

Homogenization of material properties of a combined rubber/steel sheet using modal analysis

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Abstract. The paper deals with determining of material constants of a homogenized material. The introduced homogenization technique is based on experimental modal analysis and subsequent parametric optimization using finite element method. The paper describes the basic principle of the proposed technique and presents its practical application, in which the equivalent material constants of a combined rubber/steel sheet are estimated.

Introduction

When analysing the mechanical behaviour of structures, it is necessary to determine their geometry, boundary conditions and material properties. Modern, unconventional materials, such as particulate and fibrous composites, laminates, honeycombs, metal foams, various porous materials, etc., are increasingly being used today. Additive technologies are also very popular. Subject to the build direction and machine resolution, material properties of an additively manufactured part often exhibit varying material distribution (heterogeneity) and direction dependent behaviour (anisotropy), i.e. they differ from the properties of the base material. Since, the elastic constants of the materials are fundamental to analysing the mechanical behaviour of structures, significant attention is paid to their determination in engineering practice. Replacing such materials with so-called equivalent or representative material with homogenized properties is one way to effectively describe their behaviour, especially for numerical calculations. Many homogenization techniques are known and others are still being developed [1], [2], [3], [4], [5], [6]. Homogenization techniques are very effective especially in combination with FEM for modelling of 2D-heterogeneous structures such as laminates or honeycomb plates [7]. However, their application is limited by knowing the material properties of the individual components. If these are unknown, the material has to be subjected to material tests. The paper introduces the alternative approach that is based on experimental modal analysis, the results of which are subsequently used in FEM analysis combined with parametric optimization. This approach has been successfully used to determine the Young modulus of homogeneous isotropic material and to estimate the constraint stiffness of the cantilever beam [8]. In this study, the method is applied to estimate material constants of the homogenized laminated sheet that consists of two steel plies and three rubber plies. The nominal material properties and ply layup of the sheet stated by the manufacturer is shown in Fig. 1a.

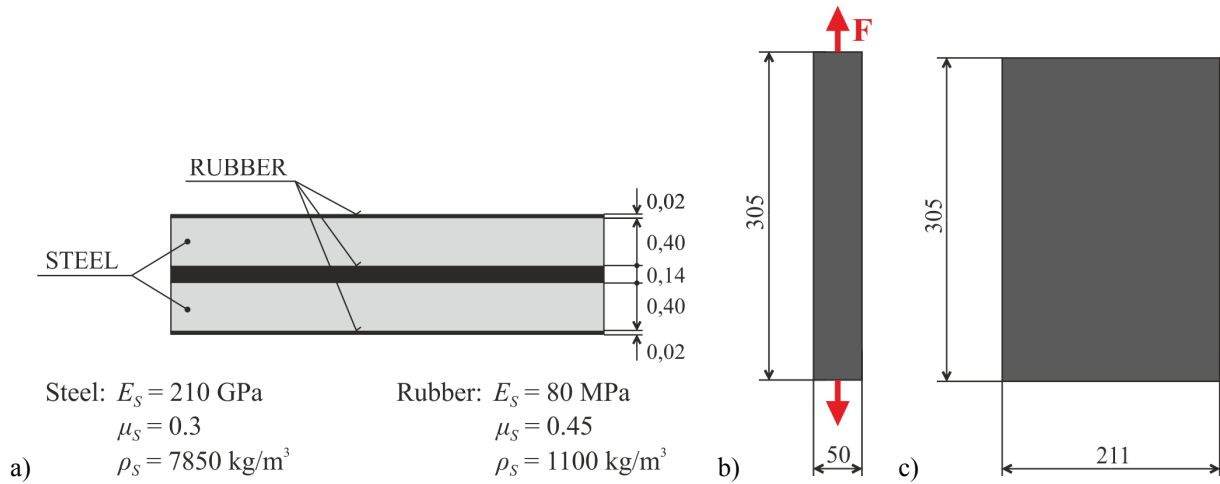


Fig. 1: a) Material properties and ply layup of the sheet, b) Dimensions of a tensile specimen, c) Dimensions of the sheet.

1. Principle of the proposed homogenization technique

The proposed homogenization technique is based on modal analysis of a material specimen. At the first phase, modal parameters, such as natural frequencies and mode shapes, are determined experimentally. The specimen should be planar and rectangular. It has to be freely supported to avoid the problem of constraint stiffness in the numerical model. The free suspension can be achieved by putting the body on a very soft pad (e.g. foam) or suspending it on soft springs. Such support is considered to be free if the highest natural frequency of rigid body modes is less than 10% of the value of the lowest deformation natural frequency [9]. At the second phase, the equivalent FE model of the same dimensions as the specimen is built. Its material properties are subsequently determined by the parametric optimization, the task of which is to tune the model so that its natural frequencies match the frequencies obtained by the measurement. Multicriterial optimization is ideal for this purpose. In this study, only one objective function is used. The optimization is based on frequency analysis, when the first natural frequency is used as an objective function. The frequency of any other mode is used as a constraint. The design variables are Young modulus and Poisson ratio that are estimated in the user defined range. However, material density must be known.

2. Verification of the principle

The principle of the homogenization technique is verified by numerical simulation of the process described in previous chapter. As the first, the homogenized properties of the equivalent material were determined from the results of FE analysis, in which the specimen shown in Fig. 1b was subjected to tensile loading. By calculation, we determine: $E = 171\,428$ MPa and $\mu = 0.3276$. These reference values will be used for verification of the proposed technique.

The natural frequencies and mode shapes of the laminated sheet (Fig. 1c) were determined by Abaqus. These frequencies can be considered as the results of an experimental modal analysis of the real sheet. The equivalent density of material was calculated by Abaqus based on nominal values. Nominal density was calculated to be 6348 kg/m³. In practice, this can be determined based on volume and weight of a specimen.

The parametric optimization was performed in program NX Nastran, in which the FE model of homogenized sheet was built. The optimization process was based on the frequency analysis SOL103 Real-Eigenvalues. The frequency of the first mode was defined as objective function. The frequency of the second mode was used as the constraint of the optimization.

The Young modulus and Poisson's ratio were design variables. The values of the objective function and design variables in the corresponding design cycles are listed in Table 1. The natural frequencies of the laminated sheet and homogenized sheet are shown in Table 2.

Table 1: Design process

Design cycle	0	1	2	3	4
f_1 [Hz]	48.49044	45.27167	47.98682	49.40555	49.55881
E [MPa]	155 000	135 818	155 818	170 576	171 637
μ [-]	0.25	0.25	0.28	0.327738	0.327738

Table 2: Natural frequencies of the sheet

Mode	1	2	3	4	5	6	7
Laminated sheet	49.561	56.630	113.21	120.98	143.46	164.02	206.96
Homogenized sheet	49.5588	55.1549	115.482	120.201	144.747	167.784	216.448

The results obtained by numerical optimization of material parameters are in good agreement with the references values. Differences in the natural frequency values are also acceptable.

3. Analysis of the real sheet

The practical use of the proposed technique is demonstrated on the real sheet. As the first, the natural frequencies of the sheet have been determined by experimental modal analysis. The sheet was putting on two very soft ribbons. Responses were measured by laser vibrometer Polytec PDV-100. The sheet was excited by impact hammer Bruel&Kjaer 8206 in 48 points. The experimental setup is shown in Fig. 2. The modal parameters are listed in Table 3.

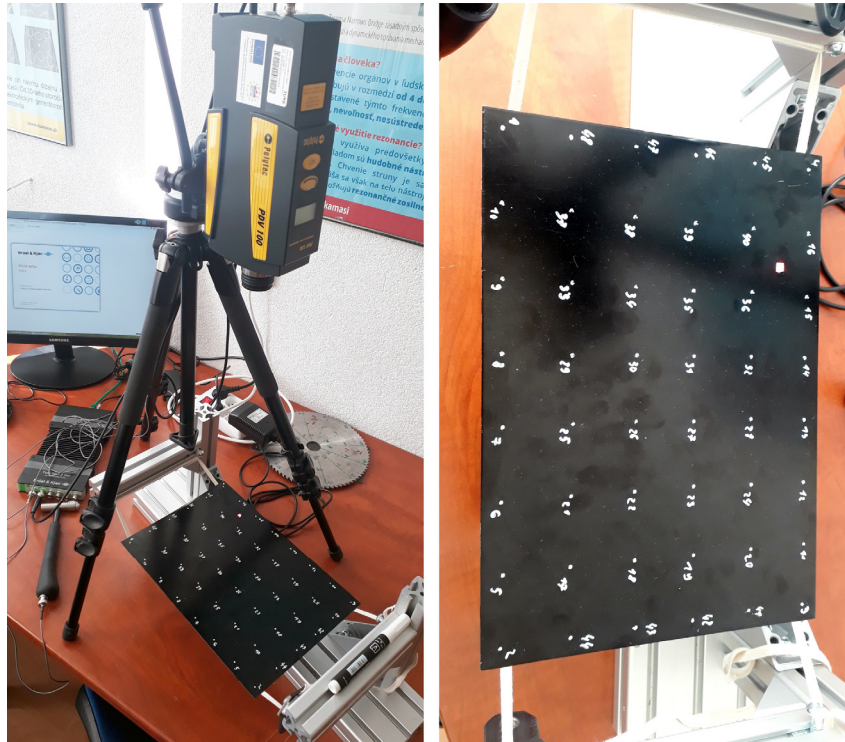
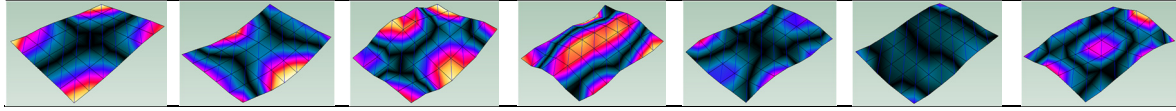


Fig. 2: Experimental setup

Table 3: Natural frequencies of the sheet

Mode	1	2	3	4	5	6	7
Frequency	45.781	80.397	108.52	120.16	141.55	191.49	205.42
Damping (%)	6.15	2.85	4.14	2.63	3.48	2.53	3.23
Complexity	0.101	0.284	0.202	0.108	0.060	0.184	0.307



It is obvious that the natural frequencies of the sheet differ from the frequencies obtained by FE analysis of the reference model that is built on nominal values. The sheet shows the relatively higher damping.

The frequencies determined by EMA were used in the parametric optimization of the homogenized models. Density of the material was determined based on volume and weight of the sheet. Its value was calculated to be 6779 kg/m^3 . The initial design values were $E_0 = 190\,000 \text{ MPa}$ and $\mu_0 = 0.3$.

The parametric optimization was performed the way described in the previous chapter with one difference, the frequency of fourth mode was used as constraint. The values of the objective function and design variables in the individual design cycles are listed in Table 4.

Table 4: Design process

Design cycle	0	1	2	3	4	5
f_1 [Hz]	49.677	50.878	51.189	48.224	46.331	45.699
E [MPa]	190 000	180 000	190 000	170 419	169 905	169 989
μ [-]	0.3	0.3	0.326	0.378	0.378	0.378

Conclusion

The paper described the homogenization technique that is based on experimental modal analysis and numerical parametric optimization where the experimentally determined value of the first natural frequency is used as an objective function and the frequency of any other mode as constraint. This two values are needed to determine two elastic material constants (Young modulus, Poisson ratio). Effective alternative is multicriterial optimization. The proposed method allows to estimate material constant of homogenized material. Its functionality was verified by numerical simulation. Practical application was presented on the combined rubber/steel sheet.

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