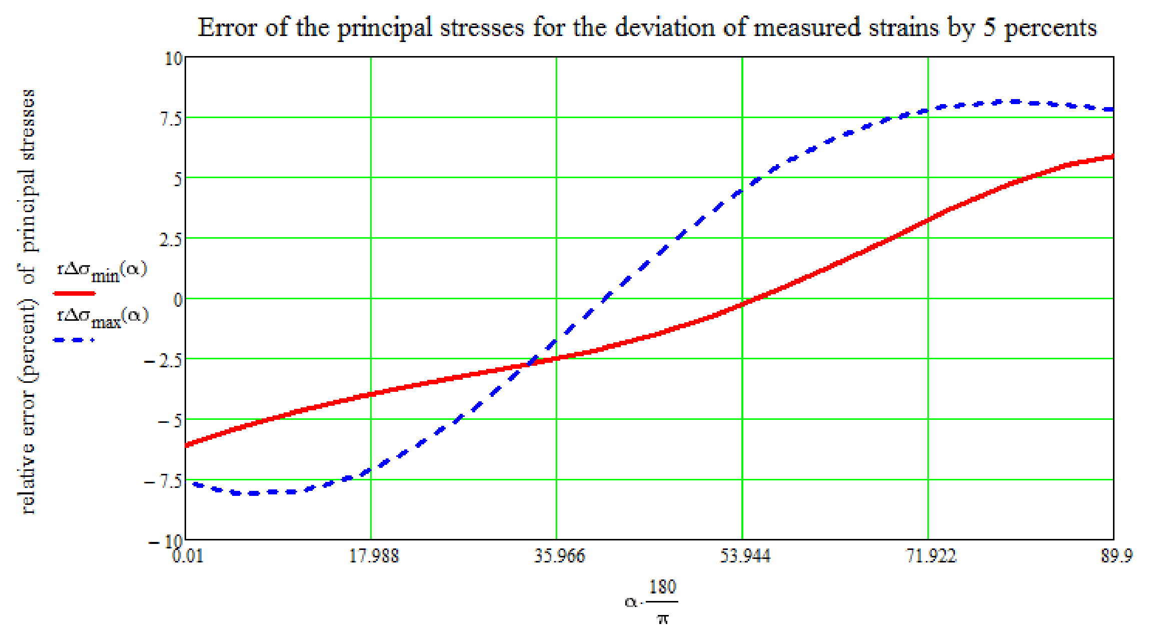

Fig.3 the angle alfa (degree) of principal stress - according to Fig.2

These drilled hole imperfections cause strain deviations, Eqs. (6) and, subsequently, also the errors of the principal stress identification Eqs. (7). The error formulations of the two principal stresses, based on the total differential principle, are introduced with the Eqs. (8,9) (absolutely) or with Eqs. (10) (relatively). The principal stress partial changes are here assessed numerically for a hole, having a concrete imperfection and an ideally precise hole, using the Eqs. (7, 8). Apart from the imperfection sizes, the imperfections of the identified principal stress inaccuracies depend on the angle a, that describes the strain gauge positions with respect to the principal directions according the Fig. 2 and also on the individual strain-gauge error combinations. It follows from the numerical experiments that the principal stress errors can be up to about 1.5 fold of the maximum strain errors of the drilled rosette strain-gauges, see Fig. 3. To the study of the hole drilling strain-gage method, standardized by the E 837 international standard, there are dedicated the diploma thesis [4, 5, 6] and the paper [8]. The papers [7, 9, 10] are aimed at the experimental method developments that are based on an universal mathematical model and is insensible on the negative influence of the experimental drilled hole eccentricity.

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