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# Computer Evaluation of the Components of the Stress Tensor in the Two-Dimensional Photoelasticity

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**Abstract:** Photoelasticity is an important optical experimental technique for the analysis of the stress and strains in the specimens. The objective of this work is to create a unique approach that allows the use of a computer-aided technique in the optical photoelasticity. The developed approach can be used to calculate the components of the stress tensor in specimens of the arbitrary shapes. The excellent agreement is established between the results of experimental technique based on the created computer procedure and the results of numerical calculations. The PATRAN program is used for numerical solution of the stress fields in the investigated specimens.

**Keywords:** Photoelasticity; Cauchy Stress Tensor; Experimental and Numerical Techniques.

## 1 Introduction

Photoelasticity is the experimental technique for analysing stress distributions in the loaded members. This method is used primarily for plane stress problems. The main principle of photoelasticity is based on the optomechanical effects called the birefringence and the polarization of light; see reference [1], [2].

The results of the photoelasticity method are isoclinic and isochromatic fringe patterns [3] that specify the principal stress directions and the principal stress differences, respectively. In proposed procedure the shear difference method is used for determine the components of the stress tensor in loaded specimens.

The photoelasticity method's usefulness lies in the fact that it is non-destructive technique, is adaptable both static and dynamic investigations [4], provides the principal stresses directions and enables determine the values of the principal stresses along the perimeter of the specimen, where the stresses are typically the greatest. Nevertheless, the determination of the components of the stress tensor within the entire specimen of measured isoclinic and isochromatic fringe patterns is very tedious and time consuming. The main goal of the submitted contribution is to create the unique procedures for computer to evaluating the photoelasticity data. Computer-processed measurement results, i.e. components of the stress tensor are also compared with the results obtained from numerical and analytical solution.

## 2 Description of the Developed Computer Procedures

The evaluation of the measurement results is in large part automatized and it is performed with a specially developed computer procedure. Components of the stress tensor are generated by taking the following steps: (1) the identification of the contour of the specimen, (2) the filtration of the measured isoclinic and isochromatic fringe patterns into the smoothed patterns, (3) the combination isoclinic smoothed patterns, (4) the calculation of the principal stress directions, (5) the determination of the principal stresses in unloaded contour of the specimen, (6) the calculation and display of the components of the stress tensor at an arbitrary point and field of the loaded specimen.

The measurement to identify contour of a specimen, isoclinic and isochromatic fringe patterns must be carry out in the static position. The contour of a specimen, the isoclinic (Fig. 1) and isochromatic (Fig. 2, Fig. 3) fringe patterns are defined by a control polygon, which is a Bézier curve. To separate stresses in a specimen the shear-difference method [5] which utilizes the integration of the equilibrium equations is used. The component of the stress tensor $\sigma_x$ can be computed using the equation

$$(\sigma_x)_i = (\sigma_x)_{i-1} - \{[(\Delta\tau_{xy})_{i-1} + (\Delta\tau_{xy})_i]/2\}(\Delta x/\Delta y)_i, i = 1, 2, 3, \dots \quad (1)$$

The term $(\sigma_x)_0$, denotes the know stress at starting point, $\Delta\tau_{xy}$ is the difference shear stress in points of the lines above and below the line of integration, $\Delta x$ and $\Delta y$ are the grid along $x$ and $y$ axis.

The grid of points (Fig. 4) is created according to the direction of the principal stress (Fig. 5). The line of the integration (lies in the horizontal or vertical direction) is passing through grid of points and in these points the components of shear stress tensor are computed.

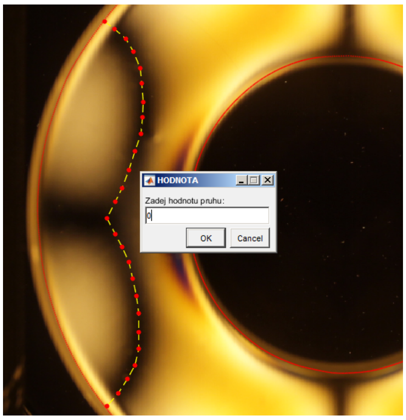


Fig. 1: Shape definition of the segment of the isoclinic fringe pattern.

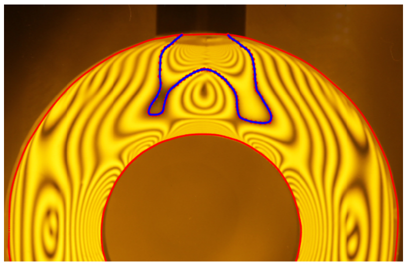

Fig. 2: Separated of the isochromatic fringe pattern and contour of the specimen.

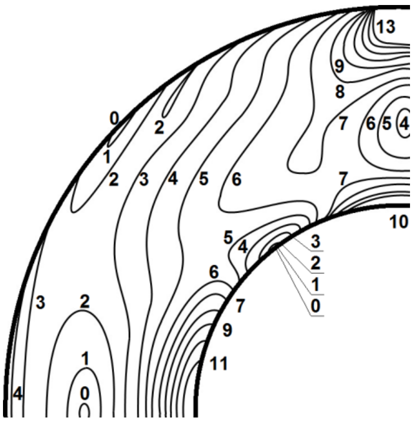


Fig. 3: Sketch of the isochromatic pattern obtained by composition of the separated pattern.

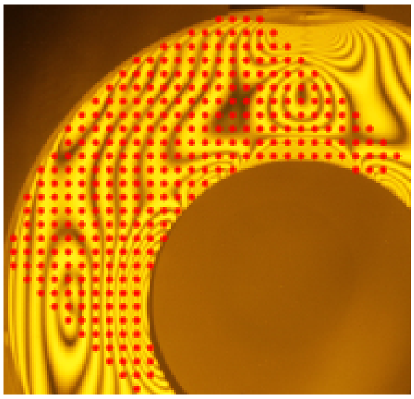

Fig. 4: Generating of the grid points by means of the developed algorithm.

The thickness of the annular disk and thin disk is equal 12.9 mm and the tested shaped beam is 10 mm thick. The experimental specimens are made of the optically sensitive material. The material of the CT 200 is used for the annular disk and thin disk and the shaped beam is made of a polycarbonate material. Young's modulus and Poisson's ratio of the CT 200 material is equal 3 124 MPa, 0.35, respectively and for the polycarbonate material is equal 2480 MPa, 0.38, respectively. The material fringe value of the CT 200 and the polycarbonate material is calibrated to 17.4 N·mm$^{-1}$ and 7.0 N·mm$^{-1}$, respectively.

Figs. 6-8 show the stress obtained by the developed procedure for the diametral compressive load equal to 1308 N of the annular disk and 997 N of the disk. The distribution of stress for the shaped beam subjected to a bending force equal to 50 N is depicted in Fig. 9.

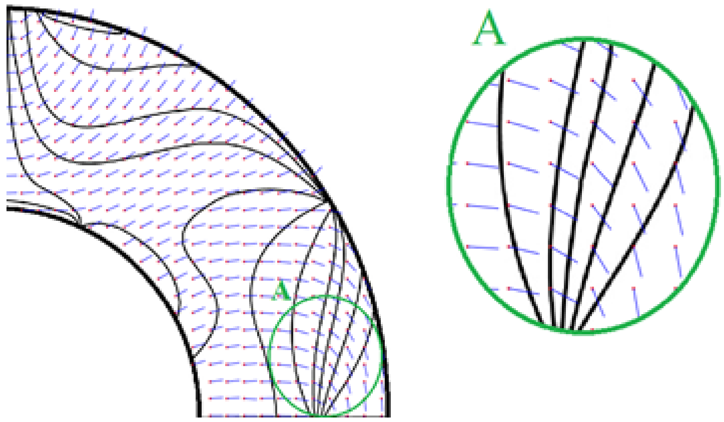


Fig. 5: Isoclinic patterns and the computed directions of the first principal stress.

## 3 Tested specimens and Experimental Results

The developed procedure is implemented in the MATLAB computing environment and its functionality is tested on the thin annular disk (Fig. 6, 7) and the thin disk (Fig. 8) subjected to a diametral compressive load and the shaped beam (Fig. 9) subject to a bending force. The measurements are performed with a test rig using the linear and circular polarization.

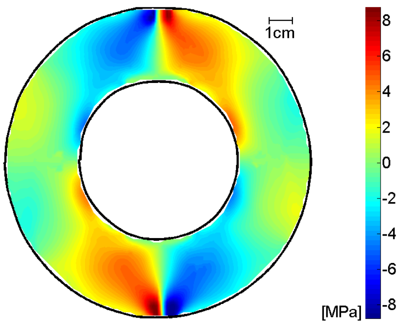


Fig. 6: Distribution of shear stress in a thin annular disk subjected to a diametral compressive load.

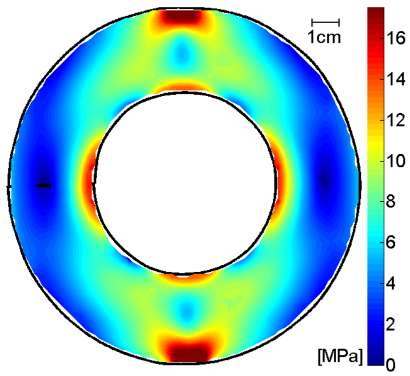


Fig. 7: Distribution of maximum shear stress in a thin annular disk subjected to a diametral compressive load.

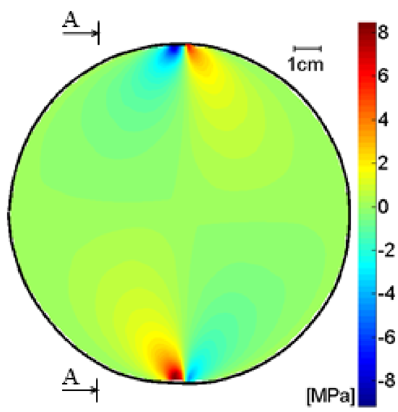


Fig. 8: Distribution of shear stress in a thin disk subjected to a diametral compressive load.

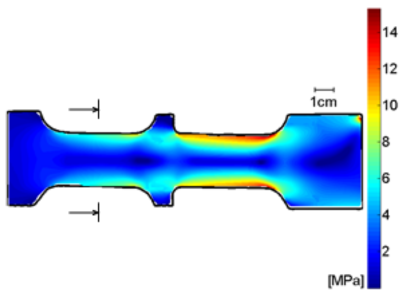


Fig. 9: Distribution of maximum shear stress in shaped beam subjected to a bending force.

The thickness of the annular disk and thin disk is equal 12.9 mm and the tested shaped beam is 10 mm thick. The experimental specimens are made of the optically sensitive material. The material of the CT 200 is used for the annular disk and thin disk and the shaped beam is made of a polycarbonate material. Young's modulus and Poisson's ratio of the CT 200 material is equal 3 124 MPa, 0.35, respectively and for the polycarbonate material is equal 2480 MPa, 0.38, respectively. The material fringe value of the CT 200 and the polycarbonate material is calibrated to 17.4 N·mm$^{-1}$ and 7.0 N·mm$^{-1}$, respectively.

Fig. 6-8 shows the stress obtained by the developed procedure for the diametral compressive load equal to 1308 N of the annular disk and 997 N of the disk. The distribution of stress for the shaped beam subjected to a bending force equal to 50 N is depicted in Fig. 9.

## 4 Comparison of the Measured and Computed Results

Results show that the stress tensor components computed by means of the analytical solution (Ana) [6] and the finite element method (FEM) are almost identical (see Fig. 10). Comparing measured (Exp) and computed results of the shear stress in the disk and of the von Mises stress in the shaped beam is depicted in Fig. 10, 11. The observed difference between the measured and the computed results is lower than 10 %.

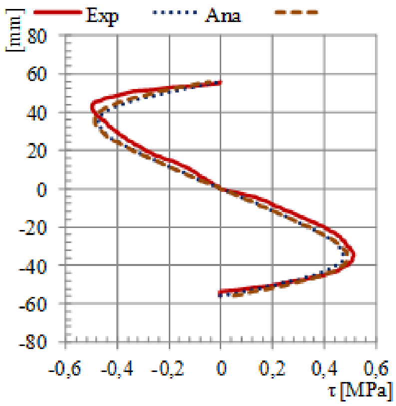


Fig. 10: Cross sectional view A-A of the distribution of the shear stress in the disk.

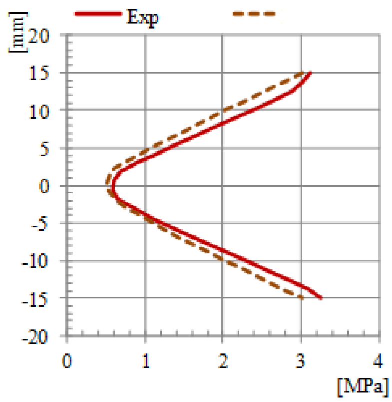


Fig. 11: Cross sectional view A-A of the distribution of von Mises stress in the shaped beam.

## 5 Conclusion

The presented paper describes the computer procedure that was used to the analysis of the isoclinic and isochromatic fringe patterns measured by the two-dimensional photoelasticity. The specimens of different shapes were tested and the results of the directions of principal stress, the components of the stress tensor, the maximum shear stress and von Mises stress were obtained. The future work will be focused on the development of algorithms for the analysis of the residual stress by means of the photoelasticity method and the photoplasticity method of forming processes.

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