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Spectral Fatigue and Probability Density Function

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Abstract. The article deals with the influence of the choice of the probabilistic model in the numerical resp. analytical calculation of fatigue life in the frequency domain. In the introduction, the authors present modern probabilistic models that are suitable for application in the mentioned problem. The article also includes an experiment where an experimental modal analysis and a lifetime test are performed on 10 test-specimens. The results of the experiment are compared at the end of the paper with the numerical solution.

Introduction

Structural and mechanical systems are often exposed to irregular loads. If these loads are known in advance (for example, they are obtained experimentally), a time domain analysis based on the values of the rainflow matrix and consequently the accumulation of linear damage is generally used to determine fatigue damage and system life [1].

In fact, structures such as a car drives on rough road or a wind turbine are exposed to random loads (e.g. road surface, wind speed). Such a random load can be seen as performing a random Gaussian process, which can be described in the frequency domain using power spectral density (PSD) representing the propagation of the mean quadratic amplitude in the frequency range [2]. Working with power spectral density has proven to be particularly beneficial when working with complicated finite element models, where the frequency response calculation is much faster than transient time domain dynamic analysis [3].

One approach is the development of frequency domain fatigue assessment methods that offer a direct link between spectrum power density and damage intensity or load cycle distribution, respectively. Therefore, most authors consider the rainflow method to be the most accurate, the frequency domain methods seek to obtain the distribution of cycles by rainflow counting over a time domain [4].

At present, the analysis of frequency fatigue is dealt with by many authors who describe relatively complex load models as multiaxial loads [5-7] resp. random non-Gaussian signals[7-9].

Theory

The paper describes methods related to the stationary Gaussian process. These methods are divided into narrowband and broadband processes, of which narrowband allows direct derivation of cycle distribution. For the broadband process, the relationship of peak distribution and cycle amplitude is much more complex. Several empirical solutions (e.g. Dirlik [4] and Zhao-Baker [10]) have been proposed, but very few completely theoretical solutions (Bishop [11]). The theoretical solution presented by Bishop is based on the Markov process theory and is computationally demanding. However, there is a slight improvement in accuracy over the Dirlik method, which is usually the preferred method.

In 2004, Benasciutti and Tovo [3] compared a group of methods in the frequency domain (Wirsching-Light [12], Zhao-Baker [10], Dirlik [4], empirical α 0.75 and Tovo-Benasciutti [3]) and found that the Tovo-Benasciutti method matches the accuracy of the Dirlik method in terms of numerically simulated power spectral densities. In Table 1 some methods with corresponding formulas are mentioned.

Dirlik model. Dirlik created an empirical expression for calculating the probability density function, which was obtained using extensive computer simulations using the Monte Carlo technique. Although, this model is more complicated than other methods, it is still only a function of four spectral moments. However, this solution has a wide range of applicability, thus surpassing other available methods. The formula for calculating the assumed cumulative damage by the Dirlik method is

$$p_D(S)=\frac{\left(\frac{D_1}{Q}\cdot \mathrm{e}^{\frac{-S_i}{Q}}+\frac{D_2\cdot S_i}{R^2}\cdot \mathrm{e}^{\frac{-S_i^2}{2R^2}}+D_3\cdot S_i\cdot \mathrm{e}^{\frac{-S_i^2}{2}}\right)}{\sqrt{m_0}} \tag{1}$$

where D_1, D_2, D_3, Q, R are constants calculated from spectral moments m_0, m_1, m_2, m_4 and skewness γ . S_i in (1) represents normalized stress amplitude.

Table 1: Overview of others formulas for solving the probability density function.

Model	Formula
Rayleigh	$p_R(S)=\frac{S_i}{m_0}\cdot \mathrm{e}^{\frac{-S_i^2}{2m_0}}$
Zhao-Baker	$p_{ZB}(S)=\frac{w\cdot\alpha\cdot\beta\cdot\hat{S}_i^{(\beta-1)}\cdot \mathrm{e}^{\left(-\alpha\cdot\hat{S}_i^{\beta}\right)}+(1-\alpha)\cdot\hat{S}_i\cdot \mathrm{e}^{\left(-\frac{\hat{S}_i^2}{2}\right)}}{\sqrt{m_0}}$
Lalanne-Rice	$p_L(S)=\sqrt{\frac{1-\gamma^2}{2\cdot\pi\cdot m_0}}\cdot \mathrm{e}^{\left(\frac{-S_i^2}{2\cdot m_0\cdot(1-\gamma^2)}\right)}+ +\frac{\gamma\cdot S_i}{2\cdot m_0}\cdot \mathrm{e}^{\left(\frac{-S_i^2}{2\cdot m_0}\right)}\cdot\left[1+\mathrm{erf}\left(\frac{\gamma\cdot S}{\sqrt{2\cdot m_0\cdot(1-\gamma^2)}}\right)\right]$
Tovo-Benasciutti	$p_B(S)=B\cdot\rho+(1-B)\cdot K$

Cumulative damage is calculated based on the formula

$$E[D]=\frac{E[P].T}{k}\cdot\int_0^\infty S^b.p(S).dS \tag{2}$$

where $E[P]$ represents the peak rate, T is a time duration, k, b are material constants.

Practical part

This part consists of an experiment and a numerical simulation combined with an analytical calculation using the Matlab program.

Experiment. It consisted of an experimental modal analysis, in which the eigenmodes, eigenfrequencies and damping of the measured specimens were determined, which were used to calculate and refine the numerical simulation. The second part of the experiment was a fatigue testing at stochastic excitation. The valid measurement was performed on 10 specimens made of aluminium alloy AL5052-H32 sheet of 4 mm thickness (Fig 1.).

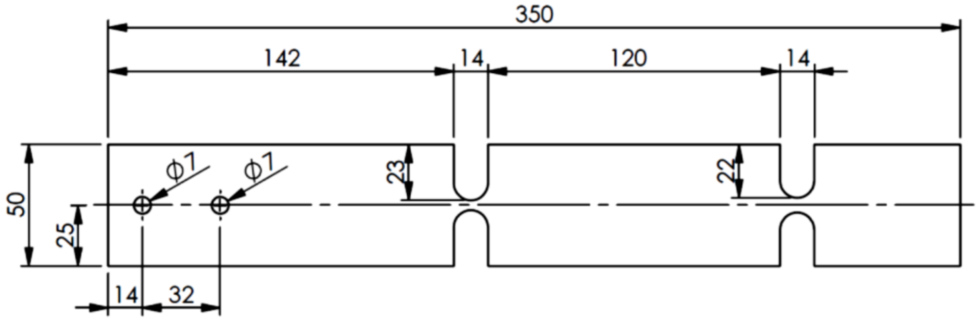


Fig. 1: Testing specimen with dimensions.

Two accelerometers were placed on the specimen. The sensor placed on the flange was acquiring the real excitation signal generated by the vibro-exciter, and the second sensor was used to measure the response of the examined specimen near the notch. A measuring configuration is shown in the Fig 2.

Experimental modal analysis (EMA) was performed by Bruel&Kjaer Pulse system. A modal hammer Type 8206 with an aluminum tip without an additional weight was used to excite the specimen. Responses were measured using a Polytec PDV100 laser vibrometer. The results of EMA are shown in Table 3 and Table 4.

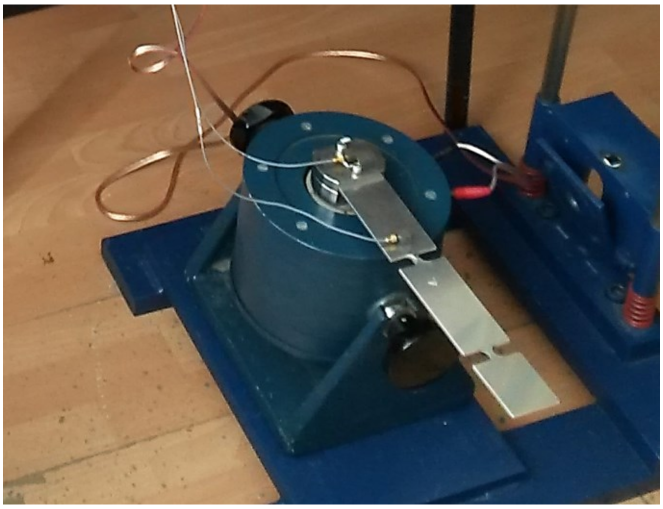


Fig. 2: Measured specimen with applied accelerometers.

Broadband white noise (1 - 1200 Hz) was used for analysis. The measurement was performed using Pulse LAN XI 3050-B-060 module and processed in LabShop software. Acceleration time histories were recorded and used in numerical analysis, the results of which were later correlated with the experiment.

Table 2: Failure times.

Specimen	Time to failure
Specimen No. 3	1040 s (17 min 20 s)
Specimen No. 5	1445 s (24 min 05 s)
Specimen No. 6	1000 s (16 min 40 s)
Specimen No. 7	815 s (13 min 35 s)
Specimen No. 8	1022 s (17 min 02s)
Specimen No. 9	820 s (13 min 40 s)
Specimen No. 10	1132 s (18 min 52 s)
Specimen No. 11	1150 s (19 min 10 s)
Specimen No. 12	1470 s (24 min 30 s)
Specimen No. 13	1058 s (17 min 38 s)

Table 2 shows the results of 10 valid fatigue tests. The time to failure ranged from 815-1470 seconds. The higher scatter of values was caused by earlier failure of two specimens (spec. No. 7 and 9), whose surface around the notch was roughened due to the application of an inappropriate technological procedure in the notch production. Therefore, these specimens were not included in the statistical processing. The remaining 8 specimens with a smooth surface were broken in the time from 1000 to 1470 seconds. Specimen after the fatigue test is shown in the Fig. 3.

Fig. 3: Specimen after failure.

Numerical simulation. The pre-simulation was carried out in SolidWorks program, where the primary calculation containing the PSD response was performed. The PSD response was obtained by the linear dynamic random vibration module.

Since the specimen was firmly attached in the flange, the model had taken all degrees of freedom at the point of contact of the specimen with the flange. The PSD function, which can be seen in the Fig. 5, was used as an excitation. The PSD was estimated using a periodogram in the Matlab program from the actually measured history of acceleration as mentioned above. The excitation was performed in a direction perpendicular to the largest specimen area.

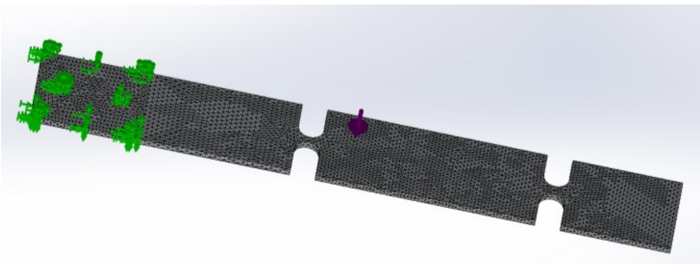


Fig. 4: Numerical model prepared to simulation

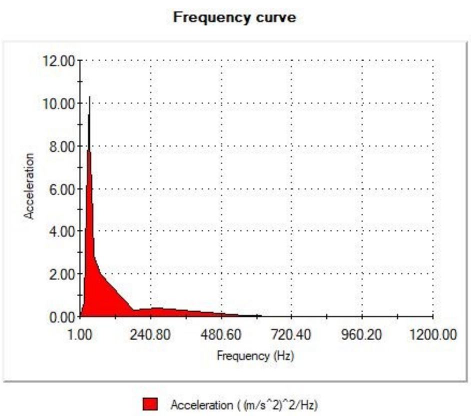


Fig. 5: PSD response

Results comparison of the numerical modal analysis with EMA can be seen in Table 3 and Table 4. The results of the numerical analysis are slightly higher. It can be caused due to the idealization of boundary conditions (perfect rigid couplings that do not exist in reality). Nevertheless, it is possible to state on the basis of the figures from Table 3 that the eigenmodes (also the numerical model of the beam) correspond to reality.

Table 3: Comparison of bending eigenmodes.

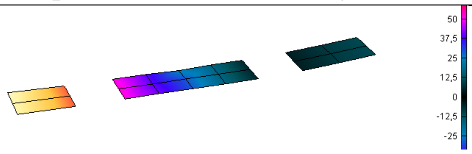
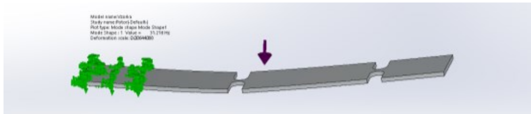
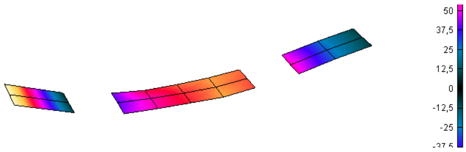
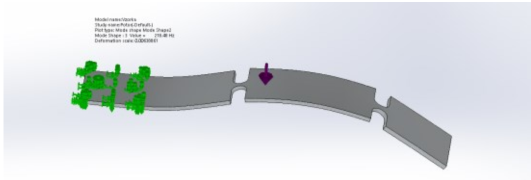
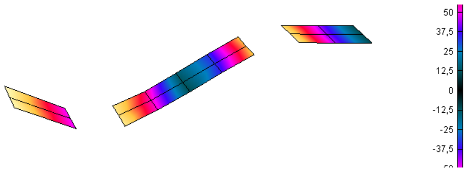
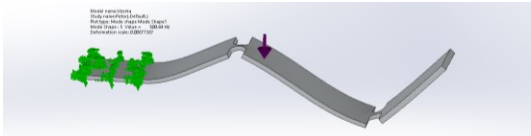
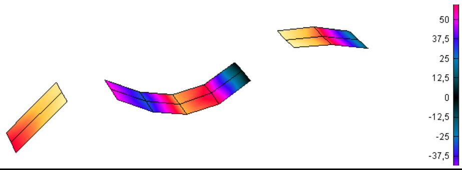
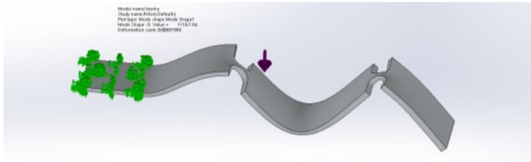
Mode	Experimental modal analysis	Simulation
1.		
3.		
5.		
8.		

Table 4: Modal parameters of the in numerical and experimental analysis.

Mode	Numerical eigenfrequency [Hz]	Experimental eigenfrequency [Hz]	Damping [%]	Mode complexity
1.	31,92	31,80	1,22684	0,01680
2.	62,21	59,32	0,51345	0,02140
3.	218,48	191,68	0,45291	0,00278
4.	283,22	279,79	0,03545	0,04141
5.	500,64	472,87	0,22234	0,00074
6.	735,78	728,28	0,31693	0,07025
7.	856,51	853,03	0,05126	0,05126
8.	1119,12	1081,9	0,09551	0,00378

Damping, based on Rayleigh damping coefficients, was used to refine the computational PSD response model.

RAYLEIGH DAMPING COEFFICIENTS:

$$\beta = \frac{2\cdot\zeta_1\cdot\omega_1 - 2\cdot\zeta_m\cdot\omega_m}{\omega_1^2-\omega_m^2} = \frac{2\cdot 0,0122684\cdot 31,8 - 2\cdot 0,0009551\cdot 1081,9}{31,8^2-1081,9^2} = 1,0999394\cdot 10^{-6}$$

$$\alpha = 2\cdot\zeta_1\cdot\omega_1 - \beta\omega_1^2 = 2\cdot 0,0122684\cdot 31,8 - 1,0999394\cdot 10^{-6}\cdot 31,8^2 = 0,7791579$$

The result of the simulation is a PSD stress field (von Misses), which is variable depending on the frequency (Fig. 6). The most endangered point (node) was selected from the critical area and this node was further subjected to a spectral fatigue analysis in the Matlab program. A PSD response of this node has been exported from SolidWorks software.

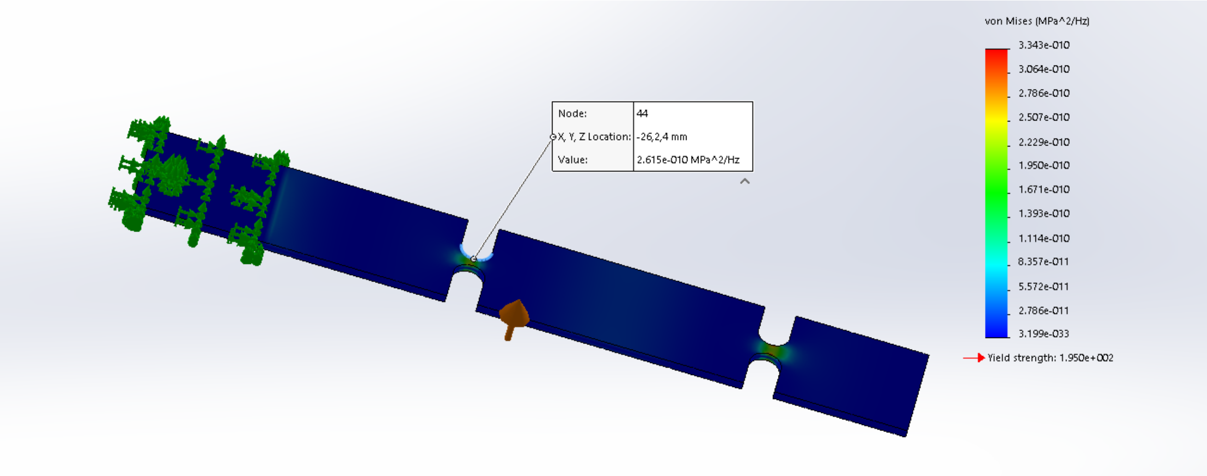


Fig. 6: Field of (von Misses) PSD stresses

Analytical calculation. A Matlab script has been programmed to determine the frequency domain lifespan using the Dirlik, Rayleigh, Zhao-Baker, Lalanne-Rice and Tovo-Benasciutti methods.

The input data for the script were the PSD response curve and the S-N curve. The program uses a common interpretation of the S-N curve based on the Basquin equation. The points of the S-N curve corresponded to real tests performed on notched standardized specimens of the AL 5052-H32 material, which was also used to produce specimens subjected to the fatigue test.

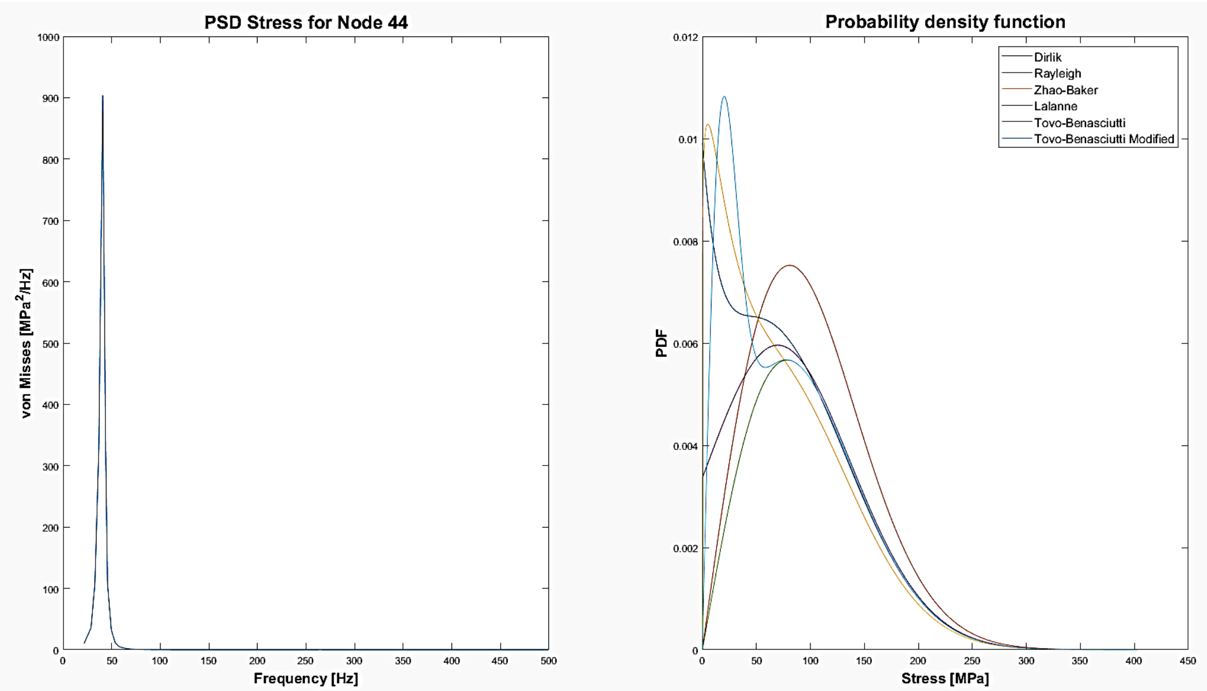


Fig. 7: PSD response for most critical node (left plot), probability density functions of all probability models (right plot)

Results are summarized in Fig. 7 and Table 5, where it is possible to see individual differences in histories of probability density functions and fatigue life estimate.

Table 5: Results of spectral fatigue analysis.

Model	Damage	Time to failure
Dirlik	0,00089622	1116 s (18 min 36 s)
Rayleigh	0,0014895	671 s (11 min 11 s)
Zhao-Baker	0,00073885	1353 s (22 min 33 s)
Lalanne	0,00092158	1085 s (18 min 05 s)
Tovo-Benasciutti	0,00090137	1109 s (18 min 29 s)
T-B Modified	0,00090163	1109 s (18 min 29 s)

Discussion

Experiment. As mentioned above, the failure of 8 smooth surface specimens occurred in the time from 1000 to 1470 s. The standard deviation in this interval was 175.9 s. In the case of specimens 5 and 12, the time to failure was much longer than in the case of other specimens. If these specimens were also excluded from the measurement, the time variance would be significantly smaller (1000 - 1150 s). When the standard deviation at this interval decreased to 55.4 s, the mean time to failure was determined based on the times of only 6 specimens (3, 6, 8, 10, 11 and 13). Thus, it can be stated that the average time to fatigue failure of the tested specimens at vibration is 1067 s (17 min 47 s).

Numerical + analytical solution. If the average failure time of the specimen is experimentally set at 1067 s, it can be stated that according to the results in Table 5, the fatigue life was the most accurately estimated by the Lalanne model. As the Dirlik and Tovo-Benasciutti models provide estimates with an error of up to 5%, this can also be taken as a positive result. From the PDF histories it can also be stated that the most damaging amplitudes were located somewhere in the range of 100-250MPa, where it can be see how the Rayleigh model is clearly higher and the Zhao-Baker model is just below the other models.

Conclusions

The paper compares models of the probability density function (Dirlik, Rayleigh, Zhao-Baker, Lalanne-Rice and Tovo-Benasciutti). The experimentally obtained average time to damage under random variable loading was 1067 s. Fatigue life was most accurately predicted by the Lalanne (error 1,68%), Tovo-Benasciutti (error 3,94%) and Dirlik (error 4,59%) models. The worst prediction of fatigue life was estimated by the Rayleigh model (error 37,11%). The differences in the results can be attributed to the ideal rigid of the specimens in the FEM program, the approximation of the PSD signal, the damping of the individual eigenmodes, and the accuracy of the S-N curves.

Acknowledges

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References

[1] F. Trebuňa, Odolnosť prvkov mechanických sústav. 1. vyd. Košice: Emilena, 2004. 980 s. ISBN 978-80-8073-148-9.

[2] P. Stoica, R.L. Moses, Spectral Analysis of Signals. [s.l.]: Pearson Prentice Hall, 2005. 490 p. ISBN 978-0-13-113956-5.

[3] D. Benasciutti, R. Tovo, Spectral methods for lifetime prediction under wide-band stationary random processes. In Int. J. Fatigue. 2005. Vol. 27, no. 8, pp. 867–877.

[4] T. Dirlik, Application of computers in fatigue analysis. Coventry, England: University of Warwick, 1985.

[5] D. Benasciutti, R. Tovo, Comparison of spectral methods for fatigue analysis of broad-band Gaussian random processes. Probabilistic Engineering Mechanics. 2006. Vol.21, no.4, pp.287-299. doi: https://doi.org/10.1016/j.probengmech.2005.10.003

[6] A. Nieslony et al., Fatigue life prediction for broad-band multiaxial loading with various PSD curve shapes. In Int. J. Fatigue. Vol.44. 2012. pp. 74–88. doi: https://doi.org/10.1016/j.ijfatigue.2012.05.014

[7] A. Nieslony, E. Macha, Spectral Method in Multiaxial Random Fatigue. Opole: Springer, 2007. pp. 152.

[8] M. Palmieri et al., Non-Gaussianity and non-stationarity in vibration fatigue. In Int. J. Fatigue. 2017. Vol.97, pp. 9-19. Doi: https://doi.org/10.1016/j.ijfatigue.2016.12.017

[9] F. Cianetti et al., The effort of the dynamic simulation on the fatigue damage evaluation of flexible mechanical systems loaded by non-Gaussian and non stationary loads. In Int. J. Fatigue. 2017. Vol. 103, pp. 60–72. doi: https://doi.org/10.1016/j.ijfatigue.2017.05.020

[10] W. Zhao, M.J. Baker, On the probability density function of rainflow stress range for statonary Gaussian processes. In Int. J. Fatigue. 1992. Vol. 14, no. 2, pp. 121–135.

[11] N.W.M. Bishop, The use of frequency domain parameters to predict structural fatigue. University of Warwick, 1988

[12] P.H. Wirsching, A.M. Shehata, Fatigue under wide band random stresses using the rain-flow method. In Published in: The Journal of Engineering Materials and Technology, July 1977, pp. 205-211.