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A Rheological Model of Fabric Elongation According to Lethersich

ŠIMIĆ PENAVA D.[1,a], PENAVA Ž.[2,b] and SKENDER, A.[1,c]

[1]Faculty of Civil Engineering, University of Zagreb, Kačićeva 26, 10000 Zagreb, Croatia

[2]Faculty of Textile Technology, University of Zagreb, Pr. b. Filipovića 28a, 10000 Zagreb, Croatia

[a]diana.simic.penava@grad.unizg.hr, [b]zpenava@ttf.unizg.hr, [c]ana.skender@grad.unizg.hr

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Abstract. Under tensile loading, tearing of fabric or unwanted viscoelastic or plastic deformations occur. In order to avoid the appearance of plastic deformations in the fabric, it is necessary to know in advance which tensile force will cause such deformations. An appropriate rheological model that describes the behaviour of fabric, depending on its geometrical and structural parameters under tensile stress, curve (F-ε), should be set. For the purpose of testing the rheological properties of fabric, various types of raw cotton fabrics have been deposited in the twill weave with same warp density and different weft densities. The samples were made on the OMNIplus 800 tt air-jet loom machine Picanol. Based on the experimental results obtained to stretch the fabric samples, a rheological Lethersich's model was introduced. Appropriate differential equations for the tested sample patterns were made and resolved, resulting in the dependence between tensile force and relative strain. The rheological model according to Lethersich describes well the process of elongation of cotton woven fabric in twill weave under static test conditions.

Introduction

The use of textile materials in various industries is increasing, and knowledge of their physical and mechanical properties is very important. When measuring, the mechanical properties of fabric are affected by nonlinear viscoelasticity, friction between fibres, yarns, fabric density, geometrical changes during connection to external forces, and changes in temperature and humidity. Tensile forces cause tearing of the woven fabric or the appearance of viscoelastic or plastic deformations [1]. Such deformations are undesirable because undesirable effects, poor quality of the finished product, e.g. woven fabric, can only be observed in the final stages of processing. Therefore, the good quality of the finished product is achieved if during the use of woven fabric such conditions are ensured in which the deformation will be within the elastic area.

In order to describe the behaviour of the woven fabric subjected to load, it is generally accepted that the woven fabric is a continuous medium, so the continuum mechanics is discussed [2, 3]. A link between stress and deformation around the arbitrary point of the fabric should be established. These connections are called the mechanical characteristics of the material. The science involved in this is called rheology [4, 5]. In order to avoid the appearance of plastic deformation in the fabric, one should know in advance at which tensile force such deformations will occur. An appropriate rheological model that describes well the fabric behaviour should be fitted. Rheology is a science that deals with the physics of deformation, and its primary goal is to establish a connection between forces, that is, stresses and their derivations over time, with the resulting deformations due to the action of these forces and their derivations over time [6]. The basic rheological properties are elasticity, viscosity and plasticity. Due to the testing of the mechanical properties of fabrics, the setting of appropriate rheological models and a good analysis of the change in load in different fabric production processes, the allowable values of the stress force of the fabrics should move within the limits of the occurrence of elastic deformations, and on the basis of this force value optimization and complete automation of the corresponding processes is performed. The aim of this paper is to set rheological model that describes relatively well the behaviour of a cotton fabric in twill weave depending on its geometrical and structural parameters at tensile load, i.e. curve (F-ε).

Relationship between stress and strain

The functional relationship between force (stress) and strain cannot be determined theoretically, but only experimentally by testing samples of fabrics. The experiment determines the relationship between forces (stresses) and strain in the form of a characteristic diagram under certain conditions. Tensile tests are carried out on woven fabric, and the experimentally measured data of force F and elongation ε are presented in the form of a characteristic force-elongation diagram (F-ε) for the woven fabric, Fig. 1. The diagram shown in Fig. 1 has several characteristic points:
F_{max} - the maximum force acting on the fabric sample
F_{pr} - ultimate force immediately before the interruption of the fabric sample
F_{inf} - force at the inflection point
F_e - force at the limit of elasticity (Hook's law applies)
ε_{max} - maximum elongation of the fabric sample at maximum force
ε_{pr} - elongation of the fabric sample at break
ε_{inf} - elongation at the inflection point
ε_e - elastic elongation (deformation) that disappears after unloading
W_{max} - area (work) below the curve to maximum force
W_{pr} - area (work) below the curve to the breaking force
W_{inf} - area (work) below the curve to the force at the inflection point

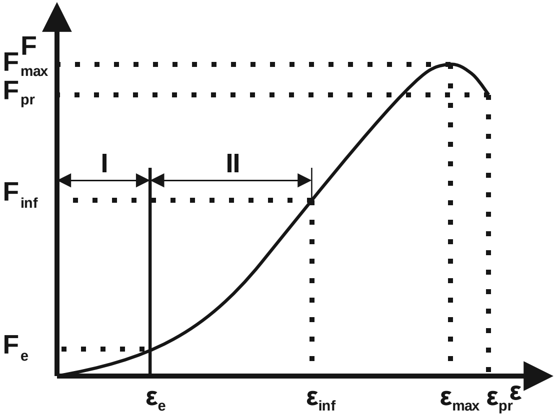


Fig. 1: The characteristic diagram of the force - elongation (F-ε) of the woven fabric

Area I is up to the elasticity limit (F_e, ε_e) and is called the elastic region that can be represented by the rheological Maxwell model. Area II is from the elasticity limit (F_e, ε_e) to the inflection point (F_{inf}, ε_{inf}) and is a viscoelastic region that can be described by the Burger's model formed by the serial connection of the Maxwell and Kelvin models.

The elasticity limits of woven fabrics are determined on the basis of the force-elongation diagram F(ε) and on the basis of F'(ε) and F''(ε). Fig. 2 represents the force-elongation function as well as its first derivation and second derivation. The maximum of the first derivation indicates the permissible load up to the limit of which the fabric shows elastic properties.

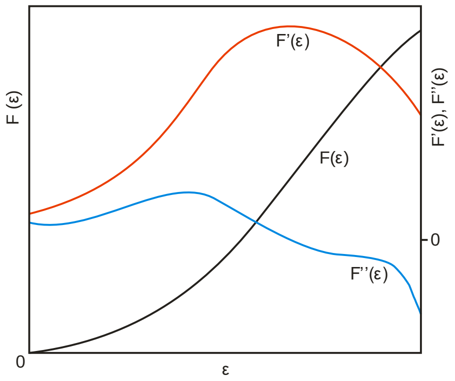


Fig. 2: Force-elongation diagram of the function F(ε), the first F′(ε) and the second F″(ε) derivation of the function

Up to this point, the woven fabric shows greater resistance to the tensile forces (F'(ε) function increases). When the function of the first derivation reaches its maximum, at that point the second derivation equals 0. Then a faster deformation of the woven fabric occurs until the destruction of the material (the function F'(ε) decreases).

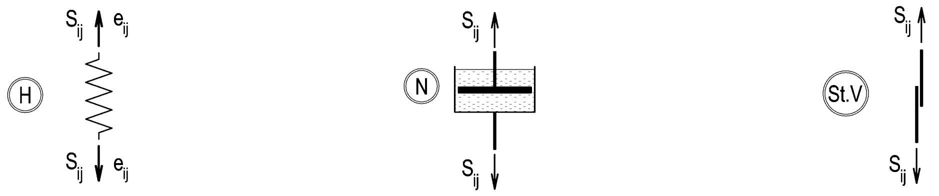
Basic rheology settings

The rheological behaviour of a material is determined by some relation containing stresses, deformations, and their derivations over time:

$$f\left(\sigma_{ij}, \dot{\sigma}_{ij}, \varepsilon_{kl}, \dot{\varepsilon}_{kl}\right) = 0 \tag{1}$$

This relationship is called rheological equation of the state of the material, Eq. (1). Rheological equations contain some scalar values that characterize the rheological properties of materials and are called rheological constants, e.g. modulus of elasticity or coefficient of viscosity. Stresses and deformations are called rheological variables. Rheological equations contain variable values: stresses, strains, strain rate and rate of change of stress [7, 8]. The basic rheological models are Hooke's, Newton's, St. Venant's model. Real bodies have at the same time properties of elasticity, viscosity and plasticity in different form and relationship. By combining simple elements, a body model can be formed to describe the behaviour of real materials [9].

The Hooke's model, i.e. the linear elastic body "H", represents the relation between the stress deviator S_{ij} and the strain deviator e_{ij}, and this relationship is linear. An ideal elastic Hooke's material is shown by an elastic spring, Fig. 3a. For this model relation $S_{ij} = 2 \cdot G \cdot e_{ij}$ is valid.



a) b) c)

Fig. 3: Basic rheological models: a) Hooke's model, b) Newton's model, c) St. Venant's model

Newton's model „N", Fig. 3b, is a viscous silencer and represents a model of a viscous fluid. For a uniaxial stress state, it is defined as $\sigma = \eta\dot{\varepsilon}$ where η is Newton's coefficient of viscosity and $\dot{\varepsilon}$ is the strain rate. Saint Venant's model „St.V", Fig. 3c, describes the plastic deformation of the material. For a uniaxial stress state, it is defined as $\sigma = \sigma_T$.

The complex rheological models (materials)

Models of base materials H, N and St.V can be interconnected in different ways, thus obtaining models of some complex materials that have complex rheological properties.

The Maxwell's model is a serial connection between Hooke's and Newton's element, also called an elastoviscous fluid having the rheological formula M = H — N, Fig. 4a. In the case of uniaxial stress state, the following expression applies:

$$\dot{\varepsilon} = \frac{1}{E}\dot{\sigma} + \frac{1}{\eta}\sigma \tag{2}$$

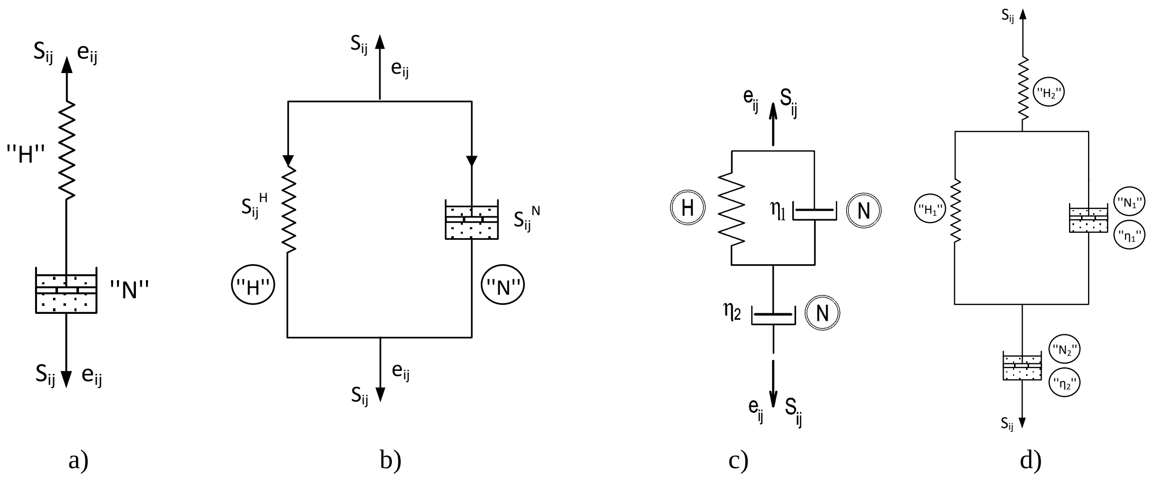


Fig. 4: Complex rheological models: a) Maxwell's model, b) Kelvin's model, c) Lethersich's model, d) Burger's model

The Kelvin / Voight model of material is formed by the parallel connecting of Hooke's and Newton's material, that is symbolically K = H │ N, Fig. 4b. It is a viscoelastic material that slowly reaches the final deformation, retains it for a long time without further noticeable increase, and when unloaded, this deformation is slowly lost, and the body returns to its original form. For uniaxial stress states the following expression is valid:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \tag{3}$$

Lethersich's model is suitable for testing the rheological properties of elongation cotton yarns, cotton woven fabrics of different types of weaves and densities, describing the behaviour of geotextiles, Fig. 4c. This material is represented by the model as a serial connection between Kelvin's and Newton's elements, so its rheological formula is L = K — N. The expression for the rate of deformation of the body according to the Lethersich model can also be written in the following form:

$$\dot{\varepsilon} = \frac{\sigma}{\eta_N} + \frac{\sigma}{\eta_K} - \frac{\sigma}{\eta_K} \cdot \exp\left(-\frac{E_K}{\eta_K}t\right) \cdot \left[\varepsilon_0 + \frac{1}{\eta_K}\int \sigma \cdot \exp\left(\frac{E_K}{\eta_K}t\right)dt\right] \tag{4}$$

where η_N is the viscosity coefficient of the Newton body, η_K is the viscosity coefficient of the Kelvin model, E_K is the coefficient of elasticity of the Kelvin model and ε_0 is the initial elongation. By derivation by time and arranging of the Eq. (4), the differential equation of the rheological model of cotton woven fabric is obtained in the form:

$$\eta_K\ddot{\varepsilon} + E_K \cdot \eta_K \cdot \dot{\varepsilon} = \dot{\sigma}(\eta_N + \eta_K) + \sigma \cdot E_K \tag{5}$$

Since in this case $\dot{\varepsilon} = const.$ and $\ddot{\varepsilon} = 0$, the Eq. (5) takes the following form:

$$\dot{\sigma}(\eta_N + \eta_K) + \sigma \cdot E_K = E_K \cdot \eta_K \cdot \dot{\varepsilon} \tag{6}$$

The solution of the differential Eq. (6) can be represented by the Eq. (7):

$$\sigma = -C \cdot \exp\left(-\frac{E_K}{\eta_N + \eta_K} \cdot t\right) + \eta_N \cdot \dot{\varepsilon} \tag{7}$$

The integration constant C is determined from the initial conditions, for $t = 0$, $\sigma = 0$. The dependence of stress on the time after the determination of the integration constant can be represented by the Eq. (8):

$$\sigma = \eta_N \cdot \dot{\varepsilon}\left[1 - \exp\left(\frac{l_0}{100 \cdot v \cdot \tau_r} \cdot \varepsilon\right)\right] \tag{8}$$

where $\tau_r = (\eta_N + \eta_K)/E_K$ is relaxation time, l_0 is the initial sample length, v is the test speed and ε elongation.

The Burger's model is applied to textile materials which have significant plastic deformation, Fig. 4d. This model is composed as a serial connection between Maxwell’s and Kelvin’s material, for its properties somewhat correspond to viscoelastic behaviour of materials. Its rheological formula is: B = M — K = (H — N) — (H ⏐ N). For the uniaxial state of stress, the following equation is obtained:

$$\left(\eta_1 \frac{\partial^2}{\partial t^2} + E_1 \frac{\partial}{\partial t}\right)\varepsilon = \left[\frac{\eta_1}{E_2}\frac{\partial^2}{\partial t^2} + \left(1 + \frac{\eta_1}{\eta_2} + \frac{E_1}{E_2}\right)\frac{\partial}{\partial t} + \frac{E_1}{\eta_2}\right]\sigma \tag{9}$$

Experimental part

In the experimental part of the paper, tests were carried out on elongation cotton woven fabric specimens in twill weave with the same warp density and different weft densities. Tensile properties of all specimens were tested according to standard ISO 13934-1:2008 using the strip method for measuring fabric strength and its elongation on a tensile strength tester Textechno Statimat M. The measurement results were collected and stored on the hard disk by a computer program of tensile strength tester. In this way, force-elongation (F-ε) curves were obtained. For the purposes of this testing standard specimens with dimensions 350 x 50 mm were cut, clamped in clamps of the tensile tester at a distance of l_0 = 200 mm with the pulling speed of 100 mm/min and subjected to uniaxial tensile load till rupture. It can be stated that the deformation rate is constant. Sample break time is 20 s. The specimens were cut in warp direction and weft direction. The direction of action of the tensile force during the test is always the same. Five tests were done for each mentioned direction of force action on the fabric specimen. Before testing all specimens were conditioned under the conditions of standard atmosphere (relative air humidity 65 ± 2%, at a temperature of 20 ± 2°C). In order to achieve balanced humidity, the yarn stood at standard conditions for 24 h before testing.

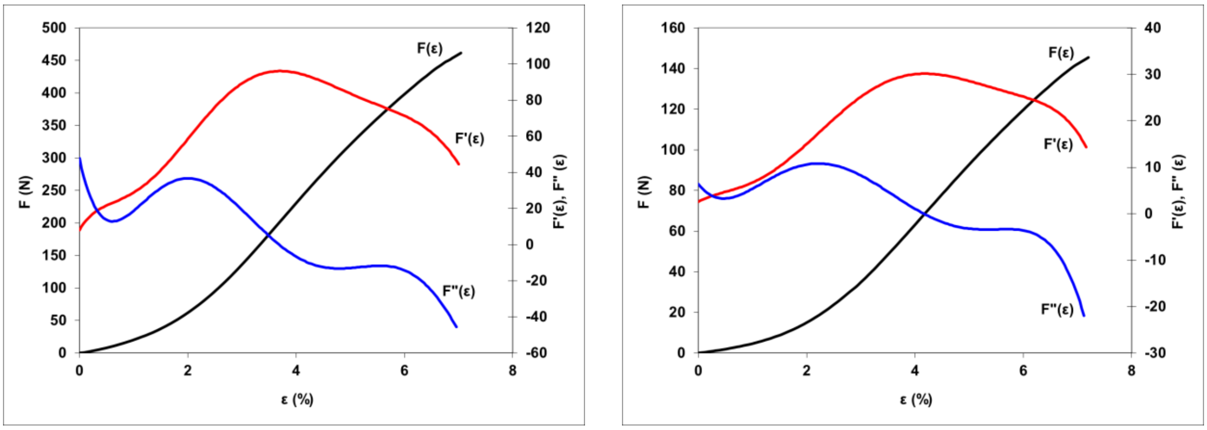
Table 1: Test results for basic fabric parameters

		Warp direction			Weft direction				
Fabric structure	Fabric tag	Yarn fibres	Yarn count (tex)	Density (cm^{-1})	Yarn fibres	Yarn count (tex)	Density (cm^{-1})	Weight (g/m^2)	Thickness fabric (mm)
	K12	Cotton	30.3	24.0	Cotton	30.3	12.0	114.89	0.362
Twill	K18	Cotton	30.3	24.1	Cotton	30.3	18.1	134.84	0.363
1/3	K24	Cotton	30.3	24.1	Cotton	30.3	24.2	156.91	0.364

Raw cotton fabrics were weaved for the purpose of testing the rheological properties of the woven fabrics. Fabrics specimens of the stated structural characteristics were made on an OMNIplus 800 tt air-jet loom Picanol. In Table 1 are the actual (measured) values of structural parameters of the raw woven fabrics. Yarn linear density was determined by the gravimetric method according to standard ISO 2060:1994. Number of threads per unit length was determined according to standard ISO 7211-2:1984. Standard ISO 5084:1996 describes a method for the determination of the thickness of fabric. The same yarn was used for the weft and the warp. Determination of the density of warp and weft threads was carried out using computer-controlled (stereo) microscope DinoLite.

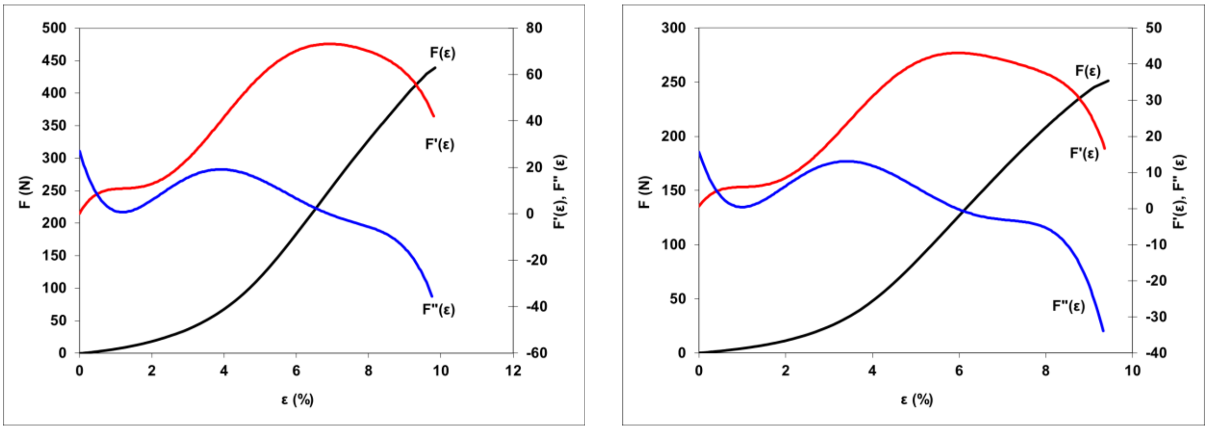
Results of testing

Specimens are tested when the tensile force acts in the warp and weft direction. As a result, another letter is introduced into the fabric labels. The first letter in the fabric label indicates the type of weave, the second letter indicates the direction of force action, and the number indicates the weft density, e.g. KP18 - twill weave, the force in the weft direction, the weft density 18.1 cm^{-1}. Diagrams of the experimental mean values of tensile force F and its longitudinal deformation (elongation) ε on fabric samples, the first derivation of the function F'(ε) and the second derivation of the function F''(ε) when the force acts in the warp and weft direction are shown in Figures 5-7.



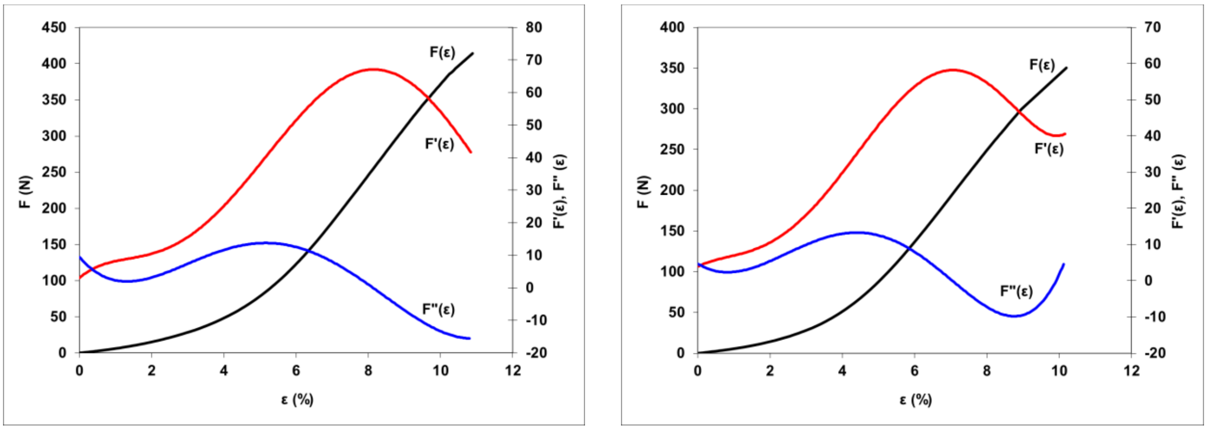
a) b)

Fig. 5: Diagram force-elongation (F-ε), the first derivation of a function F'(ε), the second derivation of a function F''(ε): a) specimen KO12, b) specimen KP12



a) b)

Fig. 6: Diagram force-elongation (F-ε), the first derivation of a function F'(ε), the second derivation of a function F''(ε): a) specimen KO18, b) specimen KP18



a) b)

Fig. 7: Diagram force-elongation (F-ε), the first derivation of a function F'(ε), the second derivation of a function F"(ε): a) specimen KO24, b) specimen KP24

The mean values of the test results of maximum force F_{max}, maximum elongation ε_{max}, work to maximum force W_{max}, ultimate force F_{pr}, elongation at break ε_{pr}, work up to break W_{pr}, force at the inflection point F_{inf}, elongation at the inflection point ε_{inf} and work up to the inflection are shown in Table 2.

Table 2: Mean values of F_{max}, ε_{max}, W_{max}, F_{pr}, ε_{pr}, W_{pr}, F_{inf}, ε_{inf}, W_{inf}

Fabric tag	F_{max} (N)	ε_{max} (%)	W_{max} (Ncm)	F_{pr} (N)	ε_{pr} (%)	W_{pr} (Ncm)	F_{inf} (N)	ε_{inf} (%)	W_{inf} (Ncm)
KO12	461.7	7.04	282.9	461.7	7.04	282.9	96.6	3.70	69.1
KP12	145.5	7.20	85.7	145.5	7.20	85.7	30.1	4.17	26.1
KO18	438.9	9.84	311.9	438.9	9.84	311.9	73.2	6.92	97.0
KP18	252.2	9.48	182.8	252.2	9.48	182.8	43.1	5.96	40.6
KO24	414.4	10.88	313.6	414.4	10.88	313.6	67.2	8.15	117.8
KP24	350.6	10.20	260.7	350.6	10.20	260.7	58.7	7.04	94.3

Setting rheological model of woven fabric to the inflection point

Predicting the behaviour of cotton woven fabrics to the inflection point when a tensile force is acting can be represented by rheological material according to the Lethersich model. When extending on a tensile tester, the deformation rate has a constant value. The initial length of the test tube is 0.2 m. The pulling speed of the clamps of the tensile tester is 100 mm/min. By introducing these data into equation (8) and assuming that the viscosity coefficients are equal $(\eta_N = \eta_K)$, Eq. (10) is obtained:

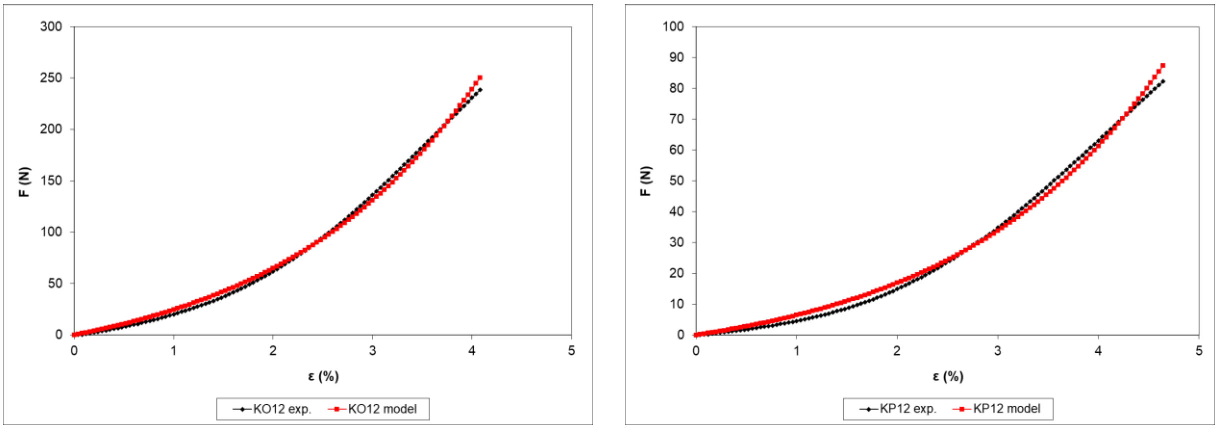
$$\sigma = a \cdot [1 - e^{-b \cdot \varepsilon}] \tag{10}$$

By fitting the experimentally obtained data in the form of Eq. (10), the values of the coefficients "a" and "b" were determined [10, 11]. In Table 3 are the values of the coefficients "a" and "b" for models which are obtained on the basis of a rheological model based on experimental data for fabrics obtained by extending woven fabric specimens.

Table 3: The coefficients of the model to the inflection point

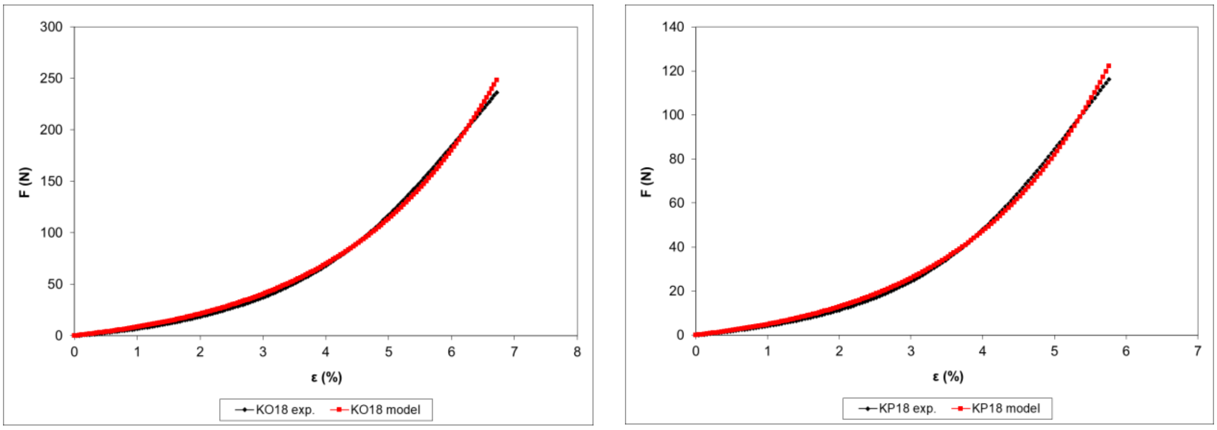
Fabric tag	a	b	Determination coefficient R^2	Root Mean Square Error RMSE (N)
KO12	-3842.4	-0.49443	0.99671	4.150
KP12	-1032.3	-0.48431	0.99480	1.813
KO18	-1649.1	-0.41312	0.99776	3.274
KP18	-772.1	-0.49015	0.99767	1.641
KO24	-1662.5	-0.35088	0.99833	2.890
KP24	-1559.3	-0.37338	0.99656	3.624

Diagrams of the mean values of the F-ε results obtained experimentally and according to the Lethersich's model are shown in Fig. 8-10.



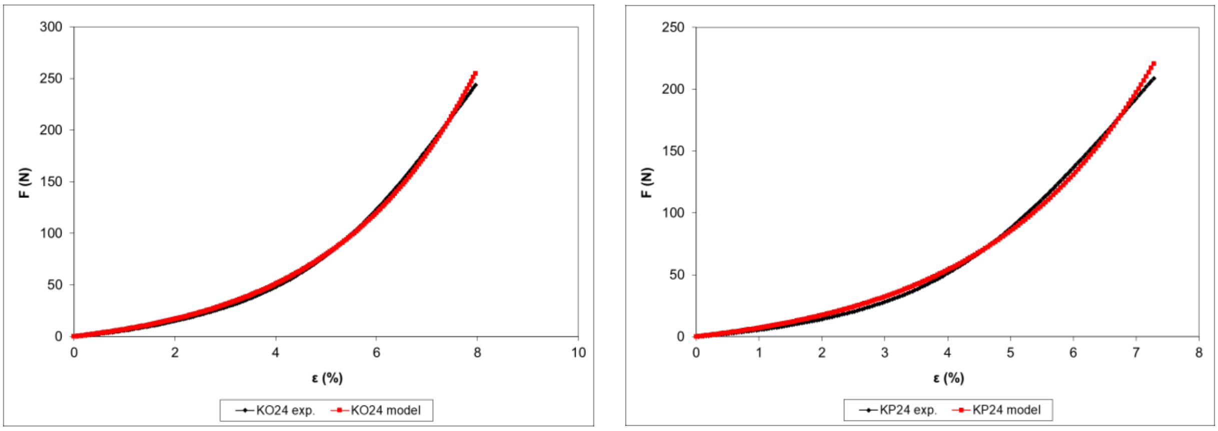
a) b)

Fig. 8: Diagram force-elongation (F-ε) obtained experimentally and by model: a) for specimen KO12, b) for specimen KP12



a) b)

Fig. 9: Diagram force-elongation (F-ε) obtained experimentally and by model: a) for specimen KO18, b) for specimen KP18



a) b)

Fig. 10: Diagram force-elongation (F-ε) obtained experimentally and by model: a) for specimen KO24, b) for specimen KP24

By reviewing the diagrams from Fig. 8 to Fig. 10, formed on the basis of experimental results (black) and the results obtained using the model (red) it is noted that they do not have a linear form, which confirms that the woven fabrics do not behave ideally elastic even at low stresses. Based on the experimental results and the shape of the force-elongation curves, it can be concluded that viscoelastic behaviour of the fabrics is observed in the analysed area. The elastic limit is essentially the limit to which elastic deformations in the material dominate. After the elastic limit, a higher rate of material deformation occurs, and the fabric structure is disrupted. Therefore, the fabric elasticity limit represents the boundary load at which the deformations occurring in the fabric will not significantly affect the stability of the structure and the durability of the fabric. By analysing the diagram, it can be concluded that the Lethersich's model, Fig. 4c, correctly describes the behaviour of the cotton woven fabric in twill weave in the zone of elastic deformation.

Conclusions

The mechanical properties of the woven fabrics are starting parameters for the design and adjusting the parameters in the textile industry. By defining the limit of elasticity of fabrics, one comes to the knowledge of the boundary intensities of the forces to which the fabrics can be subjected, without compromising their quality. The anisotropic woven fabric structure contributes to different, sometimes difficult to explain, fabric behaviours during elongation. The term "elastic limit" defines the limit to which elastic deformations in the fabric dominate. This is the limit when the material begins to deform faster under stress and should therefore be considered as the limit of permissible loads.

The mechanical properties of fabrics can be predicted using rheological models. Each type of deformation of real materials is described by a basic model, or the fabric behaviour is represented by complex models created by a combination of basic models.

Based on the experimental results obtained for elongation fabric specimens under standard test conditions, a combination of basic rheological models, set rheological models of fabric elongation, the corresponding differential equations for the tested fabric specimens were derived and solved, thus obtaining the dependence between the tensile force (stress) and the elongation. Rheological models describe the process of elongation cotton woven fabric relatively well under static test conditions.

Lethersich model was used to describe the behaviour of cotton woven fabrics in twill weave. This model can describe the behaviour of cotton fabrics subjected to the action of the tensile force in the zone dominated by elastic deformation. In the zone of elastic deformation, that is, up to the inflection point, the force-elongation relation for cotton woven fabrics is defined. The estimation error between the model and the experimentally obtained values is very low, so it can be concluded that the developed method can serve to predict the behaviour of cotton woven fabrics in the zone of elastic deformation. The obtained determination coefficients (R^2) show a high correlation ($R^2 > 0.99$) between relation of model and the experimentally obtained results, and they are slightly different (only in the third decimal place).

The force-elongation curve estimation error expressed through the root mean square error (RMSE) is the smallest for the model of woven fabric in twill weave (KP18) and is (164.1 cN) and the highest is for model of woven fabric in twill weave (KP24) whit highest density and is (362.4 cN). Mechanical properties of fabrics depend on their structural solutions, as well as on technological conditions of manufacture. The structural and physical-mechanical characteristics of the fabrics, the surface mass of the fabrics and the parameters of the weaving process play the most important role. Knowledge of the interconnectedness of the mechanical characteristics of fabrics provides the possibility of their proper design depending on the future purpose, which can contribute to the savings of raw materials and energy. Developed Lethersich’s model can be used to predict the limit load of cotton woven fabric after which irreversible deformations of these materials occur.

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