Bending of Sandwich Beam with Visco-elastic Core

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Abstract: An analysis of viscoelastic bending of sandwich beam, consisting of elastic bearing layers and viscoelastic core will be given. The analysis is based on Volterra integral equations with exponential kernels and Volterra resolvent operators. Theory is verified by experimental data from three- point bending.

Keywords: viscoelasticity; sandwich beam; light core.

1 Introduction

Polymer foam-core sandwich panels and beams are increasingly used for load-bearing components in buildings. Sandwich panels and beams are offering a high stiffness per unit weight. But such structures from polymers creep at room temperature, what is limiting their use in structural applications. In this study, we combine the viscoelastic model for foam creep with the analysis of deflection of a sandwich beam to develop an expression for the creep of a sandwich beam with a polymer foam core. The results are compared with data from a series of tests on sandwich beams with polymer based composite and rigid foam core.

2 Experimental evaluating of creep for the core

For mechanical testing has been delivered a sandwich plate with bearing layers from composite, reinforced by glass fibres and polyester matrix. First, the foam creep has been tested on cuboid testing samples, which were loaded in compression by constant load in a set-up developed and manufactured in Klokner Institute. Transversal displacements were measured by LVDTs and strains by strain gauges. Load and unload application time was at least 24 hrs. The course of strain for 24h loading and 24 h unloading is shown on Fig.1.

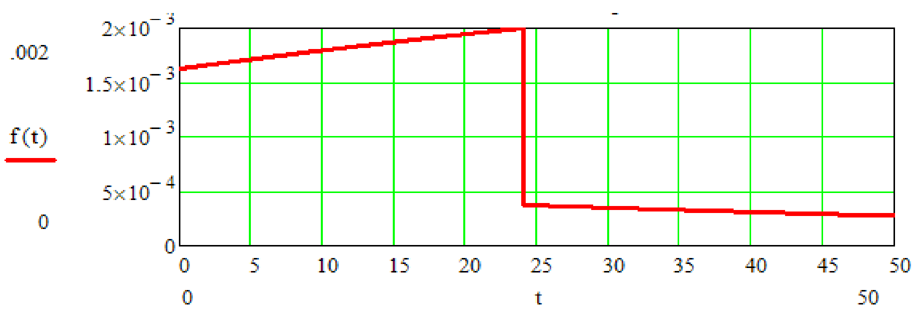

Fig. 1 Strain vs time in compression of sandwich core

Theoretical course of strain vs time according to Poynting- Thomson model has relationship

$$\varepsilon = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{E_2}\left(1 - e^{-\frac{E_2}{K}t}\right)$$

and strain is depending on three material parameters E, which can be evaluated by collocation method. For the material in Fig.1 the following values have been found:
E1=24.615 N/m2, E2=45.714 N/m2, K=1961.0 N/m2.hr

3 Experimental evaluating of vertical deflection at tree-point bending

Testing samples were put into a set-up for three point testing and loaded by a constant load 300, 600 and 800 N. Distance between supports was 200 mm, length of samples 300 mm, width 50 mm. Vertical deflection has been measured by LVDT sensors and data acquisition system National Instruments NI 1052 in program environment LabWindows. Measured values have been evaluated by MS Excel and compared with theoretical values. The comparison of results shows good agreement (< 10%). Results of long-term testing of sandwich beam are shown in Fig.2.

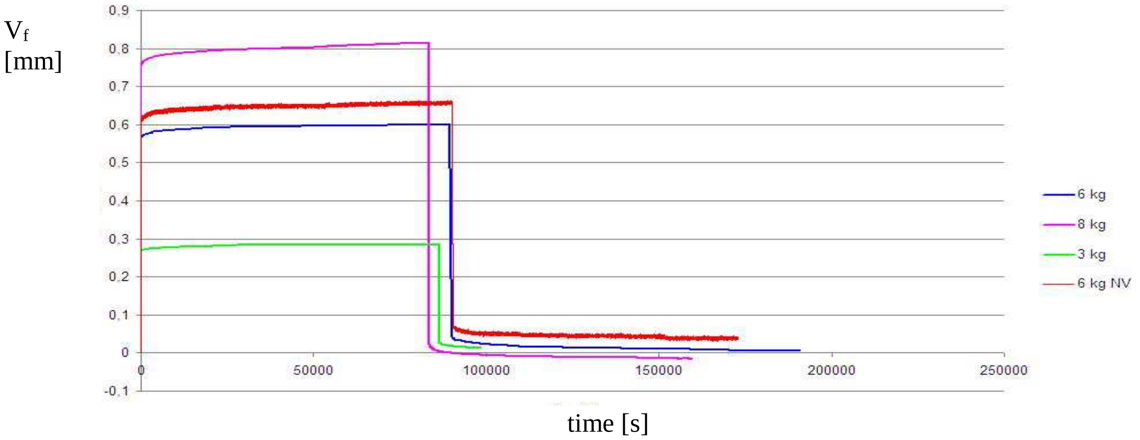

Fig. 2 Vertical displacement vs time in bending of sandwich beam

4 Formulation and solution of the problem

Paper deals with integral equations Volterra and kernels mostly of exponential type. Further, for simplicity, a symmetrical structure of the simply supported beam is considered. The load $q(x,t)$ is acting in direction of beam thickness z according to relation
$q(x,t) = g(x)\,\Phi(t)$ where $\Phi(t)$ is Heaviside function.
Below, for the simplicity, the symmetric structure of the simply supported beam is considered.

Let us treat by beam core, where the shear stress τ is of main importance and define $\tau = \varphi_1(x,t)$, $\sigma_z = \varphi_2(x,t) - z\,\varphi_{1,x}$ where $\varphi_2 = 0$ can be considered. The relations satisfy the equilibrium condition in z direction z. Further

$u_{z,z} = \dfrac{\hat{C}}{E_c}\left(-z\,\varphi_{1,x}\right)$, $u_{x,z} + u_{z,x} = \dfrac{\tilde{C}}{G_c}\,\varphi_1$, where time integral operators are of form
$C = 1 + \lambda\ C^*(\alpha)$, and parameters λ, α are changing in dependence on adopted mechanical model. After integration we obtain

$$u_x = z\left(\frac{\tilde{C}}{G_c}\varphi_1 - w_{,x}\right) + \frac{z^3}{6}\frac{\hat{C}}{E_c}\varphi_{1,xx}$$

On this basis the similar relations for bearing layers are derived. Here we will define the integral operators S, S^{-1} for a model with structural equation $E - (E / K)$.

Decisive component of stress in bearing layer is σ_x . Relation $\sigma_x = E S \varepsilon_x$ is valid and after some arrangement

$$\sigma_x = E S \left[\pm s \left(\frac{\tilde{C}}{G_c}\varphi_{1,x} - \varphi_{3,xx}\right) \mp \frac{s^3}{6}\frac{\hat{C}}{E_c}\varphi_{1,xxx} - z w_{,xx}\right]$$

kde $S = I - \frac{E}{K} E^*\left(-\frac{2E}{K}\right)$ $\qquad S^{-1} = I + \frac{E}{K} E^*\left(-\frac{E}{K}\right)$

More advantageous formulation we can get on basis of generalized function $\omega(x,t)$, where we consider

$$\varphi_1 = s_0 S \omega_{,xxx} , \qquad w = -\frac{\omega}{E r} + \frac{s S C_2}{G_c}\omega_{,xx} - \frac{s^3 S C_1}{3 E_c}\omega_{,xxxx}$$

Now one basic equation is satisfied identically and the second is transformed into form

$L \omega = -q$, where operator L is expressed by relation

$$L = \frac{1}{6} S \frac{d^4}{dx^4}\left[12 s_0^2 + r^2 - \frac{r^3 s E S}{G_c}\frac{d^2}{dx^2}\left(C_2 - \frac{s^2 G_c C_1}{3 E_c}\frac{d^2}{dx^2}\right)\right]$$

Boundary conditions for simple support are given by relations

$$\frac{d^{2k}}{dx^{2k}}\omega = 0 \qquad k = 0,1,2,3$$

At solving the problem for load $q(x,t) = g(x)\,\Phi(t)$ we will apply Fourier expansion of functions $g(x,t)$ a $\omega(x,t)$.

$$g(x) = \sum_m g_m \sin\frac{m\pi x}{l} , \qquad \omega(x,t) = \sum_m \omega_m \Phi(t) \sin\frac{m\pi x}{l} \qquad (m = 1,2,3, \dots)$$

By substitution into the basic equation we obtain

$$\omega_m = \frac{6\frac{l^4}{m^4\pi^4} g_m}{12 s_0^2 + r^2 + \frac{r^3 s}{G_c} E S \frac{m^2\pi^2}{l^2}\left(C_2 + \frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2} C_1\right)} \frac{1}{S\Phi(t)}$$

As illustration we give expression for vertical displacement for the middle of the beam ($x = \frac{l}{2}$).

$$w = -\sum_m \left[\frac{1}{E r} + \frac{s}{G_c} m^2 \frac{\pi^2}{l^2} S\left(C_2 + m^2\frac{\pi^2}{l^2}\frac{s^2}{3}\frac{G_c}{E_c} C_1\right)\right] \Phi(t)\,\omega_m \sin\frac{m\pi}{2}$$

$m = 1,2,3, \dots.$

After substitution we obtain the relation

$$w = -\frac{6l}{\pi^4 E}\sum_m g_m \frac{1}{m^4}\frac{\frac{l}{r}+\frac{s}{l}\frac{E}{G_c}m^2\pi^2 S\left(C_2+\frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2}C_1\right)}{\frac{12{s_0}^2+r^2}{l^2}+\frac{r^3 s}{l^4}\frac{E}{G_c}m^2\pi^2 S\left(C_2+\frac{s^2}{3}\frac{G_c}{E_c}\frac{m^2\pi^2}{l^2}C_1\right)}\frac{\Phi(t)}{S\Phi(t)}\sin\frac{m\pi}{2}$$

For concentrated load we can introduce a generalized function $\omega(x,t)$ instead of functions $\varphi_1(x,t)$, $w(x,t)$ by substitution $\omega_{,xxx} = -2s_0\,\varphi_1$ and further

$$w = \frac{\omega}{2Ers_0^{\,2}} - \frac{\tilde{C}}{2G_c s_0}\omega_{,xx} - \frac{s_0\hat{C}}{12E_c}\omega_{,xxxx}$$

By applying the boundary conditions $\omega = \omega'' = 0$ we receive for the middle of the beam

$$w\left(\frac{l}{2},t\right) = \frac{\bar{P}\,l^3}{48E.2rs_0^{\,2}} + \frac{\tilde{C}\,\bar{P}\,l}{8G_c s_0}$$

Operator $\tilde{C}$ is of the form $\tilde{C} = 1 + \lambda\ C^*(\alpha)$. Parameters λ, α are changed according to adopted mechanical model. For the Poynting-Thomsonův model (linear solid) and arrangement $G_c - G_x/K_2$ it will be valid

$$\tilde{C}\,\Phi(t) = \Phi(t) + \frac{G_c}{K_2}\int_0^t e^{-\frac{G_x}{K_2}(t-\tau)}\Phi(\tau)\,d\tau = 1 + \frac{G_c}{G_x}\left(1-e^{-\frac{G_x}{K_2}t}\right)$$

Relation $\tilde{C}\,\Phi(t)$ will be substituted into the above equation:

$$w\left(\frac{l}{2},t\right) = \frac{\bar{P}\,l^3}{48E.2rs_0^{\,2}} + \frac{\tilde{C}\,\bar{P}\,l}{8G_c s_0} = \frac{Pl}{8bs_0}\left(\frac{l^2}{12Ers_0}+1+\frac{G_c}{G_x}\,(1-e^{-\frac{G_x}{K_2}t})\right)$$

Conclusion

Experimental evaluation of the analysis of bending for sandwich beams of a symmetrical structure with elastic bearing layers and viscoelastic core is given. Three point testing by a constant load has shown good agreement with theoretical values.

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