

# An Influence of Combined Mean Stresses on Lifetime under High-cycle Fatigue

JANOUŠEK Jaromír<sup>1,a</sup>, BALDA Miroslav<sup>2,b</sup> and CHOCHOLOUŠEK Michal<sup>1,c</sup>

<sup>1</sup>Research Centre Rez, Hlavní 130, 250 68 Husinec-Řež, Czech Republic
<sup>2</sup>Research and Testing Institute Plzen, Tylova 46, 301 00 Plzeň, Czech Republic
<sup>a</sup>jas@cvrez.cz, <sup>b</sup>balda@vzuplzen.cz, <sup>c</sup>cho@cvrez.cz

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Abstract. This study builds on experiments with different combinations of tension and torsion pre-stresses which were published in [1]. The results were particularized by new tests for combined pre-stresses in normal and shear components. The ratio between normal and shear pre-stresses was 3:2 in agreement with the coefficient  $k_c$ . The same material structural low carbon ČSN 41 1523.1 steel after normalization annealing was used for specimens. The results were compiled and displayed in a three-dimensional Haigh diagram with normalized coordinates and interlaid by corresponding terminal lines.

## Introduction

There is no doubting the importance of mean value of stress during a damage process. It changes the value of fatigue limit  $s_c$  and corresponding fatigue life (cycle count on fatigue limit)  $N_c$  [1]. In spite of that fact, the investigation of an influence of stress mean values is very expensive and time consuming because every point in the Haigh diagram represents one S-N curve. If a S-N curve should be determined according to standard, it is necessary to use at least 10 specimens. [2] The situation is more complicated for values in both stress components of combined loads because it belongs among unsolved problems due to insufficient information on material behaviour for this loading type.

At zero mean stress, the allowable stress amplitude is the effective fatigue limit for a specified fatigue life. As the mean stress increases, the permissible stress amplitude decreases. The allowable cyclic amplitude is zero at a mean stress equal to the ultimate tensile strength limit of the material. [3] If the mean value is higher, the damage produced by maximal stresses in a cycle would be higher too [4], and therefore the positive mean stress decreases the fatigue limit. The lifetime should prolong in the case of negative normal pre-stress because of its closing effect on the crackfront. This hypothesis was published in [5] without an experimental confirmation.

## State of the Art

The following analytical function seems as a good approximation in certain occurrences for regression of the Haigh diagram:

$$s_{cm} = s_c \left[ 1 - \frac{s_m}{s_F} \right]^{k_H}, \tag{1}$$

where  $s_F$  is so called fictive stress and coefficient  $k_H$  is used to determine experimentally. For  $k_H = 1$ , the previous function becomes a straight line that goes through the points  $(0, s_c)$  and  $(s_F, 0)$ , where  $s_F$  is often defined as strength limit  $R_m$ . If the safe area of tenseness could be obtained, it would be necessary to accomplish intersection of this line with line  $s_m + s_a = R_e$ , where the amplitude is done as  $s_a = (s_{max} - s_{min})/2$ . The constructions proposed in this way should not show any fatigue failure.

The possibilities how to apply loading as a combination of tension and torsion are based on the general equation for complex damaging stress

$$\sigma_d(t) = \sigma_m + ik_c \tau_m + \sigma_a \sin(2\pi f_z t) + ik_c \tau_a \sin(2\pi f_z t + \varphi), \tag{2}$$

where the constant  $k_c$  is a ratio of fatigue limits in tension-pressure and in shear without mean values,  $\sigma_m$  and  $\tau_m$  are normal and shear mean values,  $\sigma_a$  and  $\tau_a$  are normal and shear amplitudes, t is time of loading and  $f_z$  is loading frequency and  $\varphi$  is a phase angle. The direction of the crack propagation is controlled only by the character of the stress component [6]. The static component has an influence on crack propagation speed because in the case of positive pre-stress crack front opens and it affects the overall life.

#### **Experimental Program**

The experiments were carried out on an electrohydraulic computer-controlled testing machine Inova ZUZ 200-1. The specimens were tubular with 30 mm diameter, wall thickness 2 mm and a notch in the shape of one-sided transverse hole of diameter 3 mm positioned in the middle of the specimen. The specimens were made of structural low carbon ČSN 41 1523.1 steel after normalization annealing. The American equivalent of this material type is ASTM A623 [7].

The results published in [8] provide information on sinusoidal in-phase stress components and tenseness distribution. The conception of experiments was to find out a biaxial fatigue limit with static pre-stress in both stress components and with one dynamical sinusoidal component with frequency 10 Hz.

This study builds on experiments that have been performed at the workplace of the Institute of Thermomechanics in Pilsen. The step test method similar to [9] was used for the evaluation of the fatigue limit with a 90 % band of reliability. The value of fatigue limit in tension-pressure was experimentally established as value 120 MPa and 80 MPa in torsion. The ratio of these values gives the coefficient  $k_c = 1.5$  that informs on damage effect distribution between normal and shear components. The mean values in normal and shear components were proposed in the same ratio. The stress mean values  $\sigma_m = 150$  MPa and  $\tau_m = 100$  MPa were chosen for the first tests which provided the level of fatigue limit on 85 MPa. Another experiment with mean values  $\sigma_m = 200$  MPa and  $\tau_m = 133.3$  MPa was performed at the level of fatigue limit about magnitude 45 MPa.

A new experiment with stress mean values  $\sigma_m = 90$  MPa and  $\tau_m = 60$  MPa provided the level of fatigue limit on 90 MPa from 5 specimens. Cycle count on fatigue limit (breaking point from slant to horizontal branch in Fig. 1) corresponds to 4 201 458. The shape of straight line equation of slanted branch for combined pre-stress is defined by an equation that was found by probabilistic approach as

 $\log N_a = 20.9795 - 7.3461 \log \sigma_a$ .

If the slanted branches of all three curves with pre-stresses are compared, it can be found out that the directions are considerably different in spite of the fact that herein [10] the results are mentioned on the basis of the hypothesis that the direction is a material constant.



Fig. 1. S-N curve for low carbon steel ČSN 41 1523.1 with applied combined pre-stress in normal component  $\sigma_m = 90$  MPa and in shear component  $\tau_m = 60$  MPa.

### **Evaluation of Results**

Results from the previous experiments published in [1, 6] were used for the complementation of the three-dimensional Haigh diagram which is displayed in Fig. 2. All coordinates are normalized therefore it was necessary to find only one parameter  $k_H$  of the regression curve. The fitting curve coefficients were obtained by a Fletcher's version of the Levenberg-Maquardt algorithm for the minimization of a sum of squares of equation residuals. The program was created in numerical computing environment and programming language MATLAB. The vector of residuals was defined as an anonymous function in the program. The guess  $p_0$  of a number 0.5 was selected as starting solution. The final result was obtained in three iterations as a parameter p multiplied by starting guess  $p_0$ . The final value for coefficient  $k_H$  of curve in the Haigh diagram for points of combined pre-stress components was computed as 1.0585.



Fig. 2. Haigh diagrams for shear and tensile mean stresses and for their combination in ratio of  $3\sigma_m$  to  $2\tau_m$  that are plotted in normalized coordinates in the three-dimensional graph.

## Conclusions

The results from the field of uniaxial and multiaxial pre-stresses were summarized in this paper. Structural low carbon ČSN 41 1523.1 steel after normalization annealing was used for specimens. A three-dimensional graph was constructed and all points of Haigh diagrams were interlaid by terminal lines in the form of an analytical expression of equation (1). Terminal lines were determined by using the Fletcher's version of the Levenberg-Maquardt algorithm for the minimization of a sum of squares of equation residuals. Numerical computations were performed in the environment and the MATLAB programing language.

A new experimental point for Haigh diagram of combined pre-stresses was added. The ratio 3:2 between normal and shear pre-stresses was proposed in agreement with the coefficient  $k_c$ . A new experiment with stress mean values  $\sigma_m = 90$  MPa and  $\tau_m = 60$  MPa provided the level of fatigue limit on 90 MPa from 5 specimens. The final value for coefficient  $k_H$  of curve in the Haigh diagram for points of combined pre-stress components was computed as 1.0585.

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