

Heat Transfer Analysis of MRE Under Compressive Cyclic Loading

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Abstract: The self-heating phenomenon is one of the essential parameters in the design and analysis of composites, as the accumulated heat can dominate the fatigue process under particular circumstances. This paper presents an uncoupled finite element (FE) simulation of the heat accumulation inside an isotropic magnetorheological elastomer (MRE) under compressive cyclic loading. The FE model was solved solely as a heat transfer problem based on dissipated energy data from an experiment. The theoretical analysis, FE modeling, and results were presented and discussed. The simulation results showed good agreement with experimental data.

Keywords: magnetorheological elastomer; finite element method; self-heating; energy dissipation.

1 Introduction

Magnetorheological elastomers (MREs) are smart materials whose mechanical properties can be modified under the presence of a magnetic field. MREs are usually fabricated by mixing micron-sized iron particles with a rubber-like matrix before the matrix is fully cured. MREs can be either isotropic or anisotropic according to applying a magnetic field during the curing process [1]. Under the presence of a magnetic field, the particles will be regularly aligned and form an anisotropic MRE. In contrast, the particles will be distributed randomly without a magnetic field, creating an isotropic MRE [2]. MREs are used in applications where a variable stiffness is required, such as tunable vibration absorbers, sensors, vehicles seats, and electronic devices development [3,4].

Many studies have investigated the temperature effect on the deterioration of mechanical properties and referred is as thermal fatigue. Ratner et al. [5] illustrated the difference between thermal and mechanical fatigue by presenting a criterion for critical failure stress. This criterion compares the failure stress with the critical one to decide whether the dominant failure mode is thermal or mechanical. So that understanding the heat transfer mechanisms, portions of dissipated energy, and the temperature distribution are crucially essential for fatigue and design purposes.

During compressive cyclic load, the temperature rises steeply at the first cycles then gets stabilized when the dissipated energy is fully transferred to heat [6]. In literature, the ratio of dissipated heat to the total dissipated energy is referred to as the conversion ratio. Pichon et al. [6] demonstrated that a portion of dissipated energy is dissipated at the first cycles as a change of the material microstructure. Then, it is totally dissipated to heat when the temperature stabilizes. Rittel et al. [7] illustrated the dependence of conversion ratio on the strain and frequency.

There are several studies in the literature that used FE modeling to investigate the self-heating phenomenon. Generally, there are two types of FE Models: the first one is the fully coupled approach. The heat transfer problem is solved simultaneously with the mechanical one, and the mechanical properties deterioration with temperature rise is evenly updated. While in the second approach, the problem is solved solely as a heat transfer one with information from the experiment regarding the hysteresis loops areas or sample temperature distribution. Bazkiaei et al. [8] illustrated the different approaches in more detail and proposed a FE model with different heat source distributions.

In this study, an uncoupled finite element model was presented and compared with experimental data from Nam et al. [4]. Firstly, the experimental setup of Nam et al. had been illustrated, and the used mechanical and

thermal analysis were described, then the Numerical FE model was explained in detail. Finally, the results, discussion, and conclusion were presented.

2 Theory

In this study, the Experimental setup proposed by Nam et al. [4] was simulated solely as a heat transfer problem. Nam et al. [4] investigated the self-heating phenomenon of isotropic MRE filled with carbonyl iron particles (CIPs) under compressive cyclic loading. The CIPs were added with a volumetric fraction of 0.27 and subjected to compressive cyclic loading for two hours, at variable strain amplitude (1%–5%), variable pre-strain (5%–20%), and variable frequency (10 Hz–50 Hz).

Assuming an incompressible rubber and normalized stress response ($\sigma(t)$) upon the strain amplitude ($\Delta\varepsilon$) during cyclic loading, then the stress response can be estimated as [9]:

$$\sigma(t) = \frac{F(t)}{A} = \sigma_0 + \Delta\varepsilon [E' \sin(2\pi ft) + E'' \cos(2\pi ft)], \quad (1)$$

where E' and E'' are the storage and loss moduli, which can be evaluated using the lag angle (δ) between the applied stress and the resulted strain, as shown in Eqs. 2 and 3.

$$E'(\varepsilon_0, f, \Delta\varepsilon) = \frac{\Delta\sigma}{\Delta\varepsilon} \cos(\sigma), \quad (2)$$

$$E''(\varepsilon_0, f, \Delta\varepsilon) = \frac{\Delta\sigma}{\Delta\varepsilon} \sin(\sigma). \quad (3)$$

The storage modulus represents the strain energy stored within the sample, while the loss modulus represents the dissipated energy. The volumetric dissipated energy (q_{diss}) can be estimated from the stress-strain loops areas [4], as shown in Eq. 4:

$$q_{diss} = \int_0^T \sigma(t) \dot{\varepsilon}(t) dt = \pi \Delta\varepsilon^2 E''. \quad (4)$$

Then the conversion ratios (η) can be estimated from the temperature distribution and by applying the First Law of thermodynamic (Conservation of energy principle) on the sample boundaries, as shown in Eq. 5 [10]:

$$\dot{q}_g = \frac{\rho C_p \Delta T}{t} = \dot{q}_{net} - \dot{w}_{net}, \quad (5)$$

where \dot{q}_g represents the variation of the total energy stored in the system. In this study, it is considered the accumulated heat during the experiment. While ρ , C_p and ΔT are sample density, specific heat, and the temperature difference. The net heat done on the system (\dot{q}_{net}) is considered the summation of conduction, convection, and radiation heat rates. In contrast, the net work done by the system (\dot{w}_{net}) is regarded as the dissipated energy (q_{diss}) multiplied by the conversion ratio (η) divided by the time interval (t). So that, Eq. 5 can be rewritten as:

$$\dot{q}_g = \eta \dot{q}_{diss} - \dot{q}_c - \dot{q}_{con} - \dot{q}_r. \quad (6)$$

Using this one-dimensional equation with information about temperature distribution, one can predict the conversion ratio of dissipated power to heat (η). A similar approximation was used by Balandraud et al. [11], Samaca et al. [12] and Loukil et al. [13].

3 Finite Element Model

In this study, a transient 2d finite element model was built using Comsol Multiphysics [14] to simulate the experimental data of Nam et al. [4]. The geometric structure in a 2D axisymmetric configuration and the thermal boundary conditions were described, as shown in Fig. 1. The used MRE sample is cylindrical with 13 mm height and 28 mm diameter. FE meshing has been applied to the MRE sample, Teflon platens, and steel grips. The following boundary conditions were used:

- Heat conduction between MRE sample and Teflon platens, and between Teflon platens and steel grips.

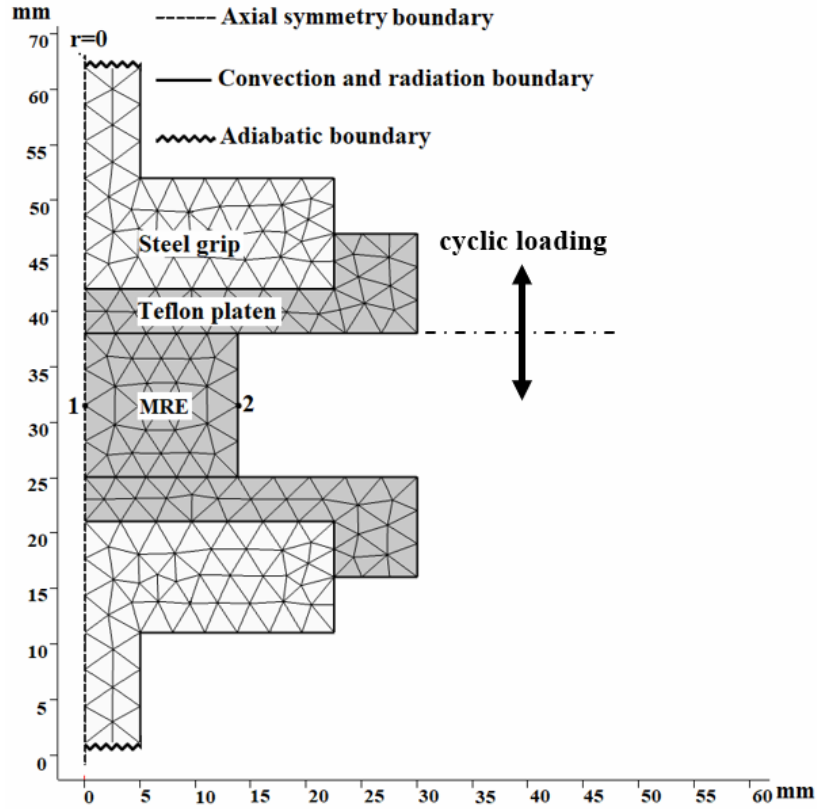


Fig. 1: FE Model as described by Nam et al. [4].

- Convection and radiation on the exposed surfaces.
- Adiabatic at the end of steel grips, where the temperature is close to the ambient one.
- The initial temperature of all components was considered as the ambient temperature.
- Axial symmetry.

The thermal contact resistances were ignored as the Teflon platens have a soft surface and are in pressure contact with the steel and MRE sample [10]. For horizontal exposed surfaces, the convection coefficient (h) was calculated as [10]:

$$h_u = 5.15 \left(\frac{T_s - T_\infty}{LT_\infty} \right)^{1/4}, \quad (7)$$

$$h_l = 1.5 \left(\frac{T_s - T_\infty}{L^2 T_\infty} \right)^{1/5}. \quad (8)$$

The subscripts (u) and (l) represent upper and lower according to the surface direction. In contrast, for vertical surfaces, the convection coefficient is estimated as [10]:

$$h_v = \frac{0.025}{0.01295} \left[0.825 + 17.04 \left(L^3 \frac{T_s - T_\infty}{T_\infty} \right)^{1/6} \right]^2, \quad (9)$$

where (L) is the characteristic length which can be estimated by dividing area over the perimeter or volume over the area. Tab. 1 illustrates each component's conductivity, emissivity, and specific heat [4].

4 Results and discussion

The experimental data of stress-strain loops areas were used as a transient variable for heat generation inside the MRE sample multiplied by the conversion ratio (η), and the generated heat was assumed to be evenly distributed through the sample [6–8]. The internal temperature and surface temperature were compared with

Tab. 1: Thermal properties of MRE sample, Teflon platens, and steel grips.

Material	Emissivity	Conductivity	Specific heat
Emissivity [-]	0.95	0.22	0.85
Conductivity [W/mK]	0.775	44.5	0.24
Specific heat [J/KgK]	585.08	475	1,050

experimental ones, as shown in Figs. 2, 3, and 4. The comparison was performed at point (1) and point (2) mentioned in Fig. 1.

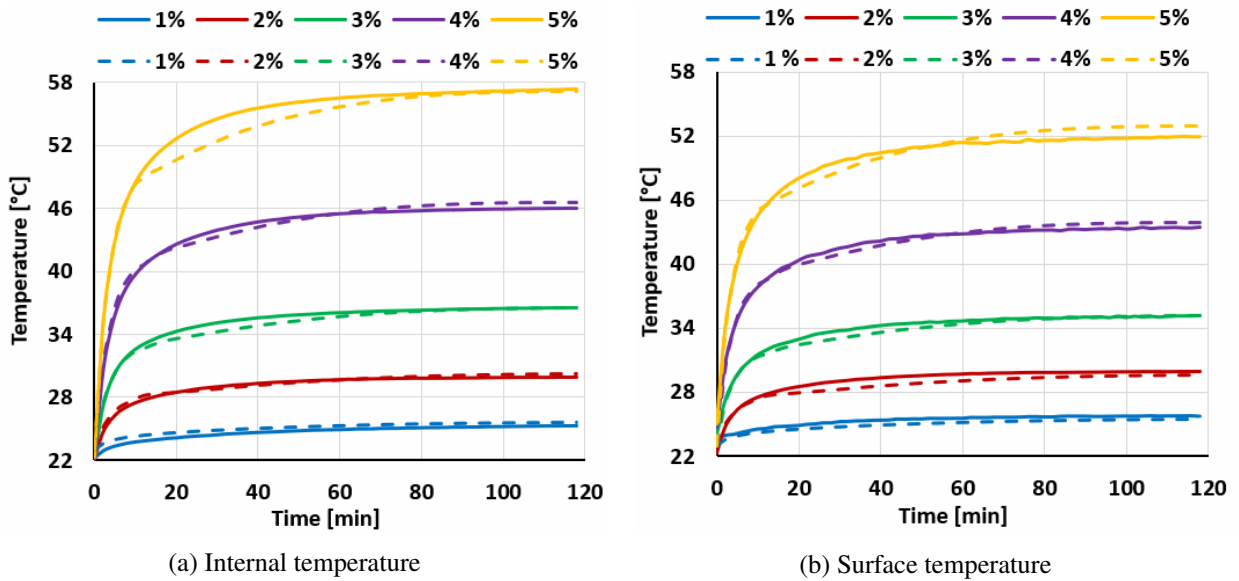


Fig. 2: Internal and surface temperature at different strain amplitudes (1% to 5%), pre-strain of 20%, and frequency of 40 Hz. Solid lines (experimental) and dashed lines (simulation).

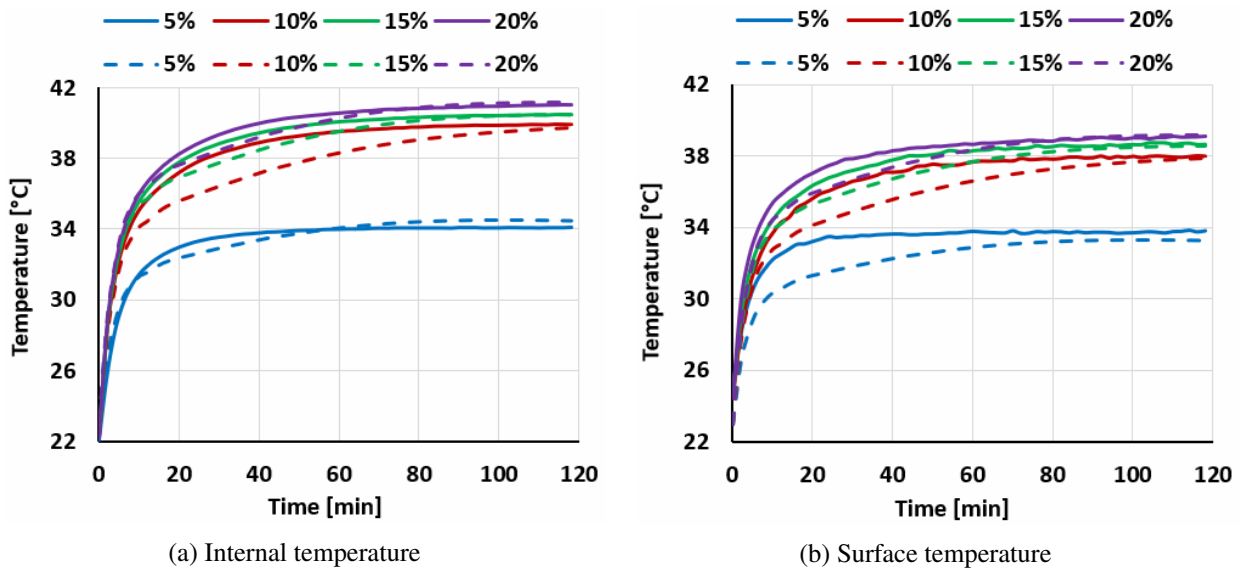


Fig. 3: Internal and surface temperature at different pre-strains (5% to 20%), strain amplitude of 3%, and frequency of 50 Hz. Solid lines (experimental) and dashed lines (simulation).

The results showed good agreement with the experiment and literature. The average relative error was less than 2% for all cases, and the maximum absolute error was observed at the pre-strain effect testing group with a value of 8%. Most of the uncoupled FE studies mentioned in the literature were able to predict the

final internal and surface temperatures accurately and underestimate transient temperatures. Because these studies either ignored the radiation effect, horizontal plane of symmetry, or used a constant conversion ratio, such approximations could result in a significant error as the test time interval is relatively long. While in this study, the transient self-heating was successfully predicted by solving the problem solely as a heat transfer one. Moreover, the temperature contour lines shown in Fig. 5 showed good agreement with the infrared camera data mentioned by Nam et al. [4].

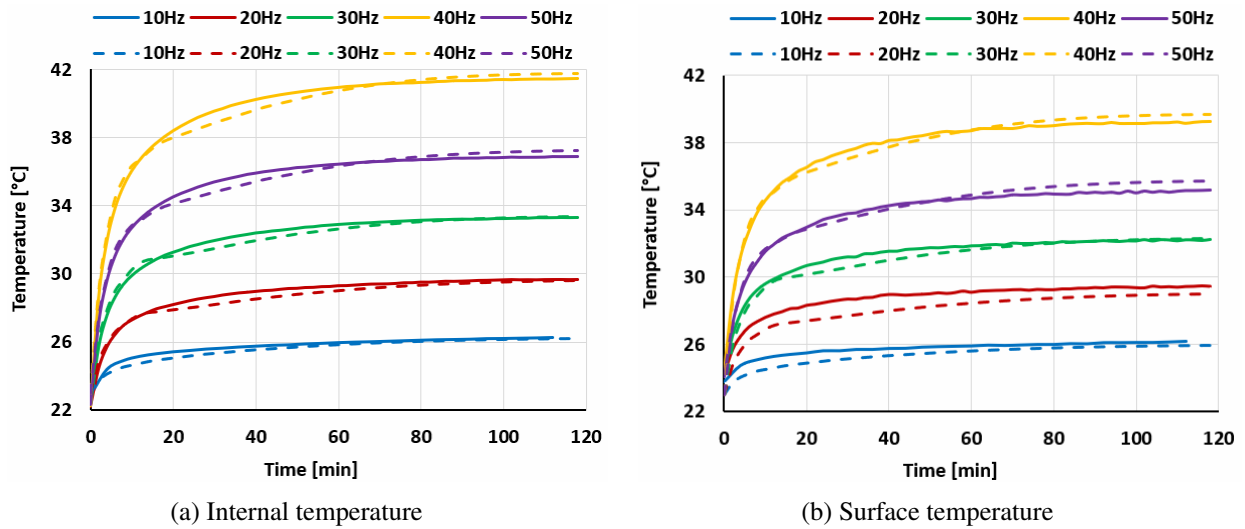


Fig. 4: Internal and surface temperature at different Frequencies (10 Hz to 50 Hz), pre-strain of 20%, and strain amplitude of 3%. Solid lines (experimental) and dashed lines (simulation).

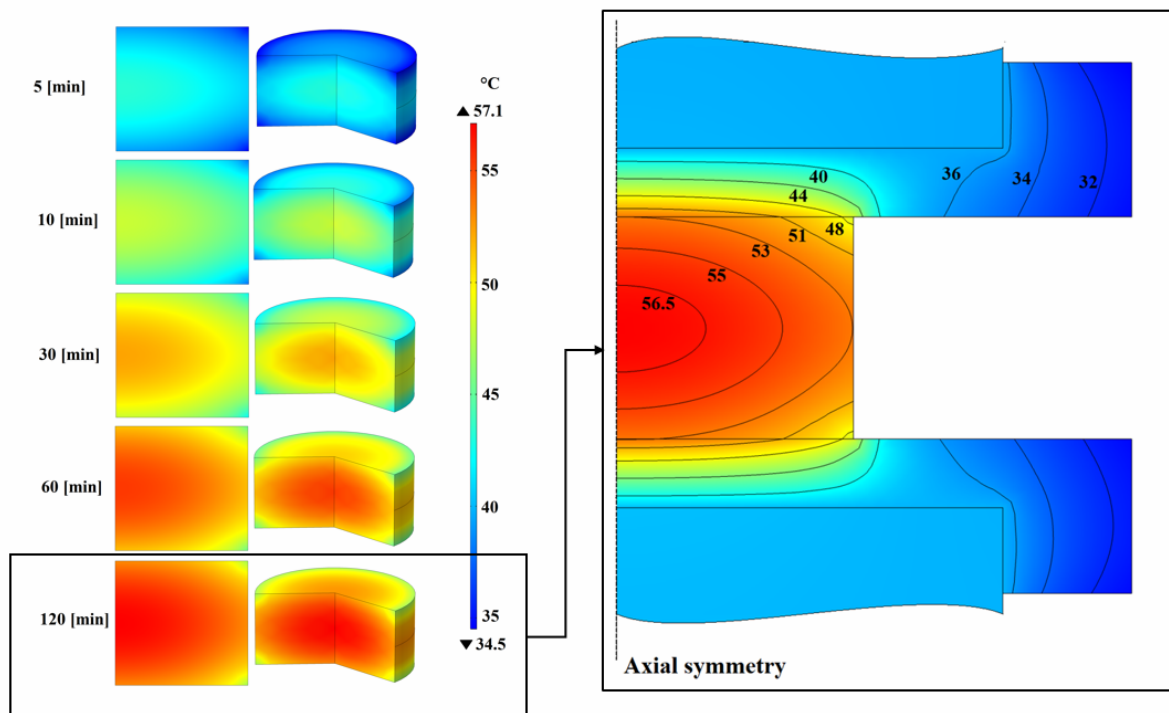


Fig. 5: Temperatur evolution with time and contour lines at a frequency of 40 Hz, pre-strain 20%, and 5% strain amplitude.

5 Conclusion

In this study, an uncoupled 2d FE simulation was performed to investigate the self-heating phenomena of isotropic MRE under compressive dynamic loading. A 1d-model was proposed using the First Law of thermodynamics to calculate the conversion ratios based on the experimental temperature distribution. The stress-strain loop areas were inserted simultaneously multiplied by the conversion ratios into the heat transfer problem to simulate the transient effect accurately.

Moreover, the conversion ratios significantly depend on the frequency, which agrees with Rittel et al. [7] study. Also, this study shows that the transient effect was predicted with a small error by considering precise boundary conditions and temperature-dependent thermal coefficients.

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References

- [1] T. Liu and Y. Xu, Magnetorheological Elastomers: Materials and Applications, *Smart Funct. Soft Mater.*, 2019, doi: [10.5772/intechopen.85083](https://doi.org/10.5772/intechopen.85083).
- [2] S.S. Kang, K. Choi, J. Do Nam, and H.J. Choi, Magnetorheological elastomers: Fabrication, characteristics, and applications, *Materials (Basel)*, vol. 13, no. 20, pp. 1–24, 2020, doi: [10.3390/ma13204597](https://doi.org/10.3390/ma13204597).
- [3] R. Ahamed, S.B. Choi, and M.M. Ferdous, A state of art on magneto-rheological materials and their potential applications, *J. Intell. Mater. Syst. Struct.*, vol. 29, no. 10, pp. 2051–2095, 2018, doi: [10.1177/1045389X18754350](https://doi.org/10.1177/1045389X18754350).
- [4] T.H. Nam, I. Petříková, B. Marvalová, and M.Y. Hdaib, Self-heating and dynamic mechanical behavior of silicone rubber composite filled with carbonyl iron particles under cyclic compressive loading, *J. Compos. Mater.*, p. 002199832110370, 2021, doi: [10.1177/00219983211037055](https://doi.org/10.1177/00219983211037055).
- [5] S.B. Ratner, V.I. Korobov, and S.G. Agamalyan, Mechanical and thermal fracture of plastics under cyclic strains, *Sov. Mater. Sci.*, vol. 5, no. 1, pp. 66–70, 1972, doi: [10.1007/BF00721313](https://doi.org/10.1007/BF00721313).
- [6] P.G. Pichon, M. Boutaous, F. Méchin, and H. Sautereau, Measurement and numerical simulation of the self heating of cross-linked segmented polyurethanes under cyclic loading, *Eur. Polym. J.*, vol. 48, no. 4, pp. 684–695, 2012, doi: [10.1016/j.eurpolymj.2012.01.005](https://doi.org/10.1016/j.eurpolymj.2012.01.005).
- [7] D. Rittel, On the conversion of plastic work to heat during high strain rate deformation of glassy polymers, *Mech. Mater.*, vol. 31, no. 2, pp. 131–139, 1999, doi: [10.1016/S0167-6636\(98\)00063-5](https://doi.org/10.1016/S0167-6636(98)00063-5).
- [8] A.K. Bazkiaei, K.H. Shirazi, and M. Shishesaz, Thermo-hyper-viscoelastic analysis of a rubber cylinder under cyclic deformation, *J. Rubber Res.*, 2021, doi: [10.1007/s42464-020-00068-2](https://doi.org/10.1007/s42464-020-00068-2).
- [9] B. Marvalová, I. Petříková, and T.H. Nam, Fatigue of filled rubber due to hysteretic heating, *Const. Model. Rubber XI - Proc. 11th Eur. Conf. Const. Model. Rubber*, 2019, pp. 374–378, 2019, doi: [10.1201/9780429324710-65](https://doi.org/10.1201/9780429324710-65).
- [10] F.P. Incropera, *Fundamentals of Heat and Mass Transfer*. Hoboken, NJ, USA: John Wiley & Sons, Inc., 2006.

- [11] X. Balandraud and J.B. Le Cam, Some specific features and consequences of the thermal response of rubber under cyclic mechanical loading, *Arch. Appl. Mech.*, vol. 84, no. 6, pp. 773–788, 2014, doi: [10.1007/s00419-014-0832-3](https://doi.org/10.1007/s00419-014-0832-3).
- [12] J.R. Samaca Martinez, J.B. Le Cam, X. Balandraud, E. Toussaint, and J. Caillard, Mechanisms of deformation in crystallizable natural rubber. Part 2: Quantitative calorimetric analysis, *Polymer (Guildf.)*, vol. 54, no. 11, pp. 2727–2736, 2013, doi: [10.1016/j.polymer.2013.03.012](https://doi.org/10.1016/j.polymer.2013.03.012).
- [13] M.T. Loukil, G. Corvec, E. Robin, M. Miroir, J.B. Le Cam, and P. Garnier, Stored energy accompanying cyclic deformation of filled rubber, *Eur. Polym. J.*, vol. 98, no. August 2017, pp. 448–455, 2018, doi: [10.1016/j.eurpolymj.2017.11.035](https://doi.org/10.1016/j.eurpolymj.2017.11.035).
- [14] C. Multiphysics, Introduction to COMSOL multiphysics®, COMSOL Multiphysics, Burlington, MA, accessed Feb, vol. 9, p. 2018, 1998.