EAN 2014
52nd conference on experimental stress analysis

Application of Decomposition Theory of the Hole-Drilling Method for Measuring by Semiconductor Strain Gauges

VÍTEK Karel[1]

[1] CTU in Prague, Faculty of Mechanical Engineering, Department of Mechanics, Biomechanics and Mechatronics, Technicka 4, 166 07 Prague 6, Czech Republic

karel.vitek@fs.cvut.cz

Keywords: Hole-drilling method, residual stress, decomposition theory, regression coefficients.

Abstract. When determining stress states by the hole-drilling method, it is necessary to take into account the experiment performance accuracy. The drilled holes eccentricity appears as a frequent imperfection, which influences essentially the reliability of their stress state assessment. This paper presents the hole-drilling measurement method corresponding to the E 837 standard method, but, at the same time, it is more universal. This method transforms the full stress tensor of the drilled hole position by the regression coefficients and describes the state of strains released in the hole surrounding, based on the hole center distance and its depth. The regress coefficients are not defined in the method concretely for the rosette but they are universal both for the isotropic Hooke's materials and for the other measuring elements. The method defines the way for the processing of the released strains measured with a defined measuring element and involves naturally the influence of the drilled hole eccentricity and so it is possible, in the hole-drilling method, to apply measuring elements more simply, without determining their specified regression coefficients. Modification of Decomposition theory for semiconductor hole drilling rosette with a greater sensitivity is advantageous.

Introduction

The experimental semi-destructive hole drilling principle for the stress state identification is based on the assumption, that the free surface is one of the principal planes and the stress state in the surface layer thus can be only a uniaxial or plane one. The impair of the inner force equilibrium of a strained structure by drilling of the relatively small cylindrical hole perpendicularly to the surface induces a change of a strain state in its close vicinity. These released strain changes are calibrated with respect to the uniaxial stress-state existing originally in the drilled hole axis. For isotropic Hooke's materials, the released strains measured can be formulated by using the superposition of the two principal stresses of the drilled surface layer and so the original stress-state to identify [2]. The theory of this experimental principle take advantage of the analytical Kirsch's stress-state solution of a thin plate with a hole drilled through perpendicularly and uniaxially loaded by principal stress [1]. The thin plate in Cartesian coordinates x, y, z under the loading by principal stress σ_x is depicted in Fig. 1. On the surface of this plate are defined polar coordinates R, α, stresses σ_r, σ_θ, τ and strains ε_r, ε_θ, γ, ε_z . We define the relative radius $r = R/R_0 \geq 1$ in a radius R direction according to [1, 2, 3]. If the hole of the radius R_0 has not been drilled yet, which is loaded by principal stress σ_x, is loaded by stresses $\sigma'_r, \sigma'_\theta, \tau'$ in planes defined by r and α polar coordinates and marked by indices of their normal lines r, θ. The stresses are determined in (1) from an elementary equilibrium. The Kirsch's equations (2) describe the state of plane strain in the vicinity of the through hole of radius R_0 (Fig. 1). The change of straining induced by the hole drilling in comparison to the original state is defined by the difference of corresponding components of (1) and (2) in (3). In comparison with (1), the (2) include terms dependent on the drilled hole, which are left in the (3) that are otherwise of a character similar to (1) and (2). If E stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_θ, τ can be used for any isotropic material for a calculation of changes related to strains ε_r, ε_θ, γ and ε_z (see Fig. 1) in a point on the plate using the Hooke's law (4) and further modified to (5).

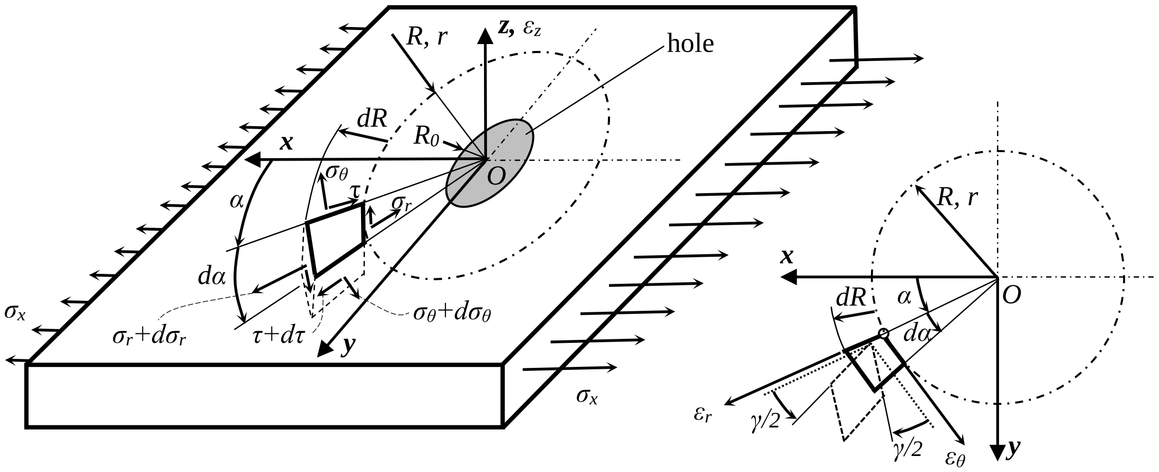

Fig. 1. Components of the stress tensor and strain tensor in the drilled hole vicinity.

$$\left\{\begin{aligned} \sigma'_r &= \frac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma'_\theta &= \frac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau' &= \frac{\sigma_x}{2}\sin 2\alpha \end{aligned}\right\} \tag{1}$$

$$\left\{\begin{aligned} \sigma''_r &= \frac{\sigma_x}{2}(1-\frac{1}{r^2})+\frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma''_\theta &= \frac{\sigma_x}{2}(1+\frac{1}{r^2})-\frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha \\ \tau'' &= \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \tag{2}$$

$$\left\{\begin{aligned} \sigma_r &= \sigma''_r-\sigma'_r = \frac{\sigma_x}{2}(-\frac{1}{r^2})+\frac{\sigma_x}{2}(\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha \\ \sigma_\theta &= \sigma''_\theta-\sigma'_\theta = \frac{\sigma_x}{2}(\frac{1}{r^2})-\frac{\sigma_x}{2}(\frac{3}{r^4})\cos 2\alpha \\ \tau &= \tau''-\tau' = \frac{\sigma_x}{2}(-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha \end{aligned}\right\} \tag{3}$$

The Hole drilling strain-gage method used for the residual stress state identification is currently standardized by the E 837 international standard [2]. This hole drilling method theory is based on two parameters adjusted for particular designs of drilling rosettes and requires very accurate the experimental hole drilling. It is valid for isotropic Hooke's materials with a known strain response to the drilling of the hole. The response is measured by strain gauges assembled to a drilling rosette. The response function is similar to strains identified in the Kirsch's solution of the thin plate with a hole as described in (4) and (5).

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma\\ \varepsilon_z\end{bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\\ -\nu & -\nu & 0\end{bmatrix}\cdot\begin{bmatrix}\sigma_r\\ \sigma_\theta\\ \tau\end{bmatrix}=\sigma_x\left\{\begin{matrix}\left[\frac{-(1+\nu)}{2E}\right]\cdot\left[\frac{1}{r^2}-\frac{3\cdot\cos 2\alpha}{r^4}+\frac{4\cdot\cos 2\alpha}{r^2(1+\nu)}\right]\\ \left[\frac{-(1+\nu)}{2E}\right]\cdot\left[-\frac{1}{r^2}+\frac{3\cdot\cos 2\alpha}{r^4}-\frac{4\nu\cdot\cos 2\alpha}{r^2(1+\nu)}\right]\\ \left[\frac{2\cdot(1+\nu)}{2E}\right]\cdot\left[-\frac{3}{r^4}+\frac{2}{r^2}\right]\cdot\sin 2\alpha\\ \left[\frac{4\nu}{2Er^2}\right]\cdot\cos 2\alpha\end{matrix}\right\} \tag{4}$$

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma\\ \varepsilon_z\end{bmatrix}=\frac{\sigma_x}{2E}\left\{\begin{matrix}\left\{\left[-\frac{1}{r^2}-\frac{1}{r^2}\nu\right]+\left[\frac{1}{r^4}3+\frac{1}{r^4}3\nu-\frac{1}{r^2}4\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[+\frac{1}{r^2}+\frac{1}{r^2}\nu\right]-\left[\frac{1}{r^4}3+\frac{1}{r^4}3\nu-\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[-\frac{1}{r^4}3+\frac{1}{r^2}2-\frac{1}{r^4}3\nu+\frac{1}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\}\\ \left\{\left[\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\end{matrix}\right\} \tag{5}$$

$$\bar{\varepsilon} = \sigma_x(\bar{A}+\bar{B}\cos 2\alpha)+\sigma_y(\bar{A}-\bar{B}\cos 2\alpha) \tag{6}$$

A simplification of the goniometric function (5) describing a response of an ideal strain gauge placed with a deviation of angle α from a direction related to the principal stress σ_x is standardized by E 837 standard, used (see (6)). Standardized theory E 837 defines the measured relaxed strain (6) as a signal of a complete hole drilling rosette strain gauges placed in the ideal position. Standard constant variables $\bar{A}=-\bar{a}(1+\nu)/2E$ and $\bar{B}=-\bar{b}/2E$ related to the particular design of the drilling rosette are used within the superposition of wanted principal stresses σ_x and σ_y. Both constants $\bar{a},\bar{b}$ are tabulated in E 837 standard for particular types of drilling rosettes. The measurement properties of the rosettes during the hole drilling according to are considerably dependent on the accuracy of compliance with conditions of the experiment. Precision drilling experiment, it is difficult to observe in terms of practice. A frequent imperfections of the drill holes is eccentricity to their ideal position, which standardized theory E 837 requires to measuring hole drilling rosette. Informations that are needed to transform the signals measuring elements hole drilling rosette $\bar{\varepsilon}$ from (6) with respect to their actual position relative to the drilling hole standard method considered. Since the measurement of released deformation $\bar{\varepsilon}$ is not objective due to the eccentricity of drill hole, so the algorithm standardized method causes errors in the state of stress identification.

Computing Model of Regression Coefficients

Then the hole drilling experiment as formulated by E 837 standard cannot be used for any more complex determination of the strain state in the vicinity of the drilled hole, which would be necessary for any eventual improve corrections. Therefore, this paper on the elimination of the influence of the drilled hole eccentricity on the stress evaluation by the drilling method is actual [4, 5, 6, 7].

When derived Decomposition Theory hole drilling method [4, 8, 9] was assume that the measuring element positions are defined with both the drilling rosette and the measured eccentricity of the hole drilled. There was used decomposition and subsequent discretization measuring gauge rosettes to measure elements of gauges. The released strains can be locally transformed in the measured direction and the measuring strain-gauge responses can be objectively formulated. The regression model (7) released deformations in the vicinity of the experimental drill hole is formed on the analogy with the plane Kirch theory (1-5). The regression coefficients are independent of both the isotropic Hooke's materials and the structures of specified measuring elements, i.e., they are universal (general).

$$\left\{\begin{aligned}&\sigma_r(\alpha,r,\bar{c}_1,\bar{c}_2)=\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha=\bar{\sigma}_r(\alpha,r,z)\\&\sigma_\theta(\alpha,r,\bar{c}_4,\bar{c}_5)=\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha=\bar{\sigma}_\theta(\alpha,r,z)\\&\tau(\alpha,r,\bar{c}_6)=\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha=\bar{\tau}(\alpha,r,z)\end{aligned}\right\}\tag{7}$$

$$\left\{\begin{aligned}&\min\sum_i(\sigma_r-\bar{\sigma}_r)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right]_i^2=\min F1(\bar{c}_1,\bar{c}_2)\\&\min\sum_i(\sigma_\theta-\bar{\sigma}_\theta)_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta(\alpha,r,z)\right]_i^2=\min F2(c_4,c_5)\\&\min\sum_i(\tau-\bar{\tau})_i^2=\min\sum_i\left[\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha-\bar{\tau}(\alpha,r,z)\right]_i^2=\min F3(\bar{c}_6)\end{aligned}\right\}\tag{8}$$

$$\left\{\begin{aligned}&\frac{\partial}{\partial\bar{c}_1}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right\}\cdot\left(-\frac{\sigma_x}{2r^2}\right)\right]_i=0\\&\frac{\partial}{\partial\bar{c}_2}F1=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{1\cdot\bar{c}_1(r,z)}{r^2}\right)+\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cdot\bar{c}_2(r,z)\cdot\cos 2\alpha-\bar{\sigma}_r(\alpha,r,z)\right\}\cdot\left(\frac{\sigma_x}{2}\left(\frac{3}{r^4}-\frac{4}{r^2}\right)\cos 2\alpha\right)\right]_i=0\\&\frac{\partial}{\partial\bar{c}_4}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(\frac{\sigma_x}{2r^2}\right)\right]_i=0\\&\frac{\partial}{\partial\bar{c}_5}F2=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(\frac{1\cdot\bar{c}_4(r,z)}{r^2}\right)-\frac{\sigma_x}{2}\left(\frac{3\cdot\bar{c}_5(r,z)}{r^4}\right)\cdot\cos 2\alpha-\bar{\sigma}_\theta\right\}\cdot\left(-\frac{3\sigma_x}{2r^4}\cos 2\alpha\right)\right]_i=0\\&\frac{\partial}{\partial\bar{c}_6}F3=2\sum_i\left[\left\{\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\cdot\bar{c}_6(r,z)\cdot\sin 2\alpha-\bar{\tau}(\alpha,r,z)\right\}\cdot\left(\frac{\sigma_x}{2}\left(-\frac{3}{r^4}+\frac{2}{r^2}\right)\sin 2\alpha\right)\right]_i=0\end{aligned}\right\}\tag{9}$$

The five regression coefficients $\bar{c}_1,\bar{c}_2,\bar{c}_4,\bar{c}_5,\bar{c}_6$ of the regularized model can be obtained by using the least squares method in (8), which minimizes the residual errors between the analytical method ($\sigma_r,\sigma_\theta,\tau$) and numerical method ($\bar{\sigma}_r,\bar{\sigma}_\theta,\bar{\tau}$ - numerical method with using a numerical simulation of the experiment drilling and measured relaxed strains) in the comparative points *i* of the numerical model of the thin plate from Fig. 1. This task can be transformed by the minimization of three independent functionals F1, F2 and F3, yielding the five linear equation system in the form of (9). The conditions for the minimization of the functionals F1, F2 and F3 can be separated into the three independent linear equation systems as stated in (10).

$$\left\{\begin{array}{l}
\begin{bmatrix} \sum\limits_i -\dfrac{1}{r^4} & \sum\limits_i (\dfrac{3}{r^6}-\dfrac{4}{r^4})\cos 2\alpha \\ \sum\limits_i -(\dfrac{3}{r^6}-\dfrac{4}{r^4})\cos 2\alpha & \sum\limits_i (\dfrac{9}{r^8}-\dfrac{24}{r^6}+\dfrac{16}{r^4})\cos^2 2\alpha \end{bmatrix} \cdot \begin{Bmatrix} \bar{c}_1 \\ \bar{c}_2 \end{Bmatrix} = \dfrac{2}{\sigma_x} \cdot \begin{Bmatrix} \sum\limits_i \dfrac{\bar{\sigma}_r(\alpha,r,z)}{r^2} \\ \sum\limits_i \bar{\sigma}_r(\alpha,r,z)\cdot(\dfrac{3}{r^4}-\dfrac{4}{r^2})\cos 2\alpha \end{Bmatrix} \\[3ex]
\begin{bmatrix} \sum\limits_i \dfrac{1}{r^4} & \sum\limits_i -\dfrac{3\cos 2\alpha}{r^6} \\ \sum\limits_i \dfrac{\cos 2\alpha}{r^6} & \sum\limits_i -\dfrac{3\cos^2 2\alpha}{r^8} \end{bmatrix} \cdot \begin{Bmatrix} \bar{c}_4 \\ \bar{c}_5 \end{Bmatrix} = \dfrac{2}{\sigma_x} \cdot \begin{Bmatrix} \sum\limits_i \dfrac{\bar{\sigma}_\theta(\alpha,r,z)}{r^2} \\ \sum\limits_i \dfrac{\bar{\sigma}_\theta(\alpha,r,z)}{r^4}\cos 2\alpha \end{Bmatrix} \\[3ex]
\bar{c}_6 \cdot \sum\limits_i (\dfrac{9}{r^8}-\dfrac{12}{r^6}+\dfrac{4}{r^4})\sin^2 2\alpha = \dfrac{2}{\sigma_x}\sum\limits_i \bar{\tau}(\alpha,r,z)\cdot(-\dfrac{3}{r^4}+\dfrac{2}{r^2})\sin 2\alpha
\end{array}\right\} \tag{10}$$

Elimination of Drilled Hole Eccentricity Effects

We expect location measuring via basic Cartesian coordinates $\bar{x}, \bar{y}$ (see Fig. 2). The real position of the drilling hole centre O can deviate from the ideal centric position $\bar{O}(\bar{x}=0, \bar{y}=0)$ to a new position $O(\bar{x}=x_0, \bar{y}=y_0)$, set with eccentricity components x_0, y_0, where are implemented parallel Cartesian coordinates x, y. The semiconductor strain gauge i has the shape of a thin rod, because it is modeled in contact with the surface geometry as a line segment divided into several measuring points j. The strain gauge i position is defined by the its origin W_i, from which are defined local winding Cartesian coordinates s, g and also by the angle ψ_i from the axis $\bar{x}$ or x to the winding orientation axis g. The direction of the local winding axis g is here identical with the measuring orientation of the surface strain ε_j. A set of j winding points marked M_j is defined by s_j, g_j coordinates of the strain gauge winding. Due to implementation hole drilling principle theory are here applied polar coordinates R_i, ω_i for the expression geometry, more precisely polar coordinates r_i, ω_i, where the radius around the drilling hole is formulated by the radius ratio $r = R/R_0$. The position of the origin W_i of the i-th strain gauge is defined in relation to the real center of the drilled hole shifted by the eccentricity components x_0, y_0 from the ideal position by Cartesian coordinates x_i, y_i and polar coordinates R_i, ω_i, or r_i, ω_i by the formula (11).

$$\left\{\begin{array}{l}
x_i = \bar{x}_i - x_0 = R_i \cdot \cos\omega_i \quad \text{and} \quad y_i = \bar{y}_i - y_0 = R_i \cdot \sin\omega_i\,, \\
\text{where} \quad R_i = \sqrt{x_i^{\,2} + y_i^{\,2}} = \sqrt{(\bar{x}_i - x_0)^2 + (\bar{y}_i - y_0)^2}\;, \\
r_i = R_i / R_0 \text{ and} \quad \omega_i = \arcsin(y_i/R_i) \quad \text{for} \quad x_i \geq 0\,, \\
\text{or} \quad \omega_i = \pi - \arcsin(y_i/R_i) \quad \text{for} \quad x_i \prec 0
\end{array}\right\} \tag{11}$$

M_j point on the centerline of the winding of strain gauge i then has coordinates x_j, y_j by the (12), derived from Cartesian coordinates x_i, y_i and local coordinates s, g of the i-th strain gauge.

$$\left\{\begin{aligned}
&x_j = x_i + s_j \cdot \sin\psi_i + g_j \cdot \cos\psi_i = R_i \cdot \cos\omega_i + \\
&+ s_j \cdot \sin\psi_i + g_j \cdot \cos\psi_i = R_j \cdot \cos\omega_j \\
&y_j = y_i - s_j \cdot \cos\psi_i + g_j \cdot \sin\psi_i = R_i \cdot \sin\omega_i + \\
&- s_j \cdot \cos\psi_i + g_j \cdot \sin\psi_i = R_j \cdot \sin\omega_j
\end{aligned}\right\} \tag{12}$$

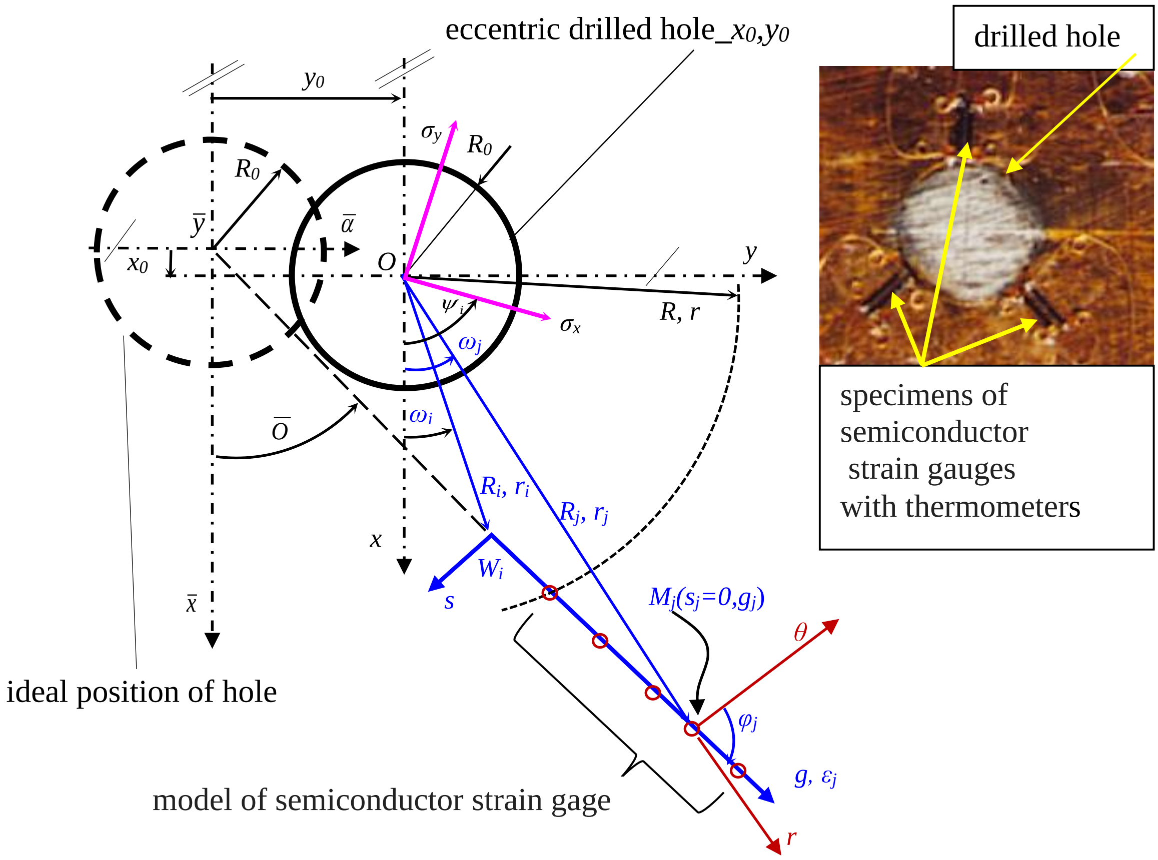

Fig. 2. Semiconductor strain gauge model and specimens.

The polar coordinates R_j, ω_j, or r_j, ω_j of the M_j point in (13) are set from (12) in analogy with (11). The angle φ_j of g axis from θ axis is described by the (14) in the local coordinate system r,θ (Fig. 2), which here also represents the direction of the dominant part of the strain gauge i winding.

$$\left\{\begin{aligned}
&R_j = \sqrt{x_j^{\,2} + y_j^{\,2}} \;\;, \text{where} \;\; \omega_j = \arcsin\left(y_j / R_j\right) \\
&\;\; \textit{for} \;\; x_j \geq 0 \\
&\text{or} \;\; \omega_j = \pi - \arcsin\left(y_j / R_j\right) \;\; \textit{for} \;\; x_j \prec 0 \\
&\text{and} \;\; r_j = R_j / R_0
\end{aligned}\right\} \tag{13}$$

$$\varphi_j = \pi/2 - (\psi_i - \omega_j) = \pi/2 + \omega_j - \psi_i \tag{14}$$

These complete components of the stress state change induced by drilling the hole can be transformed to the strain components. If the isotropic Hooke's material is evaluated, then the strain components can be computed by (15) in analogy to (5). The state of strain on planes perpendicular to the surface can be set by an angular transformation, where the use of the first three components ε_r, ε_θ, γ in (15) is sufficient, because of the principal strain ε_z does not have any effect on it. Figure 2 defines the coordinates s_j, g_j of the point M_j at axes s, g. The strain ε_j tangential to the winding direction at the point M_j, or more precisely aligned to the direction of the axis g, is derived from ε_r, ε_θ, γ strains according to the transformation (16) for an acute angle φ_j. Subsequently it is expressed using goniometric functions of a double angle $2\varphi_j$. The latter statement is a consequence of the fact that strain gages primarily measure along the winding tangent. We expect the direction of the principal stress σ_x given by the angular parameter $\bar{\alpha}$ measured from either x or $\bar{x}$ axis (see Fig. 2). For an examined point M_j of the i-th strain gauge winding is its angular position to principal stress σ_x determined by the difference of angles $\bar{\alpha}-\omega_j$. The angular position of this M_j point is shifted by a right angle $\bar{\alpha}-\omega_j+\pi/2$ in relation to the other principal stress σ_y.

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma\end{bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\end{bmatrix}\cdot\begin{bmatrix}\sigma_r\\ \sigma_\theta\\ \tau\end{bmatrix}=\frac{\sigma_x}{2E}\begin{bmatrix}\left\{\left[-\frac{\bar{c}_1}{r^2}-\frac{\bar{c}_4}{r^2}\nu\right]+\left[\frac{\bar{c}_2}{r^4}3+\frac{\bar{c}_5}{r^4}3\nu-\frac{\bar{c}_2}{r^2}4\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[+\frac{\bar{c}_4}{r^2}+\frac{\bar{c}_1}{r^2}\nu\right]-\left[\frac{\bar{c}_5}{r^4}3+\frac{\bar{c}_2}{r^4}3\nu-\frac{\bar{c}_2}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[-\frac{\bar{c}_6}{r^4}3+\frac{\bar{c}_6}{r^2}2-\frac{\bar{c}_6}{r^4}3\nu+\frac{\bar{c}_6}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\}\end{bmatrix} \tag{15}$$

$$\begin{aligned}\varepsilon_j(\varphi_j)&=\varepsilon_\theta\cos^2(\varphi_j)+\varepsilon_r\cos^2(\pi/2-\varphi_j)+\gamma\sin(\varphi_j)\cdot\cos(\varphi_j)=\varepsilon_\theta\cos^2(\varphi_j)+\varepsilon_r\sin^2(\varphi_j)+\gamma\sin(\varphi_j)\cdot\cos(\varphi_j)=\\ &=\frac{\varepsilon_\theta+\varepsilon_r}{2}+\frac{\varepsilon_\theta-\varepsilon_r}{2}\cos(2\varphi_j)+\frac{\gamma}{2}\sin(2\varphi_j)\end{aligned} \tag{16}$$

The bonded strain gauge reads the strain field of the contact surface. Therefore, the deformation under the strain-gauge, at a specified section of its winding, is proportional to the contribution of this winding section into the total signal measured with the strain-gauge. We set a unit vector in the direction of the principal stress σ_x under the $\bar{\alpha}$ in the first case and in the direction of the stress σ_y under angle $\bar{\alpha}+\pi/2$ (Fig. 2). Relieved strain $\bar{\varepsilon}_j$ is multiplied with a unit dummy load vector introduced in the direction of principal stress and transformed to the winding direction using the strains $\bar{\varepsilon}_r, \bar{\varepsilon}_\theta, \bar{\gamma}$ of the point j (see (16)). The both considered sensitivities t_i of the i-th strain gauge to the strains relieved during the drilling can be formulated by average strain in the direction of the strain gauge winding according to (17). The curvilinear integral of strain along the winding length w_i has an argument including strain $\bar{\varepsilon}_j$. The angle φ_j from (14) is function of the particular position of the point M_j on the winding and is not a function of the parameter $\bar{\alpha}$.

$$t_i(\alpha)=\frac{\oint_{w_i}\bar{\varepsilon}_j(\bar{\alpha})\cdot dw_i}{\oint_{w_i}dw_i},\quad\text{or}\quad t_i(\alpha+\pi/2)=\frac{\oint_{w_i}\bar{\varepsilon}_j(\bar{\alpha}+\pi/2)\cdot dw_i}{\oint_{w_i}dw_i} \tag{17}$$

$$\varepsilon_i=\sigma_x\cdot t_i(\bar{\alpha})+\sigma_y\cdot t_i(\bar{\alpha}+\pi/2) \tag{18}$$

The strains $\bar{\varepsilon}_j, \bar{\varepsilon}_r, \bar{\varepsilon}_\theta, \bar{\gamma}$ normed by a unit vector are goniometric functions (see (11)-(14)) of the particular position of the point M_j on the winding and of parameter $\bar{\alpha}$ defining the position of the unit vector introduced to the direction of the principal stress. The system of at least three independent (18) of i-th strain gauge signals ε_i read in the vicinity of the drilled hole for unknown principal stresses σ_x, σ_y and the angle of their position $\bar{\alpha}$ (system is similar to (6)). A superposition including effects of both principal stresses is done. The relative radius $r_j = \rho_j / R_0$ in M_j integration point on the strain gauge winding is identified. The integrands of (17) are set in (19) and (20) by a substitution ε_r, ε_θ, γ from (15) to (16). The equivalences $\cos 2(\bar{\alpha} + \pi/2) = -\cos 2\bar{\alpha}$ and $\sin 2(\bar{\alpha} + \pi/2) = -\sin 2\bar{\alpha}$ is used (20).

$$\bar{\varepsilon}_j(\bar{\alpha}) = \frac{1}{4E}\left\{\begin{aligned} &\left[-\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2} + \frac{\bar{c}_1}{r_j^2}\nu - \frac{\bar{c}_4}{r_j^2}\nu\right] + \left[+\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2} + \frac{\bar{c}_1}{r_j^2}\nu + \frac{\bar{c}_4}{r_j^2}\nu\right]\cdot\cos 2\varphi_j + \\ &+\left[+\frac{\bar{c}_2}{r_j^4}3 + \frac{\bar{c}_5}{r_j^4}3\nu - \frac{\bar{c}_3}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4\nu\right]\cdot\cos 2\bar{\alpha} + \\ &+\left[-\frac{\bar{c}_2}{r_j^4}3 - \frac{\bar{c}_5}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4\nu\right]\cdot\cos 2\varphi_j\cdot\cos 2\bar{\alpha} + \\ &+\left[-\frac{\bar{c}_6}{r_j^4}3 + \frac{\bar{c}_6}{r_j^2}2 - \frac{\bar{c}_6}{r_j^4}3\nu + \frac{\bar{c}_6}{r_j^2}2\nu\right]\cdot 2\cdot\sin 2\varphi_j\cdot\sin 2\bar{\alpha} \end{aligned}\right\} \tag{19}$$

$$\bar{\varepsilon}_j(\bar{\alpha} + \frac{\pi}{2}) = \frac{1}{4E}\left\{\begin{aligned} &\left[-\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2} + \frac{\bar{c}_1}{r_j^2}\nu - \frac{\bar{c}_4}{r_j^2}\nu\right] + \left[+\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2} + \frac{\bar{c}_1}{r_j^2}\nu + \frac{\bar{c}_4}{r_j^2}\nu\right]\cdot\cos 2\varphi_j + \\ &+\left[+\frac{\bar{c}_2}{r_j^4}3 + \frac{\bar{c}_5}{r_j^4}3\nu - \frac{\bar{c}_2}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4\nu\right]\cdot(-\cos 2\bar{\alpha}) + \\ &+\left[-\frac{\bar{c}_2}{r_j^4}3 - \frac{\bar{c}_5}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3\nu + \frac{\bar{c}_2}{r_j^2}4\nu\right]\cdot\cos 2\varphi_j\cdot(-\cos 2\bar{\alpha}) + \\ &+\left[-\frac{\bar{c}_6}{r_j^4}3 + \frac{\bar{c}_6}{r_j^2}2 - \frac{\bar{c}_6}{r_j^4}3\nu + \frac{\bar{c}_6}{r_j^2}2\nu\right]\cdot 2\cdot\sin 2\varphi_j\cdot(-\sin 2\bar{\alpha}) \end{aligned}\right\} \tag{20}$$

The constants d_k^j (k=1,…6) independent from Poisson's ratio of the drilled material but dependent on the position of the particular M_j point of the i-th strain gauge are introduced into (19) and (20). The integrands in (19) and (20) of two (17) are by the help of (21) concentrated to d_k^j constants of the M_j point in (22).

$$\left\{\begin{aligned} d_1^j &= \left[-\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2}\right] + \left[+\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2}\right]\cdot\cos 2\varphi_j \\ d_2^j &= \left[+\frac{\bar{c}_1}{r_j^2} - \frac{\bar{c}_4}{r_j^2}\right] + \left[+\frac{\bar{c}_1}{r_j^2} + \frac{\bar{c}_4}{r_j^2}\right]\cdot\cos 2\varphi_j \\ d_3^j &= \left[+\frac{\bar{c}_2}{r_j^4}3 - \frac{\bar{c}_2}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3\right] + \left[-\frac{\bar{c}_2}{r_j^4}3 + \frac{\bar{c}_2}{r_j^2}4 - \frac{\bar{c}_5}{r_j^4}3\right]\cdot\cos 2\varphi_j \\ d_4^j &= \left[+\frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3 + \frac{\bar{c}_2}{r_j^2}4\right] + \left[-\frac{\bar{c}_5}{r_j^4}3 - \frac{\bar{c}_2}{r_j^4}3 + \frac{\bar{c}_2}{r_j^2}4\right]\cdot\cos 2\varphi_j \\ d_5^j &= \left[-\frac{\bar{c}_6}{r_j^4}3 + \frac{\bar{c}_6}{r_j^2}2\right]\cdot 2\cdot\sin 2\varphi_j \rightarrow d_6^j = \left[-\frac{\bar{c}_6}{r_j^4}3 + \frac{\bar{c}_6}{r_j^2}2\right]\cdot 2\cdot\sin 2\varphi_j \end{aligned}\right\} \tag{21}$$

In (23) is (22) substituted into (17) and the E, ν material constants and the goniometric angle functions $\bar{\alpha}$ can be removed from the curvilinear integrals based only on the strain gauge winding position. The curvilinear integrals in strain gage sensitivities (23) correspond in sensitivities to constants D_k^i (k=1,…6) of the strain gauge i and they can be realized numerically in (24). This sums converge obviously with the increasing density of discrete calculation points to the real value of the relevant curvilinear integral. The (18) of a signal ε_i of the strain gauge i can be rewritten to a more specific (25) using both its t_i sensitivities from (23) and (24). The multiple terms in the parenthesis by principal stress σ_x, σ_y are further united into $K(\overline{\alpha})$ functional terms. A typical example of a use can be started from three signals ε_1, ε_2 and ε_3 of three independent strain gauges of the drilling rosette. The signals form a strain response vector relieved after drilling the hole.

$$\left\{\begin{array}{l} \bar{\varepsilon}_j(\overline{\alpha}) = \dfrac{1}{4E}\left[d_1^j + \nu\cdot d_2^j + d_3^j\cos 2\overline{\alpha} + \nu\cdot d_4^j\cos 2\overline{\alpha} + d_5^j\sin 2\overline{\alpha} + \nu\cdot d_6^j\sin 2\overline{\alpha}\right] \\ \bar{\varepsilon}_j(\overline{\alpha}+\pi/2) = \dfrac{1}{4E}\left[d_1^j + \nu\cdot d_2^j - d_3^j\cos 2\overline{\alpha} - \nu\cdot d_4^j\cos 2\overline{\alpha} - d_5^j\sin 2\overline{\alpha} - \nu\cdot d_6^j\sin 2\overline{\alpha}\right] \end{array}\right\} \tag{22}$$

$$\left\{\begin{array}{l} t_i(\overline{\alpha}) = \dfrac{1}{4E\cdot\oint\limits_{w_i} dw_i}\left[\begin{array}{l}\oint\limits_{w_i} d_1^j dw_i + \nu\oint\limits_{w_i} d_2^j dw_i + \cos(2\overline{\alpha})\oint\limits_{w_i} d_3^j dw_i + \\ +\nu\cos(2\overline{\alpha})\oint\limits_{w_i} d_4^j dw_i + \sin(2\overline{\alpha})\oint\limits_{w_i} d_5^j dw_i + \nu\sin(2\overline{\alpha})\oint\limits_{w_i} d_6^j dw_i\end{array}\right] \\ t_i(\overline{\alpha}+\pi/2) = \dfrac{1}{4E\cdot\oint\limits_{w_i} dw_i}\left[\begin{array}{l}\oint\limits_{w_i} d_1^j dw_i + \nu\oint\limits_{w_i} d_2^j dw_i - \cos(2\overline{\alpha})\oint\limits_{w_i} d_3^j dw_i + \\ -\nu\cos(2\overline{\alpha})\oint\limits_{w_i} d_4^j dw_i - \sin(2\overline{\alpha})\oint\limits_{w_i} d_5^j dw_i - \nu\sin(2\overline{\alpha})\oint\limits_{w_i} d_6^j dw_i\end{array}\right] \end{array}\right\} \tag{23}$$

$$D_k^i = \frac{\oint\limits_{w_i} d_k^j dw_i}{\int\limits_{w_i} dw_i} = \frac{\sum\limits_j \left\{ d_k^j \cdot \left| g_{j+1} - g_j \right| \right\}}{w_i} = \frac{\sum\limits_j \left\{ d_k^j \cdot \left| g_{j+1} - g_j \right| \right\}}{\sum\limits_j \left| g_{j+1} - g_j \right|} \tag{24}$$

$$\left\{\begin{array}{l} \varepsilon_i = \sigma_x\cdot\dfrac{1}{4E}\left[D_1^i + \nu\cdot D_2^i + \cos 2\overline{\alpha}\cdot D_3^i + \nu\cdot\cos 2\overline{\alpha}\cdot D_4^i + \sin 2\overline{\alpha}\cdot D_5^i + \nu\cdot\sin 2\overline{\alpha}\cdot D_6^i\right] + \\ +\sigma_y\cdot\dfrac{1}{4E}\left[D_1^i + \nu\cdot D_2^i - \cos 2\overline{\alpha}\cdot D_3^i - \nu\cdot\cos 2\overline{\alpha}\cdot D_4^i - \sin 2\overline{\alpha}\cdot D_5^i - \nu\cdot\sin 2\overline{\alpha}\cdot D_6^i\right] = \\ = \dfrac{1}{4E}\left[\sigma_x\cdot K_{i,1}(\overline{\alpha}) + \sigma_y\cdot K_{i,2}(\overline{\alpha})\right] \end{array}\right\} \tag{25}$$

$$\left\{\begin{array}{l} \varepsilon_1 = \dfrac{1}{4E}\left[\sigma_x\cdot K_{1,1}(\overline{\alpha}) + \sigma_y\cdot K_{1,2}(\overline{\alpha})\right] \\ \varepsilon_2 = \dfrac{1}{4E}\left[\sigma_x\cdot K_{2,1}(\overline{\alpha}) + \sigma_y\cdot K_{2,2}(\overline{\alpha})\right] \\ \varepsilon_3 = \dfrac{1}{4E}\left[\sigma_x\cdot K_{3,1}(\overline{\alpha}) + \sigma_y\cdot K_{3,2}(\overline{\alpha})\right] \end{array}\right\} = \frac{1}{4E}\begin{bmatrix} K_{1,1}(\overline{\alpha}) & K_{1,2}(\overline{\alpha}) \\ K_{2,1}(\overline{\alpha}) & K_{2,2}(\overline{\alpha}) \\ K_{3,1}(\overline{\alpha}) & K_{3,2}(\overline{\alpha}) \end{bmatrix}\cdot\begin{Bmatrix}\sigma_x \\ \sigma_y\end{Bmatrix} \tag{26}$$

The system of three non-linear equations (26) can be formulated in analogy to (18) and (25). The first two equations serve for determination of unknown principal stresses σ_x and σ_y as a functions of ε_1 and ε_2 strain signals and of an unknown angular parameter $\overline{\alpha}$ defining the position of the principal stress σ_x according to (27).

$$\left\{\begin{array}{l}\sigma_x = \dfrac{4E\cdot(\varepsilon_1\cdot K_{2,2}(\overline{\alpha})-\varepsilon_2\cdot K_{1,2}(\overline{\alpha}))}{K_{1,1}(\overline{\alpha})\cdot K_{2,2}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{1,2}(\overline{\alpha})}\\ \sigma_y = \dfrac{4E\cdot(\varepsilon_2\cdot K_{1,1}(\overline{\alpha})-\varepsilon_1\cdot K_{2,1}(\overline{\alpha}))}{K_{1,1}(\overline{\alpha})\cdot K_{2,2}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{1,2}(\overline{\alpha})}\end{array}\right\} \tag{27}$$

$$\varepsilon_3 = \frac{\varepsilon_1\cdot K_{2,2}(\overline{\alpha})\cdot K_{3,1}(\overline{\alpha})-\varepsilon_2\cdot K_{1,2}(\overline{\alpha})\cdot K_{3,1}(\overline{\alpha})}{K_{1,1}(\overline{\alpha})\cdot K_{2,2}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{1,2}(\overline{\alpha})}+\frac{\varepsilon_2\cdot K_{1,1}(\overline{\alpha})\cdot K_{3,2}(\overline{\alpha})-\varepsilon_1\cdot K_{2,1}(\overline{\alpha})\cdot K_{3,2}(\overline{\alpha})}{K_{1,1}(\overline{\alpha})\cdot K_{2,2}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{1,2}(\overline{\alpha})}\Rightarrow \tag{28}$$

$$\varepsilon_1\left(K_{2,2}(\overline{\alpha})\cdot K_{3,1}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{3,2}(\overline{\alpha})\right)+\varepsilon_2\left(K_{1,1}(\overline{\alpha})\cdot K_{3,2}(\overline{\alpha})-K_{1,2}(\overline{\alpha})\cdot K_{3,1}(\overline{\alpha})\right)+-\varepsilon_3\left(K_{1,1}(\overline{\alpha})\cdot K_{2,2}(\overline{\alpha})-K_{2,1}(\overline{\alpha})\cdot K_{1,2}(\overline{\alpha})\right)=0 \tag{29}$$

The substitution of (28) for ε_3 to the third (26) allows the computation of $\overline{\alpha}$ parameter from (29), while the last substitution of $\overline{\alpha}$ back to (27) leads to σ_x and σ_y principal stresses.

References

[1] S. Timoshenko, Theory of Elasticity, 1st ed., McGraf-Hill Book Company, New York and London, 1934, pp. 52-81.

[2] American Society for Testing and Materials, Standard Test Method for Determining Residual Stresses by the Hole- Drilling Strain-Gage Method, E 837 – 08.

[3] Vishay Micro-Measurements. Measurement of Residual Stresses by the Hole-Drilling Strain Gage Method, in: Tech Note TN-503, USA, Revision 09, pp. 19-33, 2007.

[4] K. Vítek, Decomposition Theory of the Hole-Drilling Method for the Stress State Identification, in: Universal Journal of Mechanical Engineering 2 (2014) pp. 112-123. Information on http://www.hrpub.org/download/20140205/UJME5-15101763.pdf

[5] D. Vangi. Residual Stress Evaluation by the Hole-Drilling Method With off-Center Hole an Extension of the Integral Method, in: Journal of Engineering Materials and Technology, doi.10.1115/1.2805978, Volume 119, pp. 79-85, 1997.

[6] J. P. Sandifer, G. E. Bowie. Residual Stress by Blind-hole Method with Off-Center Hole, in: Experimental Mechanics 18, pp. 173-179, 1978.

[7] M. Beghini, L. Bertini, L. F. Mori. Evaluating Non - Uniform Residual Stress by the Hole-Drilling Method With Concentric and Eccentric Holes Part I. Definition and Validation of the Influence Functions, in: Strain 46, pp. 337–346, DOI. 10.1111/ j.1475 - 1305.2009.00684.x, 2010.

[8] K. Vítek. Regression Model of the Hole Drilling Principle for the Stress State Identification, Proceedings of 49th International Scientific Conference Experimental Stress Analysis 2011, Brno University of Technology and Czech Society for Mechanics, Brno, pp. 423 – 428, ISBN 978-80-214-4275-7, 2011.

[9] K. Vitek. Coefficient Functions of Hole-Drilling Method, Proceedings of the 50th Annual Conference on Experimental Stress Analysis, Prague, Czech Technical University in Prague, pp. 515-524, ISBN 978-80-01-05060-6 , 2012.