Stability of Thin-Walled C-Section Rods Without Lateral Restraints

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Abstract: This paper analyzes critical forces and stability of steel thin-walled C-cross-section beams without lateral restraints. Mechanical properties of the rods material are determined by testing standard specimens in a laboratory. Based on the obtained data, the stability analysis of rods is carried out and critical forces are determined: analytically by using the theory of thin-walled rods, numerically by using the finite element method (FEM), and experimentally by testing the C-cross-section beams. The analysis of critical forces and stability shows that the calculation according to the theory of thin-walled rods does not take the effect of local buckling into account, and that the resulting critical global forces do not correspond to the actual behaviour of the rod. The FEM analysis and experimental test show that the simplifications, which have been introduced into the theory of thin-walled rods with open cross-sections, significantly affect final results of the level of the critical force.

Keywords: Critical Force; Stability; C-Section Profile; Laterally Unrestrained; Finite Element Method.

1 Introduction

Constructions which are made of thin-walled open cross-section beams are increasingly used in engineering practice because of safety requirements and economic conditions. The behaviour of such beams under specified load is considerably more complex than the behavior of beams with solid cross-sections. Thin-walled open cross-section beams tend to buckle locally because of the shapes of their cross-sections. The basic theory of thin-walled members with open cross-sections was developed by Vlasov [1]. He applied the term "sectorial coordinate" for the first time and presented the subject of mixed torsion in a most outstanding manner. The numerical procedures as well as necessary modifications were introduced later by Murray [2] and Ojalvo [3]. The aim of this study is to determine whether the assumptions and simplifications which have been introduced in the theory of thin-walled open cross-section rods significantly affect the form of loss of the stability of the element and the level of the critical force.

2 Determination of Critical Force and Form of Loss of Stability

The calculation and analysis of stability of the steel thin-walled C-section rod (C 80/40/14/1.1 and C 80/40/14/3), Fig. 1, will be carried out: experimentally, by testing the models in the laboratory [4], analytically, according to the theory of thin-walled open cross-section rods [5] and numerically, by the FEM with the computer program SAP2000 v14 ("buckling" analysis) [6]. According to the thin-walled beam theory, the critical force of beam F_{cr} is defined according to Eq. 1:

$$\begin{aligned}\left(y_A^2 + z_A^2 - r^2\right) \cdot F^3 + \left[(F_y + F_z + F_\omega) \cdot r^2 - z_A^2 \cdot F_y - y_A^2 \cdot F_z\right] \cdot F^2 - \\ -r^2 \cdot (F_y \cdot F_z + F_y \cdot F_\omega + F_z \cdot F_\omega) \cdot F + F_y \cdot F_z \cdot F_\omega \cdot r^2 = 0\end{aligned} \tag{1}$$

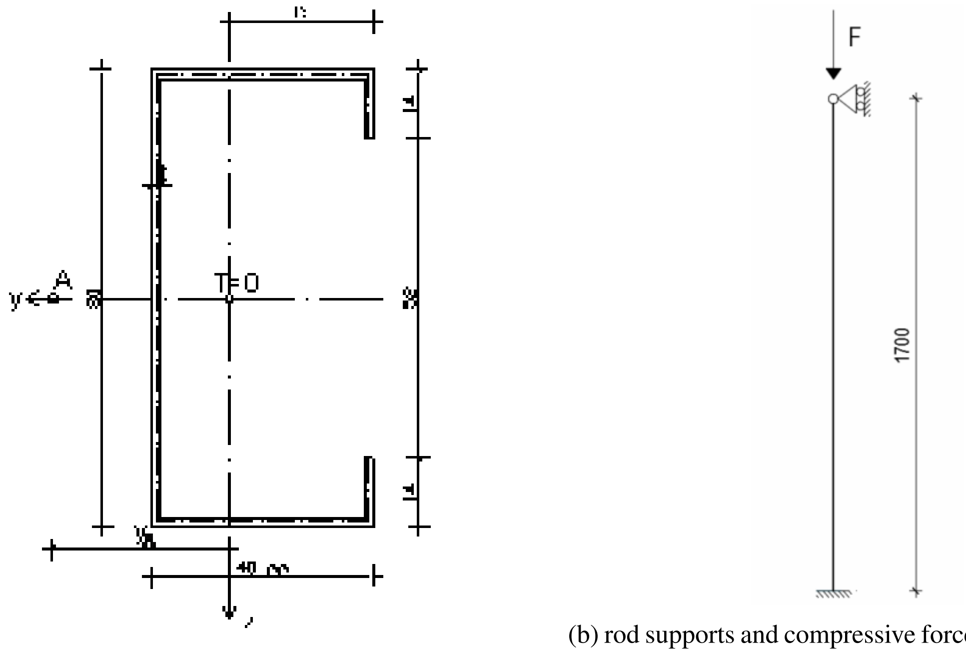
Values F_y, F_z and F_ω given in Eq. 2 are also used in Eq. 1:

$$F_z = \frac{\pi^2 \cdot E \cdot I_z}{(\mu \cdot l)^2}, \qquad F_y = \frac{\pi^2 \cdot E \cdot I_y}{(\mu \cdot l)^2}, \qquad F_\omega = \frac{1}{r^2} \cdot \left(\frac{\pi^2 \cdot E \cdot I_\omega}{(\nu \cdot l)^2} + G \cdot I_t\right) \tag{2}$$

where is

$$r^2 = \frac{I_y + I_z}{A} + y_A^2 + z_A^2 \tag{3}$$

y_A and z_A are shear centre coordinates. The values of μ = 0.7; ν = 0.7 are valid for beam fixed at one end and hinged at the other. The cubic Eq. 1 has three real and positive roots F_1, F_2, F_3. Since $F_2 < F_3$, the relevant critical force is equal to the lesser force between F_2 and F_z. If $F_z < F_2$, in-plane buckling of the rod appears, and if $F_z > F_2$, simultaneous twisting and lateral out-plane bending appear. The critical beam buckling force $F_{cr} = F_{min}$ is relevant. The thicknesses of the rod walls are t = 1.1 mm and t = 3 mm. These rods are 170 cm high.



(a) profile C 80/40/14/1.1 and C 80/40/14/3

(b) rod supports and compressive force act in the centre of gravity

Fig. 1: Thin-walled open cross section rods.

2.1 Experimental Determination of Critical Force

Mechanical properties of steel used in model fabrication were determined in a laboratory by examining a series of three standard specimens with additional strain gauges for determination of shear modulus. The specimens were prepared for testing according to EN ISO 6892-1:2009. The obtained values are: tensile strength R_M = 402.34 MPa, proof strength $R_{p0.2}$ = 321.2 MPa, modulus of elasticity E = 205.3 GPa, Poisson's ratio ν = 0.29 and shear modulus G = 79.6 GPa.

The stability of thin-walled C-section rods is tested in a static press. For laboratory testing, solid steel plates were welded on top and bottom of rods, through which compressive forces are acting. The steel plates and rods are made of the same material with the same mechanical properties. During the laboratory testing, LVDT sensors were used for measuring of the lateral displacements and strain gauges were used to measure the longitudinal strain in the rods. Disposition of measuring points is shown in Fig. 2.

Fig. 3 shows a thin-walled C-section 80/40/14/1.1 rod which is prepared for testing. The load was applied incrementally by displacement control, increment of loading for the specimens whose wall thickness is t = 1.1 mm was ΔF = 10 kN, and of those with wall thickness is t=3.0 mm increment was ΔF = 30 kN. The loss of stability of C-section 80/40/14/1.1 rod appeared under the force F_{cr} = 30.8 kN due to local buckling of the web. Fig. 4 shows the expansion of rod flanges and local buckling of web of C-section 80/40/14/1.1. Once the maximum force is achieved, strain and force are in a nonlinear relationship. Strain growths and axial stiffness of the rod fall nonlinearly. It is obvious that the global critical load will not be reached due to the local buckling. The loss of stability of the thin-walled rod C 80/40/14/3 appeared under the influence of the global critical force F_{cr} = 144.5 kN. Fig. 5 shows the deformed thin-walled rod C 80/40/14/3. Local buckling of the webs in the rods do not appear due to higher wall thickness t = 3 mm.

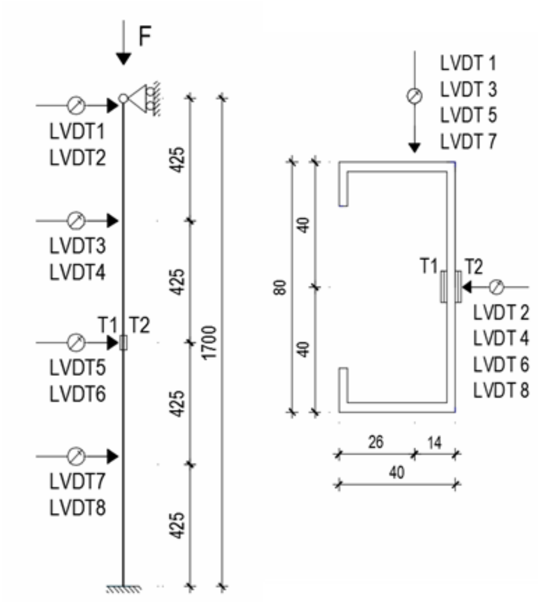


Fig. 2: Disposition of measuring points.

Fig. 3: Thin-walled C section rods prepared for testing.

Fig. 4: Local buckling of the web of rod C 80/40/14/1.1.

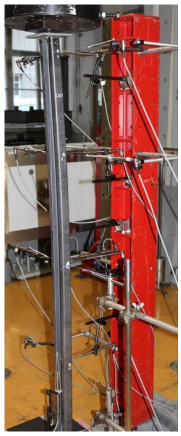


Fig. 5: Loss of stability of rod C 80/40/14/3.

2.2 Analytical Determination of the Critical Force

The values of critical forces will be determined by the theory of thin-walled open cross-section beams [5]. Geometrical properties of a thin walled *C-section 80/40/14/1.1* rod are [7]: A = 201.96 mm^2, e = 24.95 mm, I_y = 210355.6 mm^4, I_z = 48701.6 mm^4, I_t = 81.5 mm^4, i_ω = 74908776.7 mm^6. r^2 = 2469.5 mm^2; y_A = 34.45 mm; z_A = 0. Using Eq. 1 and 2, we get the critical force $F_{cr} = F_{min}$ = 42.6 kN. The loss of rod stability occurs under twisting and lateral out-plane buckling.

Geometrical properties of a thin walled *C-section 80/40/14/3* rod are [7]: A = 528 mm^2, e = 32.25 mm, I_y = 522340.5 mm^4, I_z = 114510.5 mm^4, I_t = 1584 mm^4, I_ω = 166001490.1 mm^6, r^2 = 2246.2 mm^2, y_A = 32.25 mm, z_A = 0. Using Eq. 1 and 2, we get the critical force $F_{cr} = F_{min}$= 145.5 kN. The loss of rods stability occurs under twisting and lateral out-plane buckling.

2.3 Determination of Critical Force using the Finite Element Method

The calculation of the thin-walled C- section rods is done by using a finite element method in the computer program SAP2000 v14. Thin-walled rods are modelled by shell elements. C-section rods are divided into 3400 rectangular finite elements. For the C-section 80/40/14/1.1 rod the critical force is F_{cr} = 32.7 kN. The occurrence of the local buckling of C-section rod (t = 1.1 mm) is shown in Fig. 6a. For the C-section 80/40/14/3 rod the critical force is F_{cr} = 145.8 kN. The shape of the buckling of this rod is shown in Fig. 6b.

(a) C-section 80/40/14/1.1 (b) C-section 80/40/14/3

Fig. 6: Deformed shape of thin-walled rod subjected to critical force.

Tab. 1: Comparison of critical forces.

F_{cr} [kN]	C-section rod	
	80/40/14/1.1	80/40/14/3
F_{exp} [kN]	30.8	144.5
F_{theory} [kN]	42.6	145.5
F_{SAP} [kN]	32.7	145.8

3 Conclusion

The comparison of critical forces obtained experimentally, via the thin-walled beam theory, and by finite element method, is given in Tab. 1.

In the experimental analysis of thin-walled C-section rods, the global critical force in C-section 80/40/14/1.1 rod has not been attained because of local buckling. The thin-walled beam theory offers the global critical force value without taking into account local beam buckling. The analysis based on finite element method offers the critical force value with local buckling, and shows the area of possible occurrence of local buckling, which has been proven correct through experimental testing. Theory of thin-walled beams can't be applied to the thin-walled open section rods subjected to the local loss of stability for calculating the critical force of the loss of stability, because it doesn't take the local buckling of parts of a rod into account.

References

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