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Probabilistic Assessment of Working Life for Structures

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Abstract. General principles on probabilistic approach to structural design for durability are provided in the international standard ISO „General Principles on the Design of Structures for Durability“ which is currently being implemented into the system of Czech standards. The operational use of the new procedures in practice would require specification of probabilistic durability criteria, physical models of material deteriorations, and theoretical models of basic variables. It appears that the probabilistic methods of optimisation may provide valuable background information facilitating determination of appropriate durability criteria.

Introduction

Presently the international standard ISO 13823 [1] giving the basis for design and verification of structures for durability is going to be implemented into the system of Czech standards. Complementary guidance for its national applications will be given in the new Czech standard CSN 73 0044 [3]. Moreover, some specific provisions will be developed for industrial and civil engineering structures in company standards, e.g. for energetic devices in power plants (for chimneys, cooling towers etc.). Principles for verification of the Durability limit states of structures are provided in this paper and illustrated on practical examples.

Limit States Concept

ISO 13823 [1] provides principles of limit state methods for durability. The key steps of the deterioration processes and reliability verification using concepts of limit states are indicated in Fig. 1. It is a general scheme that may be in a specific application modified depending on the actual conditions of a considered structure.

There are three vertical parts in Fig. 1 showing time axis on the left, real processes in the middle and professional practice on the right. The time axis is split into two parts by a point denoted as Durability limit state (DLS) which corresponds to the point in time when adverse environment actions have turning point, e.g. beginning of reinforcement corrosion or decay of timber. In case of concrete carbonation, it is a point when neutralized carbonation depth reaches reinforcement surface and reinforcement corrosion may start. It might be assumed that in this point in time the Durability limit state is achieved. It may be noted here that recently revised ISO 2394 [2] provides more generally the Condition Limit States (CLS) when the specific well defined and controllable limit state of a structure may be achieved without direct negative consequences.

The middle part of Fig. 1 indicates a sequence of real processes concerning structural environment and influences (rain, de-acing salts and other agents), transfer mechanisms of environmental influences and environmental effects (reinforcement corrosion, material decay). On the right part in Fig. 1 it is indicated that transfer mechanisms may be described by models or tests which may be applied in engineering practice. Two types of models are generally distinguished: conceptual (heuristic) specified on the bases of reasoning and previous experience, and mathematical (analytical) specified on the basis of theoretical assumptions, for examples concerning diffusion processes.

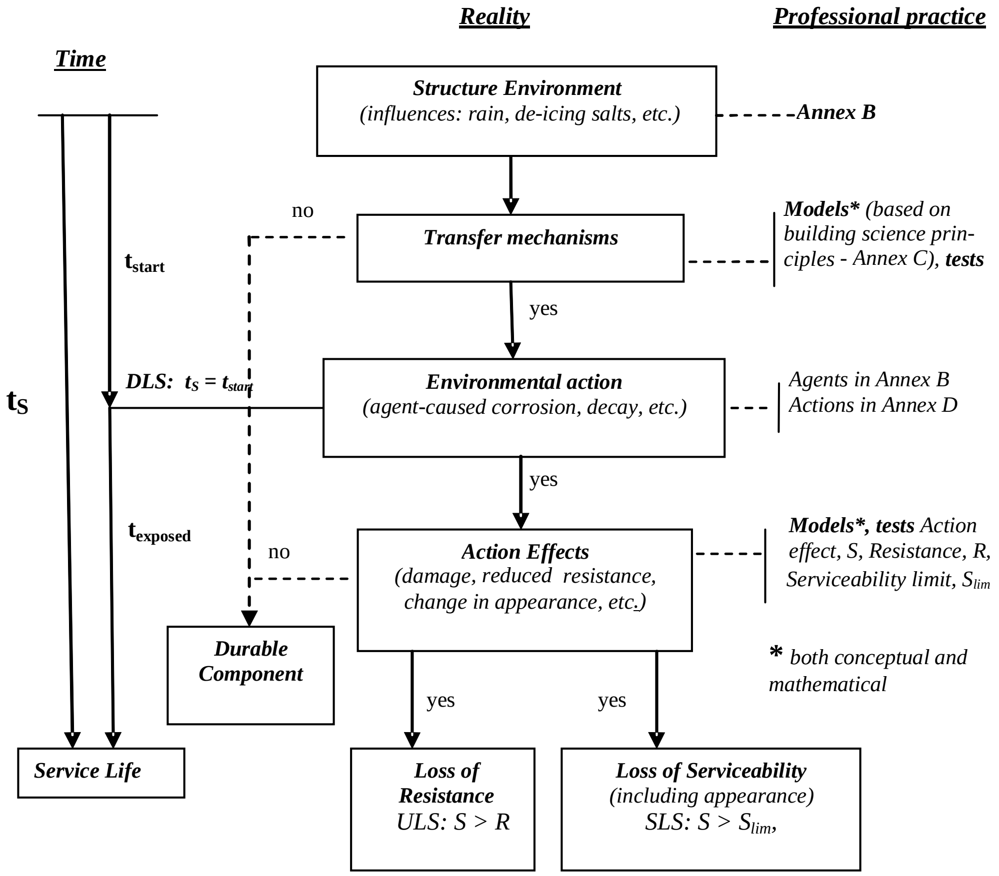

Fig. 1. Limit states method for durability.

Resulting environmental effects may then lead to the loss of resistance (bearing capacity) of structures or to the loss of their serviceability (excessive width of cracks or deformations) as indicated in the lower part of Fig. 1.

Environmental effects on structures should be combined with action effects. The load combination rules are however, not covered in ISO 13823 [1]. Therefore, supplementary guidance is developed in the new national standard CSN 73 0044 [3].

Verification of Working Life

The fundamental durability requirement is represented by a simple condition that the predicted working life t_{SP} should be greater than the design working life t_D with sufficient degree of reliability. It is well recognised that the working life t_S is dependent on a number of basic variables and is consequently a random variable having a considerable scatter. For the verification of the working life of structure, the following probabilistic condition should be analysed

$$P\{t_S < t_D\} \;<\; P_{target} \tag{1}$$

where P_{target} denotes the target probability that the working life t_S is less than design working life t_D of the structure. As a rule the design working life t_D is a deterministic quantity (for example 50 or 100 years) specified in advance.

Verification of the Limit States

Probabilistic formulation of the ultimate limit states is similar as in case of working life. For an arbitrary point in time $t \le t_D$ the following condition should be assessed

$$P_f(t) = P\{R(t) - S(t) < 0\} \; < \; P_{target} \tag{2}$$

where $R(t)$ denotes resistance and $S(t)$ action effect.

The basic probabilistic condition for the serviceability may be written analogically as

$$P_f(t) = P\{S_{lim} - S(t) < 0\} \; < \; P_{target} \tag{3}$$

Here S_{lim} denotes the limit value of the serviceability indicator, e.g. of the crack width or deflection. The Durability limit state may be verified in accordance with eq. (2) or (3) depending on the particular conditions of the structure.

Probabilistic assessment of the working life t_{SP} is illustrated in Fig. 2. It should be emphasized that Fig. 2 describes only monotonously varying load effects $S(t)$ and resistances $R(t)$. The horizontal axis denotes the time t and the vertical axis in the upper part the resistance $R(t)$ or in the lower part the load effect $S(t)$. Probability distributions of variables $R(t)$ and $S(t)$ are in Fig. 2 indicated by probability density functions.

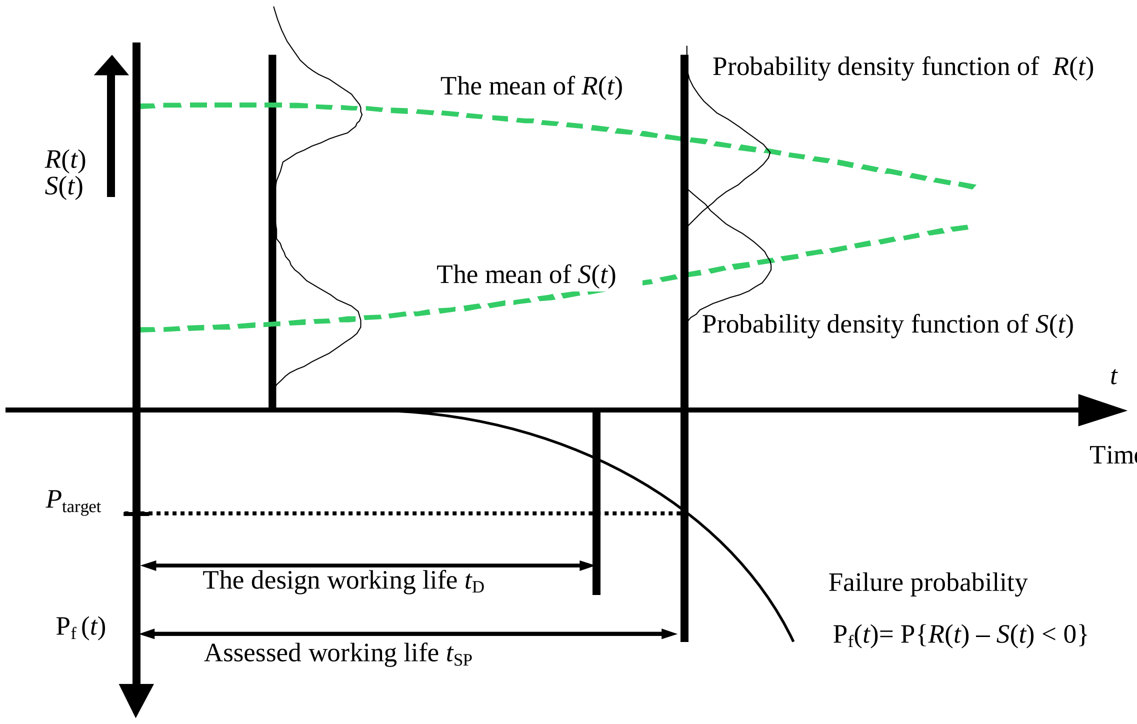

Fig. 2. Probabilistic assessment of the working life.

Obviously the failure probability $P_f(t) = P\{R(t) - S(t) < 0\}$ is an increasing time dependent function. The probabilistic assessment of the working life t_{SP} follows from the relationship

$$P_f(t_{SP}) = P\{R(t_{SP}) - S(t_{SP}) < 0\} = P_{target} \tag{4}$$

ISO 13823 [1] does not provide guidance regarding target probability P_{target}. This question remains open for national choice, therefore complementary provisions for conditions of the Czech Republic are given in CSN 73 0044 [3].

Target Reliability Level

Target reliability level, represented by the target probability P_{target} or reliability index β_{target}, depends in general on the definition of the working life time, whether the critical durability requirement concerns the Ultimate limit state, Serviceability limit state or Durability limit state and what are consequences of their infringement. In particular conditions the target reliability level may considerably vary. Table 1 provides indicative intervals for the target probability P_{target} and reliability index β_{target}.

They are derived from target values recommended in ISO 2394 [2] where additional dependence of target values on relative costs of safety measures (required for an increase of the reliability level) are also indicated.

Table 1. Indicative values of the target probability P_{target} and reliability index β_{target}.

Limit state	P_{target}	β_{target}
Ultimate limit state (ULS)	~ 10^{-4}	~ 3,7
Serviceability limit state (SLS)	0,01 to 0,10	1,3 to 2,3
Durability limit state (DLS)	0,05 to 0,20	0,8 to 1,6

A Study Case of the Durability Limit States

The Durability limit state can be well illustrated by carbonation of the concrete. The limit state is defined as a simple requirement that the carbonation depth $S(t)$ (load effect) is less than the concrete cover R (resistance). Failure probability can be then determined from the integral

$$P_{\text{f}}(t) = P\{S(t) > R\} = \int_{-\infty}^{\infty} \varphi_S(x,t)\Phi_R(x)\text{d}x \qquad (5)$$

where $\varphi_S(x,t)$ denotes probability density function of the load effect $S(t)$ and $\Phi_R(x)$ distribution function of the resistance R, see e.g. [4]).

Extensive measurements of the carbonation depth $S(t)$ on cooling towers (unprotected external concrete) [5] provided the following expressions for the mean $\mu_S(t)$, coefficient of variation $w_S(t)$ and skewness $\alpha_S(t)$

$$\mu_S(t) = 5\, t^{\,0,2} \text{ mm},\ w_S(t) = 0,1\, t^{\,0,2},\ \alpha_S(t) = 0,2\, t^{\,0,2} \qquad (6)$$

where t denotes time in years. Gamma distribution seems to be the most suitable theoretical model. For time invariant concrete cover the following parameters have been obtained

$$\mu_R = 20, 25 \text{ and } 30 \text{ mm},\ w_R = 0,35,\ \alpha_R = 0,35 \qquad (7)$$

In this case Beta distribution having the lower bound at zero seems to be the appropriate theoretical model. Note that in Annex A of ISO 13823 [1] a normal distribution is assumed for both variables $S(t)$ a R(t); this assumption may, however, provide a first approximation only.

Considering the above mentioned theoretical models and their parameters given in Eq. (6) and (7), the failure probability $P_f(t)$ analysed by Eq. (5) is illustrated in Fig. 3 that can be used to assess the working life t_{SP} using Eq. (4) for specified target probability P_{target} and the mean of concrete cover μ_R. If, for example, P_{target} = 0,10, then the mean μ_R = 20 mm corresponds to t_{SP} ~ 23 years, if μ_R = 30 mm then t_{SP} ~ 65 years. Fig. 3 confirms results of previous studies [5,6] indicating that assessment of t_{SP} is significantly dependent on theoretical models assumed for $R(t)$ and $S(t)$, and on specified target probability P_{target}. It appears that specification of the target reliability level can be solved using methods of probabilistic optimisation [4,5].

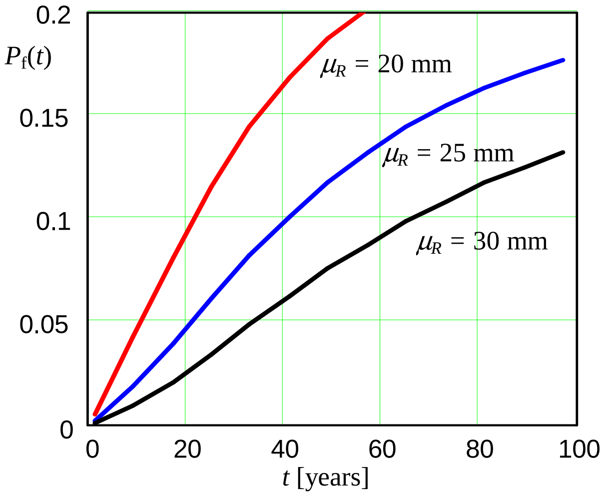

Fig. 3. Probability of failure versus time for parameters given in Eq. (6) and (7).

Probabilistic Optimization

The total costs of execution and repair of the structure due to failure (infringement of the Durability limit state) can be expressed as a function of the mean μ_R (decisive parameter)

$$C_{tot}(\mu_R,t,p) = C_0 + C_1\,\mu_R + P_f(\mu_R\,,t)\;C_f/(1 + p^t) \tag{8}$$

where C_0 denotes the initial costs independent of μ_R, C_1 expenses for a unit of μ_R, C_f expenses for durability failure and p the discount rate (assumed here around 0,03). Standardised total costs are considered as

$$\kappa_{tot}(\mu_R,t,p) = [C_{tot}(\mu_R,t,p) - C_0] \,/\, C_1 = \mu_R + P_f(\mu_R\,,t)\;C_f \,/\, [(1 + p^t)\;C_1] \tag{9}$$

The optimum mean μ_R may be then determined from

$$\frac{\partial \kappa_{tot}(\mu_R, t, p)}{\partial \mu_R} = 0 \tag{10}$$

Taking into account Eq. (9), the following condition may be derived

$$\frac{\partial P_f(\mu_R, t)}{\partial \mu_R} = -\frac{(1+p^t)C_1}{C_f} \tag{11}$$

It should be noted that in a realistic domain of μ_R from 20 to 60 mm, Eq. (11) may not have a solution and the minimum of the total costs may not exist.

Considering the Durability limit state of a structure, the standardised total costs given by Eq. (9) are shown in Fig. 4 for the design life time t = 50 years and discount rate p = 0,03. It appears that the optimum mean of concrete cover μ_R increases with increasing cost ratio C_f / C_1. For C_f / C_1 = 200 the optimum μ_R is about 18 mm (theoretical minimum is less than 20 mm), for the cost ratio C_f / C_1 = 1000 the optimum mean is μ_R ~34 mm.

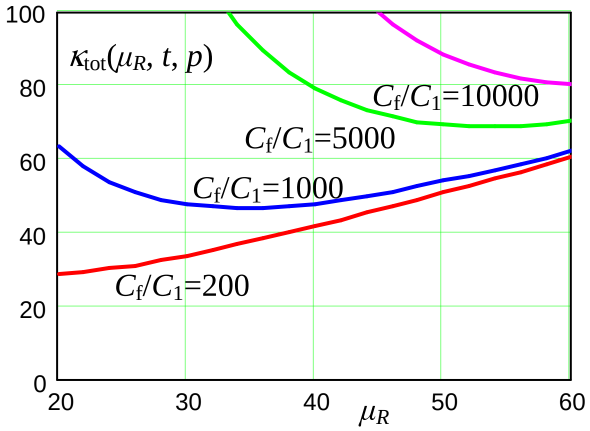

Fig. 4. The total standardised costs $\kappa_{tot}(\mu_R,t,p)$ for t = 50 years and p = 0,03.

Interactive dependence of the total costs on μ_R and p is shown in Fig. 5 for the cost ratio C_f / C_1 = 1000, t = 50 years. Fig. 5 clearly indicates that the discount rate p may significantly affect the total costs and the optimum mean μ_R. In general, with increasing discount rate p the total costs and the optimum mean μ_R decrease.

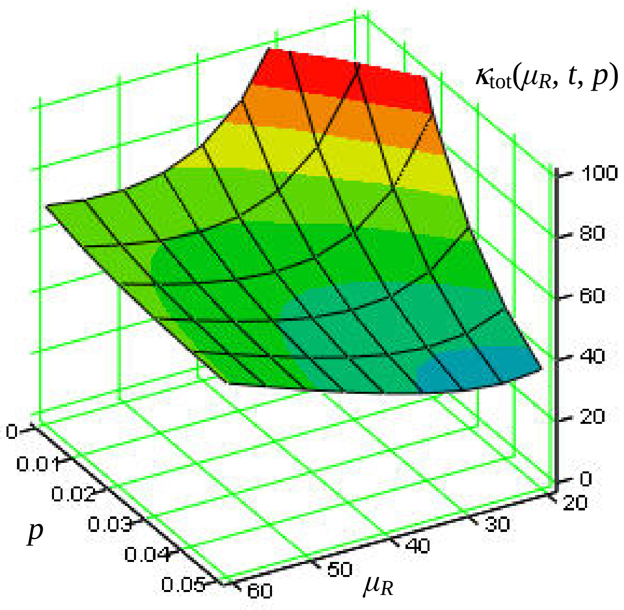

Fig. 5. The total standardised costs $\kappa_{tot}(\mu_R,t,p)$ for C_f / C_1 = 1000 and t = 50 years.

Concluding Notes

Probabilistic principles of structural design for durability or estimation of residual working life given in ISO 13823 are expected to be soon implemented into the system of Czech standards. It appears, however, that the assessment of working life is strongly dependent on the theoretical models of basic variables and specified reliability level.

The target reliability levels should be differentiated taking into account the character of the limit state, consequences of durability failure and costs of safety measures to increase the reliability level.

Methods of probabilistic optimisation may provide rational background information for specification of the target reliability level. In case of carbonation of concrete cover the total costs depend on the thickness of concrete cover, design working life and discount rate. The optimum concrete cover increases with increasing costs due to durability failure, and decreases with the increasing discount rate.

Operational use of the new procedures in practice requires specification of

- Appropriate physical models for material deterioration,
- Suitable theoretical models for basic variables,
- Differentiated probabilistic criteria for durability requirements.

Acknowledgement

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