

# Bond Graph Methodology for Modeling of Dynamical Systems and Derive the Equations of Motion

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**Abstract.** The aim of the thesis is the application of Bond Graph methodology for modeling dynamic systems. In the first section of the article the bond graph notation is defined and its underlying concept is explained. In the following parts is solved an example electromechanical model of elevator. Electromechanical model of elevator is solved by this approach at the level of its physical behavior. Technique of the bond graph theory is demonstrated and its place in the process of modeling of dynamic system of elevator behavior is discussed. From a bond graph diagram of the electromechanical system of an elevator, using a step-by-step procedure, system equations may be generated. As a starting point a model of an elevator system is taken.

## Introduction

Generally, mechatronics symbolizes the synergetic integration of mechanical engineering with electronics, informatics and control theory. On the one hand, this multidisciplinarity has many positive features, but on the other hand, the design process is complex and requires the cooperation of several engineering specialties. Each expert for the given area is however prone to see the problem from his perspective. These differences in the views often lead to misconceptions and misunderstandings between different specialists. Ultimately, this may decrease the market position and business value of the product.

In order to achieve the desired properties of the mechatronic system its inherent complexity must be underpinned already during its design. The way how to do this has been known since ancient times: divide et impera (divide and rule). The design process is usually decomposed into several parts which perform some activity. One example of a possible design process is shown in Fig. 1 [1].

We can notice that the design process is divided into two parts: abstract and concrete. In the abstract part the mechatronic system is modelled on an abstract level, which helps to understand and define the problem area, logical structure and logical behaviour of the mechatronic systems. For the description of the logical structure and logical behaviour are used formal languages such as UML (Unified Modelling Language).



Fig. 1 Mechatronics design process activity

In the concrete part an important role is played by realisation factors such as the used technologies (mechanics, electronics or software), size, power etc. Here, the physical behaviour of the bodies (motors, sensors, actuators, regulators etc.) and their networking is important. By the term "physical behaviour" we mean the behaviour of the system which depends on physical parameters such as weight, spring stiffness, inductance etc. and is mathematically described by differential respectively difference equations [3, 4].

In this article, we focused on the area of modelling of the physical behaviour of lumpedparameter mechatronic systems using performance charts (bond graphs). In traditional approaches to physical modelling multidisciplinary system, the governing equations are derived from a combination of Newton's laws, Kirchhoff's laws, Bernoulli's equations, and other fundamental governing equations in different domains of knowledge [2]. The method of bond graphs uses the opposite approach. First, the modelled dynamic system is distributed to subsystems and their mutual bonds are shown by a directed graph. At first arises the graphical simulation scheme - oriented graph, from which is then derived the mathematical model of physical behaviour [3-5] in the form of state space equations.

## **Bond Graphs Characteristic**

The bond graph technique is a graphical language of modelling. Component energy ports are connected by bonds that specify the transfer of energy between system components. Components regardless of any energy domains are connected through a lossless line called power bond. The direction in which the power flow is assigned, a positive value is indicated by a half-arrow on one end of the bond as it is shown in Fig. 2. The language of bond graphs aspires to express general class physical systems through power interactions. In the bond graph method, power consists of two variables which are known as generalized effort generalized flow denoted by e and f respectively. The power flowing in the bond is defined as the product of an effort and a flow variable. The factors of power i.e., effort and flow, have different interpretations in different physical domains. Yet, power can always be used as a generalized coordinate to model coupled systems residing in several energy domains. A subsystem is represented by a closed line with a name. This line represents the frontiers of the subsystem [2, 5].

For each energy interchange of the system with its environment we associate to it an energetic port of a defined type (mechanical energy, electrical energy, etc.). A unidirectional semi headed arrow shows the energy interchange throw this port and carries the data relative to the power transported (e: effort and f: flow). These two variables are necessary and sufficient to describe the energetic transfers inside the system. They correspond to a couple of variables in each energetic domain.

Energetic domain	Effort e	Flow f
Translational mechanics	Force	Velocity
Rotational mechanics	Torque	Angular velocity
Electricity	Voltage	Current
Hydraulic	Pressure	Volume flow rate
Thermal	Temperature	Entropy change rate
	Pressure	Volume change rate
Magnetic	Magneto-motive Force	Magnetic flow

Table 1 Effort and flow	variables in some	physical domains
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The elementary components/subsystems are classified by their energetic behavior (energy dissipation, energy storage, etc.), by their function inside the system (flow sensor, etc.).

Table 2 Effort and flow	variables in s	ome physical domains
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Junction 0:	f-f1-f2-f3=0
all efforts are equal Ex: Parallel connection in electrics.	e=e1=e2=e3
Junction 1:	f=f1=f2=f3
all flows are equal Ex: Series connection in electrics.	e-e1-e2-e3=0



Fig. 2 a) Example of 0-junction; b) Example of 1-junction

Junctions can connect two or more bonds. There are only two kinds of junctions, the 1 and the 0-junction. They conserve power and are reversible. They simply represent system topology and hence the underlying layer of junctions and two-port elements in a complete model (also termed the Junction Structure) is power conserving.

1-junctions have equality of flows and the efforts sum up to zero with the same power orientation. Such a junction represents a common mass point in a mechanical system, a series connection (with same current flowing in all elements) in an electrical network and a hydraulic pipeline representing flow continuity, etc. So, a 1-junction is governed by the following rules: The flows on the bonds attached to a 1-junction are equal and algebraic sum of the efforts is zero. The signs in the algebraic sum are determined by the half-arrow directions in a bond graph [2, 6].

0-junctions have equality of efforts while the flows sum up to zero, if power orientations are taken positive toward the junction. This junction represents a mechanical series, electrical node point and hydraulic pressure distribution point or pascalian point. So, a 0-junction is governed by the following rules: The efforts on the bonds attached to a 0-junction are equal and the algebraic sum of the flows is zero. The signs in the algebraic sum are determined by the half-arrow directions in a bond graph [5, 7].

Modeling is started by indicating the physical structure of the system. The bonds are first interpreted as interactions of energy, and then the bonds are endowed with the computational direction, interpreting the bonds as bilateral signal flows. During modeling, it need not be decided yet what the computational direction of the bond variables is. Not that, determining the computational direction during modeling restricts submodel reuse. It is however necessary to derive the mathematical model (set of differential equations) from the graph. The process of determining the computational direction of the bond variables is called causal analysis. The result is indicated in the graph by the so-called causal stroke, indicating the direction of the bond (Fig. 3) [4, 7].



Fig. 3 Determine the signal direction of the effort and flow

The last information that is necessary for simulation purposes indicated in the chart is causality. Bond graphs have a notion of causality, indicating which side of a bond determines the instantaneous effort and which determines the instantaneous flow. It is necessary to determine which of the two power signal is for that multi-port independent variable and thus to the multi-port through a performance bond entering and which is dependent variable, and thus to the multi-port through a performance bond go out. Causality is characterized with perpendicular line to binding at the site of the ports, where is input values of effort (e) as an independent variable [8-10].

Because the interaction energy is a function of two variables, so for each element of the bond graph there are two possible options for input and output (Fig. 4):

a) Effort e – input, flow f – output,

b) Effort e – output, flow f – input.

In describing by energy variables on the edge must respect the following principles illustrated in Fig. 4.



Fig. 4 Principles of describing oriented edges with bond variables

#### **Model of the Elevator Mechanism**

The aim of the simulation using Bond Graph is for us to build the model of elevator mechanism. The article describes how to use graphical formulation of system-modeling techniques for engineering systems involving power interactions which in English literature to named Bond Graphs to solve simple model of lifting equipment - lifts. In various parts of the contributions is given formalism making procedure for establishing performance graph electrical power and mechanical parts for lifts and subsequently derived an equation of state of the system [10-12]. Solving simple model of the lift consists of a DC motor, gear, drum and load Fig. 5 is shown.



Fig. 5 Model of the elevator mechanism

The process of construction of the bond graph and consequently the equation of state is described in the steps below.

Modeling is a part of the first electric motor and the mechanical part of the engine with a mechanical lift parts. The mechanical part of the lift is composed of mechanical parts acting rotary motion and mechanical parts acting translational movement [13]. Plans of components elevator model is illustrated in the following figure (Fig. 6) where:  $u_z$  - voltage source  $u_i$  - induced voltage,  $\omega_1$  - angular velocity in the motor output,  $M_h$  - drive torque at the output of the electric motor,  $I_M$  - inertia rotor engine,  $R_{11}$  - mechanical resistance of the motor,  $R_{12}$  - gearbox mechanical resistance,  $\omega_2$  - angular velocity of the gearbox output,  $I_B$  - inertia drum,  $m_K$  - cabin weight,  $m_C$  - the weight of the passenger,  $k_C$ ,  $b_C$  - stiffness and damping feet of the passenger.



Fig. 6 Parameters of the elevator mechanism with domain information

Description of the flow of current and voltage in the scheme of DC motor is shown in Fig. 7 and schematic diagram of electric and mechanic domain of the motor of the elevator model is shown in Fig. 8.



Fig. 7 Parameters of the elevator mechanism with domain information and flow of the voltage and current



Fig. 8 Schematic diagram of electric and mechanic domain of the motor of the elevator model

The electrical and mechanical part of the engine and the description of voltage and current flow are shown in Fig. 9.



Fig. 9 Schematic diagram of electric and mechanic domain of the motor

#### **Mechanic Domain of the Motor**

In this part is shown the procedure for establishing bond graphs step-by-step. We identify the individual components of the system according to the first step of construction of the bond graph [4, 11]:

 $\begin{array}{lll} \bar{S} ource \mbox{ effort, } e - u_Z & SE: u_Z \\ Inductor, L \mbox{ (Inductance)} & I:L \\ Resistor, R & R:R \end{array}$ 

Gyrator – DC motor as the inverter electrical power to mechanical power of acting rotary motion. The result is the following bond graph of the electric motor (Fig. 10).



Fig. 10 Bond graph of electric domain of the motor

We simplify bond graph by the rules of simplifying. In the simplified model we marked the integral causality with vertical lines in the Fig. 10. The parts of bond graphs within the ellipse are going to be replaced by single edge found above the ellipse in the Fig. 11.



Fig. 11 The parts of bond graphs within the ellipse are going to be replaced by single edge While  $u_i=u_R$ , we use reduction in 1-junction (Fig. 12).



Fig. 12 Mark of reduction in 1-junction

The result is the following bond-graph of the electric domain of the DC electric motor with the suggested connecting the mechanical parts (Fig. 13).



Fig. 13 The resulting bond graph of the electric part of the motor

#### The Mechanic Domain of the Motor and Elevator

In the first case we consider a model of a mechanical part of the lift, which consists of rotation and translation part of lift mechanism. The contemplated is the mechanical part of the electric motor  $(I_M, R_{11})$  [11].

According to the rules for the compilation of bond graph of mechanical systems is a model for said rotational and translational mechanical parts of the lift (Fig. 14) constructed bond graph in the next steps. Rotating part will contain axis of the winding drum and the gate transformer (gateway) and the translational part of the cord will actually exit gate [14].



Fig. 14 Schematic diagram of the mechanical elevator model

The load will affect the force of gravity there will be a source effort with SE: mg. **Step 1:** Identify all elements of the system:

Transformer - in the system shown in Fig. 14 it is a transmission  $(TF:r_2/r_1)$  and drum (TF:D/2).

**Step 2:** In the system we mark the reference angular velocity  $\omega$ ref in rotational movement and the reference velocity vref in translational motion (Fig. 15).



Fig. 15 Reference angular velocity and translational velocity

**Step 3:** In the model of the mechanical system to identify and select all junctions with different velocity (flows) and give them a unique name in Fig. 16. In the solution model to name and mark velocity:  $\omega_1$ ,  $\omega_2$ ,  $v_1$ .



Fig. 16 Mark different velocity

**Step 4:** Mark significant points with common speed (the velocity reference is not because it is zero) by the type of junctions. Velocity detected in step 3 draw using 1-junction as in the mechanical system. The reference velocity is not rendered because it has zero velocity. Velocity marked using 1-junction (Fig. 17).



Fig. 17 First skeleton of bond graph with velocity shown as 1-junctions

**Step 5:** Identified angular velocity differences and linear velocity differences. No differences velocities in the system we have.

Step 6: Mark junction 0 and construct difference velocities using the junction 0. No differences velocities in the system we have and not connect 0-junction. Junction structure is now ready and can be connected to individual elements. Connect the transformer to the junction 1 connect and source effort which is drive torque Mh and source effort the load, the gravitational force mg (Fig.18).



Step 7: All elements are connected to the appropriate junctions, as shown in Fig. 19.



Fig. 19 Bond graph of the mechanic domain of elevator - connecting the individual elements to the 1junction





Fig. 20 The resulting bond graph of the mechanical part

The following section then simplify the model so that we to reduce sliding mechanical part and get a model with only rotating mechanical parts (Fig. 21). The inertia characteristics of the slider are contained in the moment of inertia  $I_Z$  (Fig. 22). Bond graph in this section is different from the resolution model of reduced further by changing the transformer comprises a rotational motion to linear, and takes into account the source efforts sliding materials (Fig. 20). This section derives the bond graph of reduced mechanical parts, which in the next parts, we convert to the signal diagram and then the state diagram.



Fig. 21 Adjusted schematic diagram of the model of lift

The results of bond graphs applications are shown in the bond graph diagram of the mechanical part of motor and in the bond graph diagram of the rotational mechanic part of elevator (Fig. 22). In formulating the dynamic equations that describe the system, causality defines, for each modeling element, which variable is dependent and which is independent. By propagating the causation graphically from one modeling element to the other, analysis of large-scale models becomes easier.



Fig. 22 The resulting bond graph of the mechanical part

#### The Conversion from Causal Bond Graph to the State Block Diagram

Once a fully augmented bond graph is available, the equations for the system can be developed in a very orderly fashion. Frequently, when the system is small or uncomplicated in structure, state-space equations can be written down directly. However, as system size and complexity grow, the need for an organized procedure for equation generation becomes apparent.

Very general and powerful procedures are available for producing sets of system equations. There are three simple steps to be followed:

- 1. select input and energy state variables;
- 2. formulate the initial set of system equations;
- 3. reduce the initial equations to state-space form.

Selection of inputs is straightforward. For each source element write on the graph the input variable to the system. These variables will appear in the final state equations if they have any effect on system behavior. The list of input variables will be called **U**. The state variables can be placed in a vector called the state vector  $\mathbf{X}$ . The derivative of the state variables will

always be either an effort or a flow variable.

State diagram is a special case of block diagrams. In contrast to the bond graph, which shows the flow of power, in the State diagram illustrates the transmission of signals (physical quantities) represented power and energy values, collectively known as e (effort), f (flow), p (generalized momentum), q (generalized coordinates). State diagram peculiarity lies in the fact that the whole dynamics is modeled elementary energy accumulators integrators (Fig. 23a). Other blocks are block diagrams of state for constant multiplication and summation member.

Appliances energy - resistors are elements (Fig. 23b) which convert the energy into heat. Examples are electrical resistors and dampers in the mechanics. The natural direction of flow is from the source to the resistor [4].



Fig. 23 Bond graph element: a) blocks of state diagram, b) the 1-port resistor, c) the 1-port Capacitor, d) the 1-port Inductor

The capacitor accumulates and releases energy (Fig. 23c). The physical element is such a spring, flexible shaft, capacitor, open tanks, pressure vessels or electrical battery. The capacitor is usually selected flow from the source to the capacitor. Inductor accumulates and releases energy (Fig. 23d). Physically, such a factor is the material body, flywheel, or long pipe coil. For integral causality we need to enter the inductors effort (force).

Transformers (Fig. 24a) and gyrators (Fig. 24b) to performed the regulatory function of the power converter circuits that transform the performance of a single physical nature to another.



Fig. 24 Type of a) transformers and b) gyrators

## Schematic Diagram of the Electric and Mechanic System of Elevator

In the first case we consider the model of the mechanism of the lift (Fig. 25), which consists of the mechanics domain performing the rotational motion (transmission), and the mechanics domain performing the translational movement (translational motion of the elevator cabin). We consider the mechanical part of the electric motor ( $I_M$  - moment of inertia of the engine,  $R_{11}$  - viscous friction in the motor bearing). The weight of the gearbox is neglected [15, 16].



Fig. 25 Schematic diagram of the elevator

This model is simplified in that we reduce the mechanical portion acting linear motion and get a model with only mechanical parts acting rotary motion (Fig. 26). Inertial properties of the acting sliding movement are contained in the moment of inertia  $I_z$ .

According to the rules for the compilation of performance graph of mechanical systems it is for that model of the lift (Fig. 26) designed bond graph marked with integral causality.



Fig. 26 Adjusted schematic diagram model of the lift

The resulting graph of electrical power and a reduced mechanical part is after editing in the form (Fig. 27).



Fig. 27 The resulting bond graph of the DC motor and mechanical parts of the elevator acting rotational motion with designating the conflict of the causality

Conflict of the causality in bond graph of the DC motor and rotational mechanic part of the lift is shown in Fig. 28.



Fig. 28 Conflict of the causality in the bond graph of the DC motor and rotational mechanic part of the elevator

There are two ways to resolve this problem of conflict of causality. The first method assumes a certain torsion flexibility of a long shaft to the gearbox output. This is physically justified because there is a perfectly rigid body. The second way we use if we want to have a model with fewer equation of state. In this case we accept a derivative causality (Fig. 27) by drawing it in the state diagram.

#### **Expansion to Block Diagrams**

The next section describes a three-step transition from the bond graph to the block diagram for the model of the lift in which the mechanical part consists only of the rotating part (Fig. 26). Block diagram will be prepared for the bond graph in Fig. 28. We consider the ideal source effort  $SE:u_z$  and derivative causality in integrator I:  $I_M$ .

Step 1: All symbols of nodes and elements of the bond graph shall be marked circle.

**Step 2:** Individual edge bond graph will be replaced by a pair of signal edges and circled symbols nodes shall be connected by these pairs of signal edges (Fig. 29). The orientation signal edge is done in accordance with the marked causality (Fig. 28). Where it is intended that the ideal source of effort is the voltage already is a bond graph with the pair of signal edges in the form of Fig. 29.



Fig. 29 Expansion of bonds to bilateral signal flows of the elevator model

**Step 3:** All nodes are replaced circled by the block structure of Fig. 23 and Fig. 24. Using the rules set out in the literature [5, 6] will be set up state diagram in Fig. 30, which serves as a basis for drawing up the equation of state.



Fig. 30 The resulting block diagram of the model in standard form

When drawing up models of dynamic systems they are always based on certain assumptions. Any real system can be complicated. In engineering practice is an effort to assemble as possible the simplest models. But simplification must be to the extent that the model gave us the answer to the problem studied. Therefore, when compiling models it is necessary to decide whether some of inductors, capacitors and resistors can be neglected. Such a simplification, if bad, may lead to such a fit of the model to its use for the next job is difficult. In the bond graph will be arise algebraic loops or we have to use a derivation causality.

#### **Derivation of Equation from Causal Bond Graphs**

In the block diagram of Fig. 30 has the inductor  $I_Z$  differential causality. Energy variable momentum  $L_2$  is not a state variable. State variable for this dynamic system are: inductance  $\Phi$  and momentum  $L_1$ . Energy variable  $L_2$  is with state variable in algebraic relationship [17]. Therefore, before we write the equation of state, we see this relationship.

$$L_2 = I_Z . \omega_2 = I_Z . \frac{1}{n} . \omega_1 = I_Z . \frac{1}{n} . \frac{1}{I_M} . L_1,$$
(1)

$$\frac{dL_2(t)}{dt} = \frac{I_Z}{n.I_M} \cdot \frac{dL_1(t)}{dt}.$$
(2)

From block diagram (Fig. 29) for state variables we write:

$$\frac{d\Phi(t)}{dt} = u_L, \tag{3}$$

$$\frac{dL_1(t)}{dt} = M_{IM}, \qquad IM \equiv I_M, \tag{4}$$

we obtain

$$u_{L}(t) = u_{Z}(t) - u_{R}(t) - u_{i}(t) =$$

$$= u_{Z}(t) - R.i(t) - C.\omega_{1}(t) =$$

$$= u_{Z}(t) - R.\frac{1}{L}.\Phi(t) - C.\frac{1}{I_{M}}.L_{1}(t).$$
(5)

First state equation is in form:

$$d\Phi(t) = -\frac{R}{L} \cdot \Phi(t) - \frac{C}{I_M} \cdot L_1(t) + u_Z(t)$$
(6)

The state equation from block diagram (Fig. 30) could be expressed as:

$$M_{IM}(t) = M_{h}(t) - M_{RI1}(t) - M_{1}(t) =$$

$$= C.i(t) - R_{I1}.\omega_{1}(t) - \frac{1}{n}.M_{2}(t) =$$

$$= \frac{C}{L}.\Phi(t) - \left(\frac{R_{I1}}{I_{M}} + \frac{R_{I2}}{nI_{M}}\right).L_{1}(t) - \frac{I_{Z}}{n^{2}I_{M}}.\frac{dL_{1}(t)}{dt},$$
(7)

where

$$\frac{dL_{1}(t)}{dt} = \frac{C}{L} \cdot \Phi(t) - \left(\frac{R_{l1}}{I_{M}} + \frac{R_{l2}}{nI_{M}}\right) \cdot L_{1}(t) - \frac{I_{Z}}{n^{2}I_{M}} \cdot \frac{dL_{1}(t)}{dt},$$
(8)

$$\left(1 + \frac{I_{Z}}{n^{2}I_{M}}\right)\frac{dL_{1}(t)}{dt} = \frac{C}{L} \cdot \Phi(t) - \left(\frac{R_{I1}}{I_{M}} + \frac{R_{I2}}{nI_{M}}\right) \cdot L_{1}(t).$$
(9)

Second state equation is in form:

$$\frac{dL_{1}(t)}{dt} = \frac{C}{LQ} \cdot \Phi(t) - \frac{\left(\frac{R_{l1}}{I_{M}} + \frac{R_{l2}}{nI_{M}}\right)}{Q} \cdot L_{1}(t),$$
(10)

where

$$Q = \left(1 + \frac{I_Z}{n^2 I_M}\right). \tag{11}$$

The equation of state of the elevator are in the matrix form:

$$\begin{bmatrix} \frac{d\Phi(t)}{dt} \\ \frac{dL_1(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{R}{L} & -\frac{C}{I_M} \\ & \frac{R_{I_1}}{I_M} + \frac{R_{I_2}}{nI_M} \\ \frac{C}{L.Q} & -\frac{R_{I_1}}{Q} \end{bmatrix} \cdot \begin{bmatrix} \Phi(t) \\ L_1(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot u(t).$$
(12)

The state variables for this dynamic system are the inductance  $\Phi(t)$  and the momentum L1(t). Their another solution may be for example in Matlab/Simulink [18, 19].

## Conclusions

Most commonly used software systems for simulation of dynamic systems requires as input data in the form of model equations in block diagram form (Matlab/Simulink) [15]. Create equations of motion or draw the block diagrams it is not simple in more complex systems. Therefore, it is preferred to use a bond graphs methodology, it can create the bond graphs and described the equations of motion less demanding.

Modeling of dynamic systems decomposed into subsystems and their interactions shows a direct graph. This will create a first graphical simulation scheme, which subsequently derived a mathematical model that is assembled from oriented graph then put together a mathematical model. Bond graph is usually compiled easily and be accused of it better dynamic characteristics of the system than the equation of state or other graphical model specially designed for electrical, mechanical and other systems. Construction of bond graph of complex systems often helps to understand the physical nature of their activities.

Bond graphs represent a convenient tool for physical systems analysis and a precise way to represent a mathematical model. Often schematic diagrams are not entirely clear about whether certain effects are to be included or neglected in the model.

This advantage will appreciate in relation to the signal block diagram. Construction of the power graph of complex systems often helps to understand the physical nature of their activities. Formalism performance graphs and algorithmic approach to generating describing differential equations is useful for the analysis of dynamic systems with the transformation of various forms of energy (mixed systems - mechanical, electrical, hydraulic and others) occurring in mechatronic systems and there is also the largest application which is shown in the analysis of electric and mechanic domain of the elevator. Many systems involving two or more forms of energy, such as mechanical, electrical and hydraulic, there are no standard schematic diagrams that clearly indicate assumptions made in the modeling process.

This method allows a deeper insight into a dynamic system and his internal interaction and thus to better understand the behavior of the system. The inner workings of mechanical systems are better understood, visible feedback, reverse influence. A practical example of an elevator systems model is presented as the application of this methodology [20-25].

The motivation in writing this article is help the users, get familiar with the technique and develop confidence in using it. In that form it can work to serve the educational purpose of finding out about the Bond Graphs theory.

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## Nomenclature

f	: Flow
e	: Effort
uz	: Voltage source
ui	: Induced voltage
$\omega_1$	: Angular velocity in the motor output
$\omega_2$	Angular velocity of the gearbox output
$M_h$	: Drive torque at the output of the electric motor
I <sub>M</sub>	: Inertia rotor engine
I <sub>B</sub>	: Inertia drum
R <sub>11</sub>	: Mechanical resistance of the motor
<b>R</b> <sub>12</sub>	: Gearbox mechanical resistance
m <sub>K</sub>	: Cabin weight
m <sub>C</sub>	: Weight of the passenger
k <sub>C</sub>	: Stiffness
b <sub>C</sub>	: Damping feet of the passenger
$\Phi(t)$	: Inductance
$L_1(t)$	: Momentum

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