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Measurement of Vibration Responses of a Rotating Disc Using DIC Method

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Abstract. The paper deals with measurement of operational deflection shapes and modal shapes of a rotating disc using high-speed 3D DIC system. Since, the system is able to capture the motion of an object that is composed of a rigid body motion (rotation and translation) and flexible body motion (deflections caused by inertia forces or excitation forces), there is necessary to separate these two motion components to obtain the responses corresponding only to vibration. The method how to do it is explained in the theoretical part of the paper. The use of this method is presented in the practical part that describes three experiments: run-up analysis, ODS analysis, and operational modal analysis of a rotating disc.

Introduction

Rotating disk-like structures have a relevant interest in many engineering fields, where it is of paramount importance to determine their natural frequencies in order to avoid resonance problems that could cause an undesirable behavior or a critical failure. Experimental vibration analysis as well as modal analysis of a rotating object is a demanding task that requires the use of special measurement approaches such as Continuous-Scan Laser Doppler Vibrometry, Pulsed ESPI or high speed Digital Image Correlation [1]. In all these cases it is appropriate to exclusion rigid body motion. It can be realized by several ways, for example, by using a stroboscopic light source, frequency of which is synchronized with rotating frequency. The object observed by cameras then appears to be in steady state [2]. When the responses are measured by laser scanning vibrometer, the use of device called an optical derotator is necessary [3]. The derotator tracks the motion of the rotating object, resulting in a steady position of the laser beam. Another possibility is a numerical approach based on post-processing of measurement data when the rotation matrix and translation vector are calculated for each time step. There are several methods that make it possible [4-9]. The method used in this study uses singular value decomposition and least-squares estimation [9] to calculate rigid transformation between two positions of a rotating object. After the separation of rigid body components from the measured displacement fields, the remaining data correspond only to vibration responses of the disc. As will be shown below, it leads to more pronounced response spectrums in which the components of higher harmonics frequencies or higher modes are easily identifiable.

Elimination of rigid body movements

A rigid body motion from a position P into another position Q can be characterized by a translation vector $\mathbf{t}$ and a rotation matrix $\mathbf{R}$. Let $P = \{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_n\}$ and $Q = \{\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_n\}$

to be the sets of radius vectors describing the position of n corresponding points of the same body in three dimensional space. A rigid transformation that optimally aligns the two sets can be found by least-square method

$$(\mathbf{R},\mathbf{t}) = \arg\min \sum_{i=1}^{n} w_i \left\| (\mathbf{R}\mathbf{p}_i + \mathbf{t}) - \mathbf{q}_i \right\|^2 , \tag{1}$$

where $w_i > 0$ are weights for each point pair. Detailed procedure of the derivation of $\mathbf{R}$ and $\mathbf{t}$ is given in [9]. We provides only summary of the steps to computing the optimal translation vector and rotation matrix that minimize Eq. 1.

1. Compute the weighted centroids of both point sets:

$$\overline{\mathbf{p}} = \frac{\sum_{i=1}^{n} w_i \mathbf{p}_i}{\sum_{i=1}^{n} w_i}, \quad \overline{\mathbf{q}} = \frac{\sum_{i=1}^{n} w_i \mathbf{q}_i}{\sum_{i=1}^{n} w_i}. \tag{2}$$

2. Compute the centered vectors:

$$\mathbf{a}_i := \mathbf{p}_i - \overline{\mathbf{p}}, \quad \mathbf{b}_i := \mathbf{q}_i - \overline{\mathbf{q}}, \qquad i = 1, 2, ..., n. \tag{3}$$

3. Compute the $d \times d$ covariance matrix:

$$\mathbf{C} = \mathbf{A}\mathbf{W}\mathbf{B}^T , \tag{4}$$

   where $\mathbf{A}$ and $\mathbf{B}$ are $d \times n$ matrices that have $\mathbf{a}_i$ and $\mathbf{b}_i$ as their columns, respectively, and $\mathbf{W} = \mathrm{diag}(w_1, w_2, ..., w_n)$.

4. Compute the singular value decomposition $\mathbf{C} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$. The rotation we are looking for is then

$$\mathbf{R} = \mathbf{V}\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \det(\mathbf{V}\mathbf{U}^T) \end{pmatrix}\mathbf{U}^T \tag{5}$$

5. Compute the optimal translation as:

$$\mathbf{t} = \overline{\mathbf{q}} - \mathbf{R}\overline{\mathbf{p}} . \tag{6}$$

To suppress rigid body movements, we consider vectors $\mathbf{p}_i$ to be corresponding to initial position of the disc, and vectors $\mathbf{q}_i$ to all subsequent positions. Since the highest stiffness of the disc is near the center of rotation we introduced

$$w_i = \frac{1}{\|\mathbf{q}_i\|} . \tag{7}$$

After the computation $\mathbf{R}$ and $\mathbf{t}$ for all time steps, the new point coordinates eliminating rigid body movements can be determined on the basis of inverse transformation

$$\mathbf{q}_i^* = \mathbf{R}^{-1}\mathbf{q}_i - \mathbf{t} - \mathbf{p}_i \,. \tag{8}$$

The last adjustment is to shift the center of the disk to origin point and rotate the axis of rotation to match the z-axis of the coordinate system

$$\mathbf{q}_i^* = \mathbf{S}\,\mathbf{q}_i^* - \mathbf{r}_0 \,, \tag{9}$$

where $\mathbf{S}$ is a rotation matrix defining the transformation between vector $[0\ 0\ 1]^T$ and average normal vector of the disc in its initial position, $\mathbf{r}_0$ is a position vector of the center of the disk in its initial position related to the origin point. The Fig. 1 shows the disc at the same time step before and after the elimination of rigid body movements.

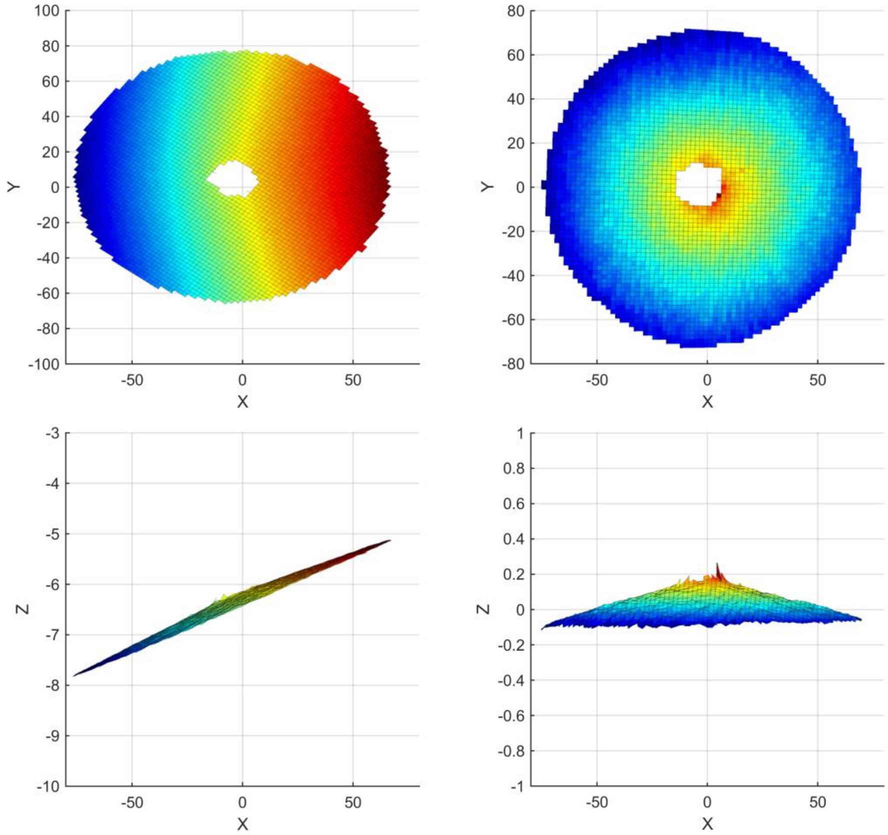


Fig. 1 Disc at the same time step before and after the elimination of rigid body movements

Experimental measurements

The object of measurement was a plane circular disc made of plastic material PS-1 commonly used in photoelasticimetry. The dimension of the disc and its material properties are shown in Fig. 2. The disc was attached to the servomotor's shaft by means of a screw connection. The servomotor was powered by DC voltage source. A thin preprinted vinyl foil with a stochastic speckle pattern was applied on the front disc's surface for the purpose of an image correlation process. The responses were measured by system Q-450 Dantec Dynamics with two high-speed cameras Phantom SpeedSense 9060. Due to high angular velocity of the disc, an extremely short shutter time 20 ns had to be used to capture sharp images. This required the use of two powerful light reflectors. The experimental setup and cameras' views can be seen in Fig. 3.

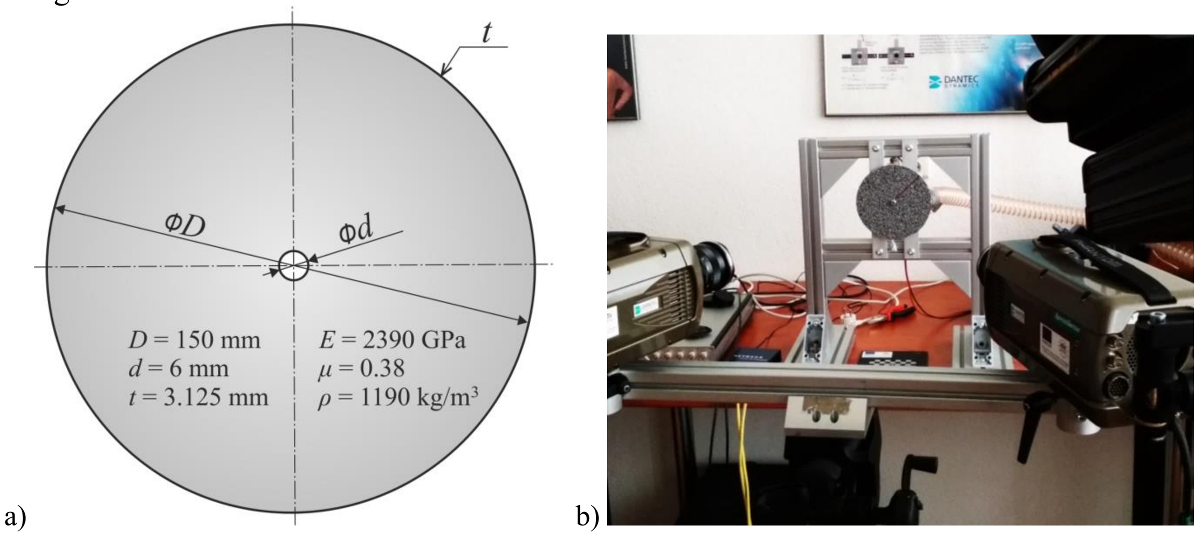


Fig. 2 a) Dimensions and material properties of the disc, b) Experimental setup

ODS analysis

The aim of the operational deflection shape analysis was to determine operational shapes of vibration of the rotating disc that is excited by inertial forces caused by its imbalance. The measurement was performed at constant revolutions 4700 rpm. A sampling frequency of the cameras was approx. 4000 fps. An image correlation analysis was carried out in Istra4D software. The obtained displacement fields were then imported to Matlab for post-processing. DICMAN 3D [10] application was used for spectral analysis and to obtain operational deflection shapes.

Fig. 3 shows the combined frequency spectrums of the responses of the rotating disc measured in z-direction. Fig. 3a relates to displacements including rigid body movements, Fig. 3b belongs to displacements after their numerical elimination. From the comparison it is apparent that the higher harmonic and sub-harmonic components are easier identifiable in the frequency response spectrum in which the rigid body movements are suppressed.

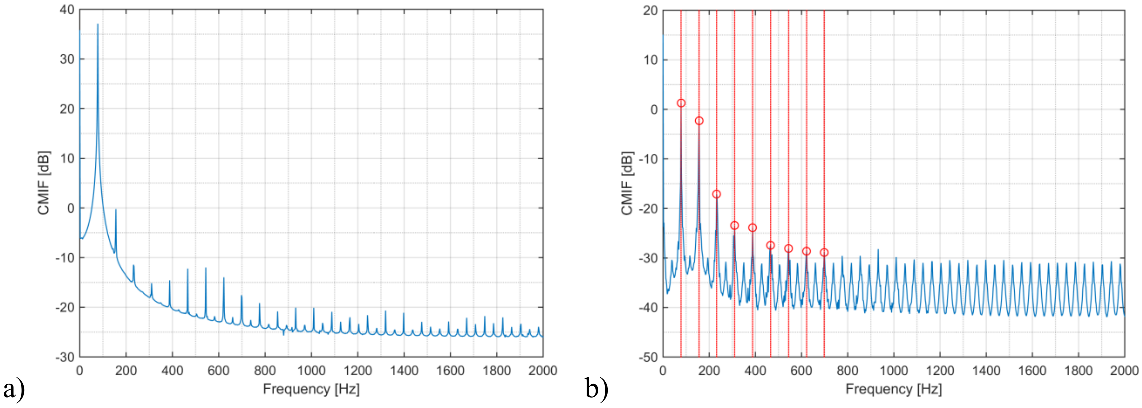


Fig. 3 Combined frequency spectrums of the responses a) before, and b) after the elimination of rigid body movements

The operational deflection shapes for the first 8 harmonic frequencies are shown in Fig. 4.

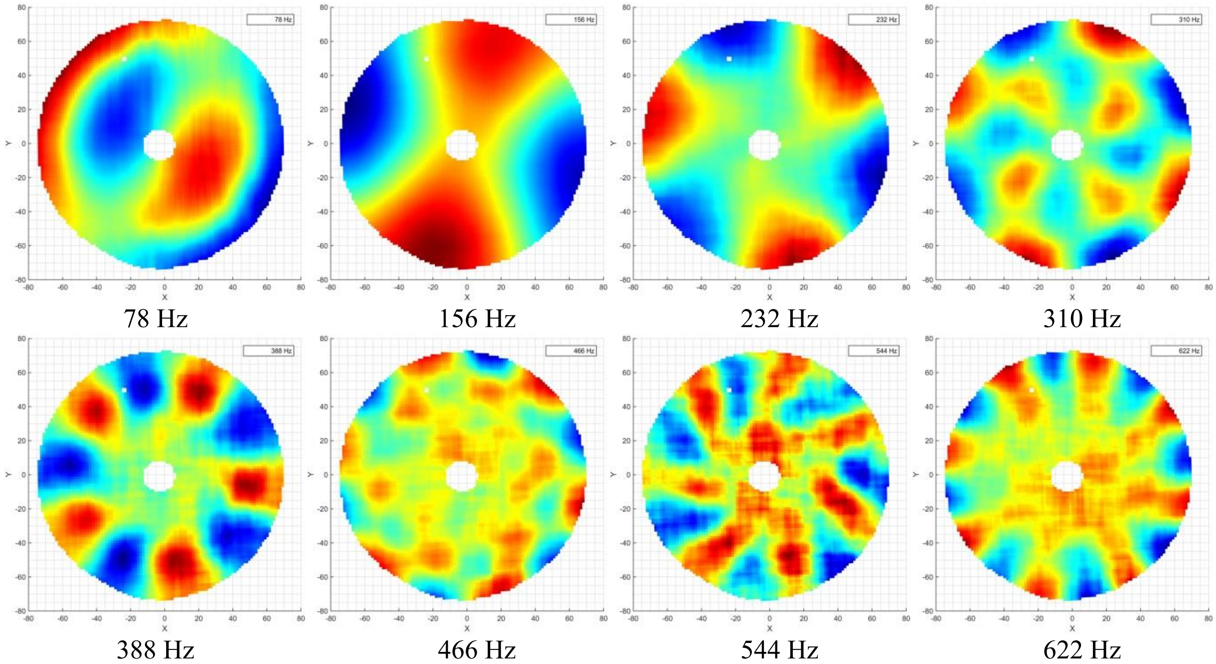


Fig. 4 Operational deflection shapes of the disc rotating at 4700 rpm

Run-Up analysis

Run-up analysis is commonly carried out to investigate dynamic behavior of rotating parts or to identify their failures. The aim of the experiment was to present the possibilities of using the presented method also in this area. The measurement was performed under the same conditions, except that the sampling frequency was 1500 fps and the maximum disc speed was about 4800 rpm. The result of the analysis is given by diagram shown in Fig. 5.

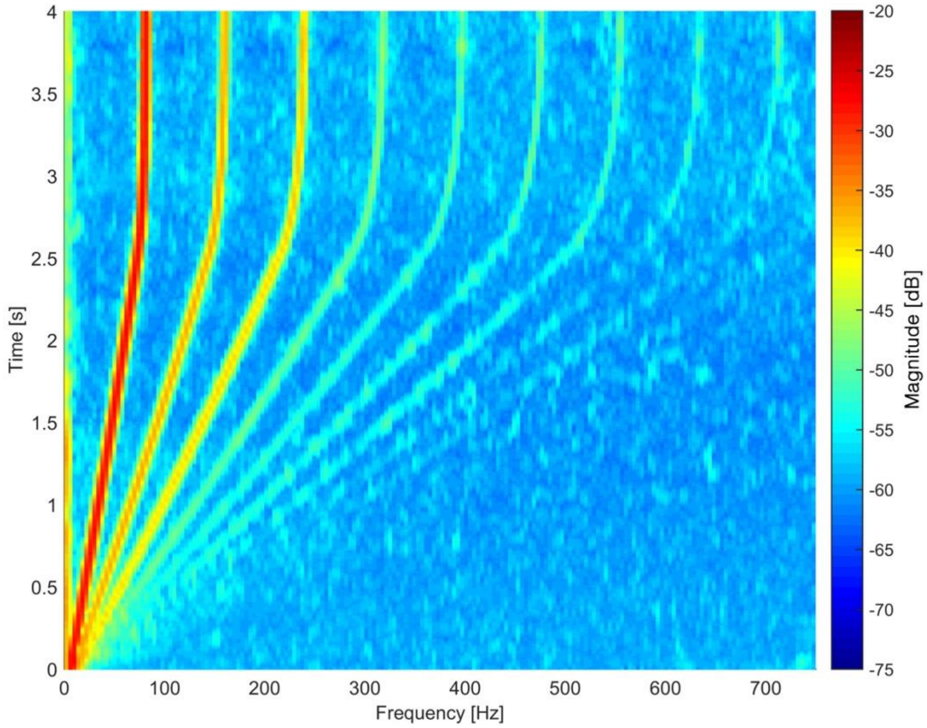


Fig. 5 Run-up diagram

The diagram shows the frequency spectrum of combined responses over time. The responses are given by displacements measured in z-direction. Their magnitudes are expressed by color field. The colored lines that can be seen in the diagram belong to fundamental frequency and its higher harmonics frequencies of the rotating disc.

Operational modal analysis

The third experiment was focused to determine modal parameters of the rotating disc. During the measurement, the disc was rotating at 4800 rpm and continuously excited from an acoustic source of sound. Fig. 6 shows the location of an excitation nozzle. A broadband excitation was ensured by the white noise signal. It should be noted that the excitation signal was not measured, i.e. modal parameters were estimated from only the response signals. The sampling frequency was set to 3000 fps.

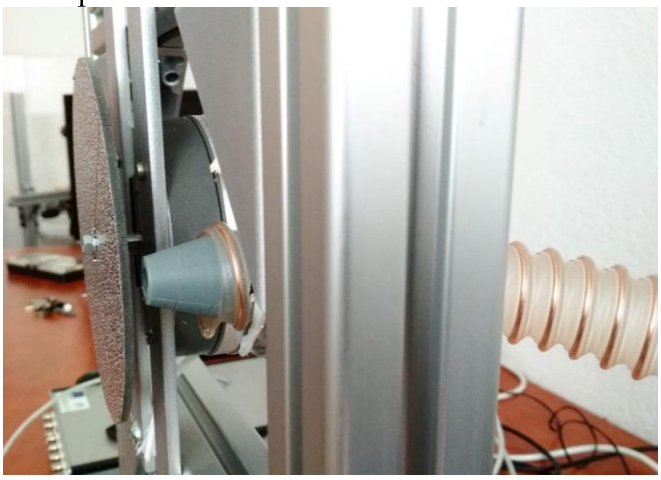


Fig. 6 Excitation of the disc

The measurement was processed in Matlab and evaluated in DICMAN 3D. Fig. 7 shows the CMIF plot of the disc. In the plot, there are peaks corresponding to modes of the disc and peaks corresponding to vibration components caused by rotation. The so called Modal Phase Collinearity (MPC) criterion can be used to distinguish them each other. The criterion expresses the linear functional relationship between the real and the imaginary parts of the unscaled mode shape vector [11]. MPC can take values from 0 to 1. The normal modes usually get values close to 1. In the frequency range up to 1500 Hz, nine modes of the disc were identified. Their natural frequencies, MPC values, and mode shapes are shown in Fig. 8.

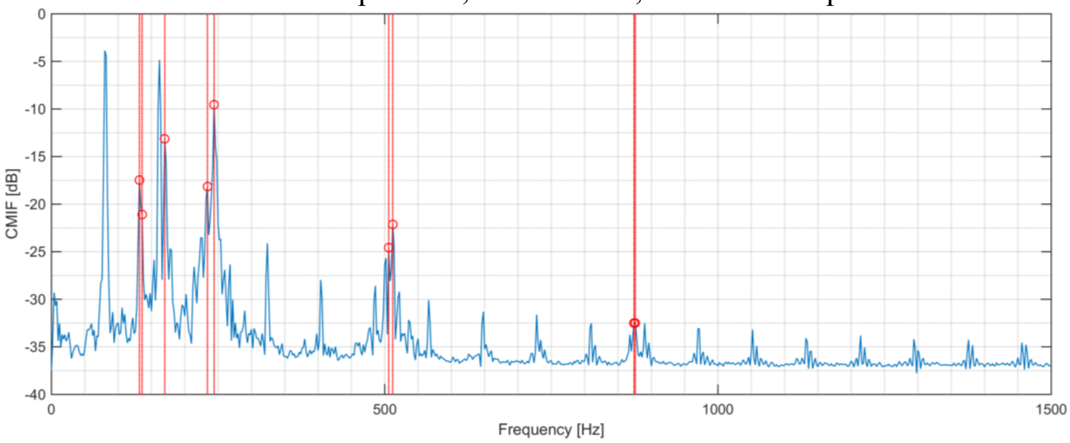
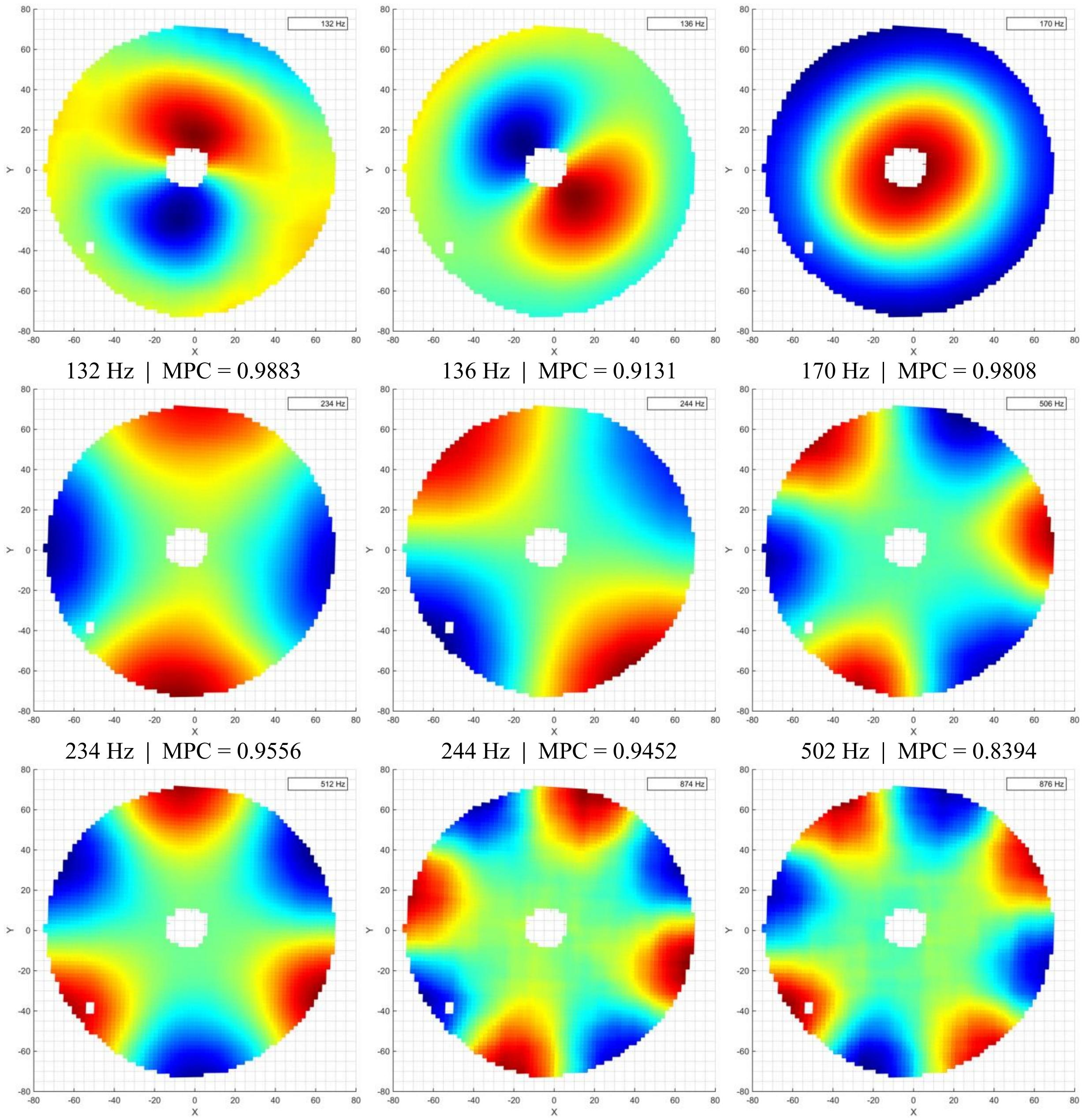


Fig. 7 CMIF plot of the rotating disc



132 Hz | MPC = 0.9883 | 136 Hz | MPC = 0.9131 | 170 Hz | MPC = 0.9808

234 Hz | MPC = 0.9556 | 244 Hz | MPC = 0.9452 | 502 Hz | MPC = 0.8394

506 Hz | MPC = 0.8268 | 874 Hz | MPC = 0.7944 | 876 Hz | MPC = 0.7712

Fig. 8 Mode shapes of the disc rotating at 4800 rpm

Conclusions

The paper presented the procedure that allows high-speed digital image correlation to be effectively used in vibration analysis of rotating objects. Its base is a suppression of rigid body movements, such as rotation and translation, which are primarily contained in measured responses. The elimination of rigid body components is given by inverse transformation of coordinates from current to initial position of the body. For that purpose, a rotation matrix and a translation vector have to be determined. The method used in this study for that purpose uses singular value decomposition and least-squares estimation. The results of the experiments described in the paper show that the elimination leads to a higher accuracy of measurement and more pronounced frequency response spectrums.

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