Determination of the Elastic Constants of a Composite Using FE Model Updating Method

Róbert Huňady[1,a], Adam Kaľavský[1,b], Peter Palička[1,c] and Štefan Kender[1,d]

[1]*Technical University of Košice; Letná 1/9, Košice, Slovakia;*

[a]*robert.hunady@tuke.sk*, [b]*adam.kalavsky@continental-corporation.com*,
[c]*peter.palicka@tuke.sk*, [d]*stefan.kender@tuke.sk*

Abstract: The paper deals with an alternative approach for determining elastic constants using the FE model update method. The object of interest is a unidirectional carbon fibre reinforced composite plate, which is subjected to experimental modal analysis. The natural frequencies of the freely supported plate are then used in an optimization process that aims to tune the homogenized numerical model of the plate to achieve modal parameters agreement. The material constants obtained in this way are compared with the results of mechanical tests.

Keywords: Elastic constants; FE model updating; carbon fibre composite; modal analysis

1 Introduction

Knowledge of the material properties is crucial for describing its behaviour in terms of elastic response, plastic response, failure or fatigue. There are several methods for determining material constants and mechanical properties, which are mainly based on material tests. Many of them are standardised. The implementation of these tests can usually be time and cost consuming, as measurements need to be performed on a larger number of samples in order to increase statistical precision. The determination of elastic constants is a relatively simple matter, especially for homogeneous isotropic materials. However, in the case of orthotropic materials, such as fibre-reinforced composites, the situation becomes more complex as the number of constants increases with the degree of orthotropy. One alternative to the material tests required to determine elastic moduli, shear moduli and Poisson's ratios is the FE model updating method [1]. Its goal is to optimize the unknown parameters of the FEM model so that its response matches the measured data. The objective function can be the displacement or stress at a certain point, or the natural frequencies of the model, etc. This is a method that has wide application in practice. It can be used in dynamic analysis to identify damping in the structure [2]. The paper [3] presents the use of the FE model updating method to determine the damage of a structure based on the PSD of the vibration response obtained by measurement. Teughels and De Roeck used measured modal parameters to identify damage to the highway bridge structure [4]. A similar approach was used by the authors in [5] to determine the bond stiffness parameters. They also demonstrated the possibility of replacing the part of the numerical model at the location of the structure partition with a flexible bond (bushing) of equivalent stiffness. The method is also applicable to the refinement of various material parameters of composites or their components [6-8]. In the study [9], the authors tuned a material model of a 3D printed structure using a genetic algorithm (GA), trying to capture the relaxation and creep of the material. The experimental-numerical approach can also be used to determine residual stresses using the inverse algorithm, as shown in [10].

In this work, the material constants of a unidirectional carbon fibre reinforced composite plate are determined using FE model updating method. Basis for this is the measured natural frequencies and plate oscillation shapes. The basis is an experimental modal analysis of the

plate. To verify the approach used, the material constants are also determined by mechanical tests. The results are compared with each other and discussed.

2 Composite sheet and tested specimens

A vacuum bag moulding method combined with hand lay-up was used to produce the composite sheet. Using this method, the pressure inside the bag is reduced and the external pressure is increased, thereby removing excess air and resin. This method helps to produce laminates with uniform consolidation, complete wetting of the fibres and better core bonding. The composite material consisted of epoxy resin (MGS L258) and unidirectional carbon fibers (200 g/m^2, 50 K) that were stacked in eight layers. Each layer was separately saturated with resin to achieve homogeneous properties through the thickness. The resulting nominal sheet thickness was approximately 2 mm. The vacuuming process can be seen in Fig. 1a. Subsequently, the composite sheet was cured in an oven for 12 hours at 60 °C. After curing, specimens for mechanical testing and modal analysis were made from the sheet by milling (Fig. 1b).

(a) (b)

Fig. 1 (a) vacuum bag moulding process, (b) milling of specimens.

3 Constitutive model of the composite

The unidirectionally reinforced composite represents a transversely isotropic material. This material is symmetric about an axis that is normal to a plane of isotropy (see Fig. 2). This transverse plane has infinite planes of symmetry and thus, within this plane, the material properties are same in all directions. This is a special case of orthotropy, where 5 independent constants in the elasticity tensor are needed to describe the linear elastic behaviour of the material.

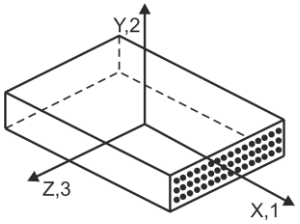


Fig. 2 Transversely isotropic material.

Assuming the 2–3 plane to be the plane of isotropy, transverse isotropy requires that $E_2 = E_3 = E_p$, $\nu_{12} = \nu_{13} = \nu_{tp}$, $\nu_{21} = \nu_{31} = \nu_{pt}$, $G_{12} = G_{13} = G_t$, where p and t stand for "in-plane" and "transverse," respectively. Thus, while ν_{tp} has the physical interpretation of

the Poisson's ratio that characterizes the strain in the plane of isotropy resulting from stress normal to it, ν_{pt} characterizes the transverse strain in the direction normal to the plane of isotropy resulting from stress in the plane of isotropy. In general, the quantities ν_{tp} and ν_{pt} are not equal and are related by $\frac{\nu_{tp}}{E_t} = \frac{\nu_{pt}}{E_p}$. The stress-strain laws reduce to [11]

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} 1/E_t & -\nu_{pt}/E_p & -\nu_{pt}/E_p & 0 & 0 & 0 \\ -\nu_{tp}/E_t & 1/E_p & -\nu_p/E_p & 0 & 0 & 0 \\ -\nu_{tp}/E_t & -\nu_p/E_p & 1/E_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_p \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix}, \tag{1}$$

where $G_p = E_p / 2(1+\nu_p)$. In the transversely isotropic case the stability relations for orthotropic elasticity are

$$\begin{aligned} &E_p, E_t, G_p, G_t > 0, \\ &|\nu_p| < 1, \quad |\nu_{pt}| < \sqrt{E_p / E_t}, \quad |\nu_{tp}| < \sqrt{E_t / E_p}, \\ &1 - \nu_p^2 - 2\nu_{pt}\nu_{tp} - 2\nu_p\nu_{pt}\nu_{tp} > 0. \end{aligned} \tag{2}$$

4 Mechanical tests

The elastic constants of the composite material were determined based on mechanical tensile and shear tests according to the relevant standards [12-14]. Loads were measured by force sensors integrated in the test machines. A Dantec Dynamics Q-400 3D correlation system was used to measure strain components (Fig. 3).

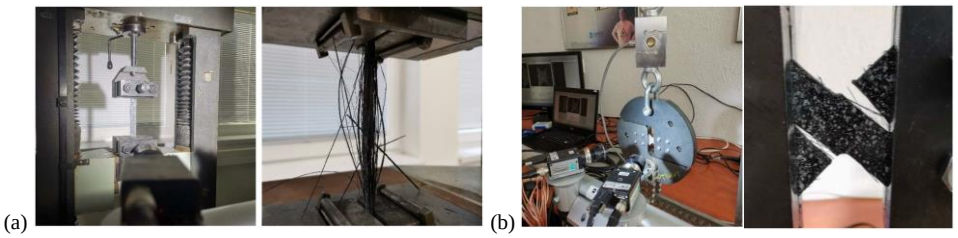


Fig. 3 Testing of material properties in tension (a) and shear (b).

From the measured data, elastic moduli, shear moduli and Poisson's ratio values were calculated using well-known procedures. To increase the statistical accuracy, 5 specimens were tested and the calculated values were averaged. The resulting material constants are shown in Table 1.

Table 1. Material constants of the composite obtained by mechanical tests.

E1 [MPa]	E2 [MPa]	G12=G13 [MPa]	G23 [MPa]	Nu12 [-]
120442	5779	3492	-	0.289

5 Experimental modal analysis

The test object was a rectangular plate with dimensions of 200.6 x 130.5 mm, whose average thickness was 1.969 mm. The plate was cut from the composite sheet at an angle of 40°. The

mass of the plate determined by weighing was 66.44 g. The density of the homogenized plate was determined by calculation to be $1.289 \cdot 10^{-9}$ t/mm^3. The plate was placed on thin elastic rubber bands so that the conditions for a free fit were met. 3D high-speed correlation system Q-450 Dantec Dynamics was used to measure vibration response (Fig. 4). A random stochastic pattern was created on the surface of the slab for the purpose of correlation measurements.

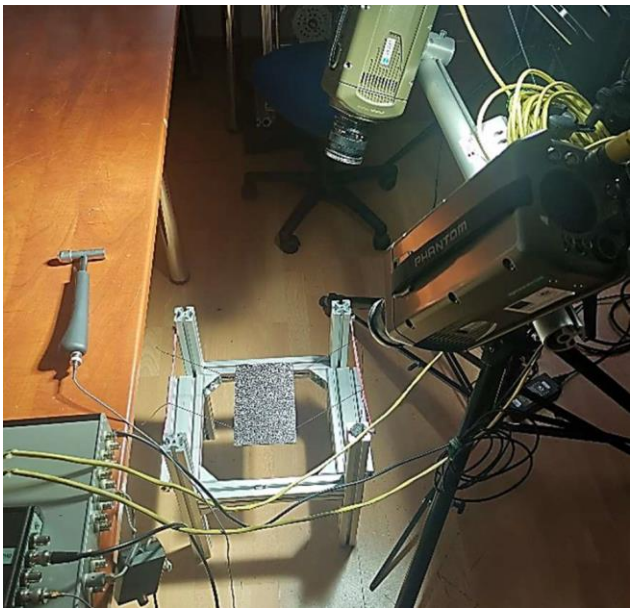
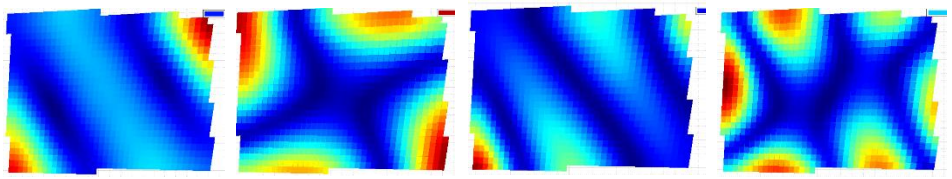


Fig. 4 Experimental configuration for measuring modal parameters of a composite plate.

The structure was excited with a Bruel&Kjaer Type 8206 modal hammer with a single impact on the bottom of the plate. The modal parameters (Fig. 5) were extracted using DICMAN 3D software [15,16]. Ten modes were identified whose frequencies were later used in the optimization process of FEM model.



1. mode: 110.4 Hz 2. mode: 197.7 Hz 3. mode: 281.3 Hz 4. mode: 452.2 Hz

Fig. 5 The first four mode shapes and natural frequencies of the composite plate.

6 FE model updating

The FEM model of the plate was created in Abaqus/CAE. The plate was modelled as a homogenised shell, so a "*Lamina"* type material model was used to define its elastic properties. The plate was considered to be free, i.e. without any constraints. A linear perturbation procedure of type "*Frequency*" was used to calculate the natural frequencies and mode shapes. Rigid body modes have been excluded. The mesh was formed by 260 linear quadrilateral elements of type S4R with 294 nodes (Fig. 6).

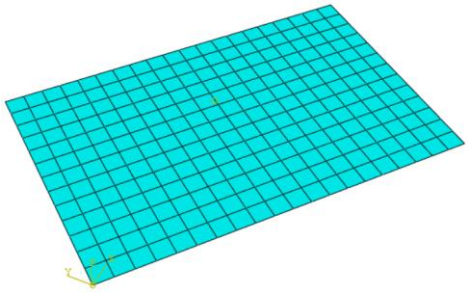


Fig. 6 FE model of the plate.

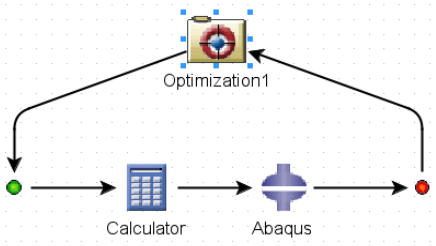


Fig. 7 Diagram of the optimization process.

Within the definition of geometric and material properties, the average density and thickness of the plate were considered according to Section 5. The material coordinate system was rotated at an angle of 40°. Other material constants were assumed to be unknown. Their values were determined by optimization in Isight. The aim of the optimization was to tune the model so that its natural frequencies matched the measured ones. A block diagram of the optimization process is shown in Fig. 7. As an optimization algorithm was used Neighborhood Cultivation Genetic Algorithm (NCGA), which is based on Pareto optimal solution [17]. Block 'Calculator' verified the fulfilment of stability conditions (2). The FEM model was included in the 'Abaqus' block. Five parameters were optimized: E1, E2, Nu12, G12=G13, G23. These parameters were defined as design variables. Their initial bounds are shown in Table 2. The natural frequencies determined by the measurements were used as objective functions (see Table 3). For the first five, optimization constraints were at the same time defined (Table 4).

Table 2. Initial and final lower and upper bounds of design variables.

Name	Initial Lower Bound	Initial Upper Bound	Final Lower Bound	Final Upper Bound
Elastic_LAMINA_1_E1	2000	200000	85000	120000
Elastic_LAMINA_1_E2	200	20000	4500	7500
Elastic_LAMINA_1_G23	200	20000	1500	7500
Elastic_LAMINA_1_Nu12	-1	1	0.1	0.35
Elastic_LAMINA_1_G13	200	20000	1500	5000

Table 3. Objective functions.

Name	Direction	Target
Step_1__History__EIGFREQ[0,1]	target	110.4
Step_1__History__EIGFREQ[1,1]	target	197.7
Step_1__History__EIGFREQ[2,1]	target	281.34
Step_1__History__EIGFREQ[3,1]	target	425.2
Step_1__History__EIGFREQ[4,1]	target	508.2
Step_1__History__EIGFREQ[5,1]	target	577.6
Step_1__History__EIGFREQ[6,1]	target	741.5
Step_1__History__EIGFREQ[7,1]	target	831.15

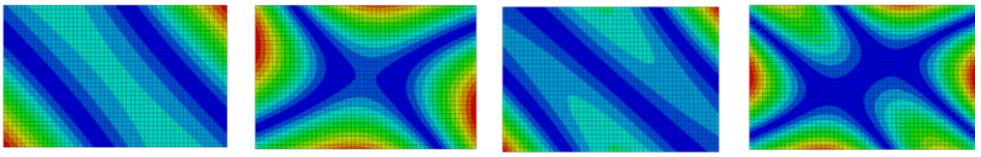
Table 4. Optimization constraints.

Name	Lower Bound	Upper Bound
Step_1__History__EIGFREQ[0,1]	105.0	115.0
Step_1__History__EIGFREQ[1,1]	190.0	210.0
Step_1__History__EIGFREQ[2,1]	270.0	290.0
Step_1__History__EIGFREQ[3,1]	415.0	435.0
Step_1__History__EIGFREQ[4,1]	490.0	520.0

It should be noted that the optimization process was run four times, with the interval of the design variables being narrowed after each run. The boundaries of the last run are shown in Table 2. The elastic constants determined by Isight are listed in Table 5.

Table 5. Material constants of the composite obtained by optimization in Isight.

E1 [MPa]	E2 [MPa]	G12=G13 [MPa]	G23 [MPa]	Nu12 [-]
115750	5367	3289	3603	0.309



1. mode: 115.7 Hz 2. mode: 203.8 Hz 3. mode: 286.6 Hz 4. mode: 435.5 Hz

Fig. 8 The first four mode shapes and natural frequencies of the optimized FE model of the composite plate.

Fig. 8 shows the mode shapes and natural frequencies of the FE model of the homogenised plate with the optimized material constants. Table 6 shows the percentage differences in the values of elastic constants and natural frequencies determined numerically and by the measurement. The measured data are considered as reference data. Comparison of the simulation and experimental results shows very good agreement, indicating high estimation accuracy.

Table 6. Percentage differences between numerical estimation and experimental measurement.

E1	E2	G12=G13	Nu12	1. mode	2. mode	3. mode	4. mode
3.9 %	7.1 %	5.8 %	-6.9 %	-4.8 %	-3.1 %	-1.9 %	-2.4 %

Conclusion

The paper presented an alternative method for determining the material constants of a composite material, which is based on the FE model updating method. The elastic parameters of the homogenized numerical model are optimized so that its natural frequencies match the frequencies determined by the experimental modal test. The accuracy of the estimation depends on several factors such as the dimensions and shape of the structure, the average density of the material, the way the structure was supported during modal analysis (bond stiffness), which must be known. If the orientation of the fibres is not known, it can be included as a design variable, in the same way the density value can be refined. As shown, the results obtained by FE model updating are in reasonable agreement with the results of the mechanical tests. With respect to the course of the optimization process, it should be noted

that the influence of the Poisson's ratio on the natural frequencies is not very pronounced. Its value could be refined by including a MAC criterion comparing the mode shapes of the structure.

Acknowledgements

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