Modal Analysis of Structural Element using Optical Experimental Methods of Mechanics

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Abstract: The content of the paper deals with a relatively new approach to the experimental determination of modal parameters (resonant frequencies, mode shapes and damping ratios) based on the measured displacements by the non-contact, full-field optical method of Digital Image Correlation (DIC). The output is an application module which, based on a three-dimensional displacement matrix from the measurement results, can compile a frequency response function (FRF) for the needs of experimental and operational modal analysis and determine the required modal parameters from it. The application module was designed in Scilab 6.1.0 software, while its script interferes directly with the ISTRA4D high-speed camera software from Dantec Dynamics company. The created application was based on the measurements of a simple steel sheet excited by an impact hammer for the simulation of experimental modal analysis. Verification of the calculation algorithm or the obtained modal parameters of the excited sheet steel plate was performed by means of a simulation in the numerical software Abaqus, whose mode shapes and resonant frequencies showed a high similarity with the results of the application.

Keywords: Digital Image Correlation; Experimental Modal Analysis; Operational Modal Analysis; Frequency Response Function; Resonant frequency; Mode shapes; Damping ratios

1 Introduction

Non-contact optical methods with the possibility of all-field analyses are among the current trends in the field of experimental mechanics. Laser Doppler vibrometry (LDV), electronic speckle interferometry, digital speckle shearography and digital image correlation fall into this field [1]. In vibration analyses, laser vibrometry is a frequently used method for practical reasons. The advantages of the laser vibrometry method are high sensitivity, frequency range, high accuracy, measurement speed and the ability to measure over large distances. The main disadvantage is the high cost of the measuring equipment. LDVs have a wide range of applications in science and research, as evidenced by the number of published papers devoted to experimental modal analysis, operational modal analysis, operational vibration waveform analysis, and vibrodiagnostics. Šároši et al. [2] used the LDV method for a comprehensive analysis of the disc during its rotation. Specifically, they dealt with experimental modal analysis of the disc in a stationary state, oscillation shape analysis at operating speed 5000 revolutions per minute and run-up analysis, which aimed to investigate the effect of the rotation speed on the change of the natural frequencies of the disc. A laser Doppler vibrometer and an optical derotator were used as measuring equipment. Zucca et al. [3] used the LDV technique to determine the frequency response functions and operating responses of 24 turbine blades at a specific angular velocity. Stanbridge et al. [4] streamlined the sensing approach which allowed modal shapes to be obtained from a relatively small set of measured data. Trebuňa et al. [5] used a laser vibrometer to identify the source of excessive

vibration of a gas compressor discharge line. The use of a laser vibrometer was necessary in this case as the vibration level limited the use of accelerometers. The operational modal analysis is also addressed using LDV in the following papers [6,7].

Digital image correlation (DIC) is another method that has potential for use in vibration analysis [8]. However, for these purposes, a measurement system with high-speed cameras must be used. Although the DIC method is primarily used for component testing and determination of material properties [14-16], various publications report its use for vibration analysis and experimental modal analysis as well. There is also known publication where DIC is used for in-plane or in-space motion analysis [17].

2 Application module for modal analysis

The ISTRA4D software is capable of recording and evaluating measurement data using the DIC optical method. It offers various options for the needs of laboratories and industrial practices. It allows further extensions in the form of post-process applications. These applications, also called application modules, are programmed in Scilab software and interfaced with ISTRA4D software, where the modules transform the measurement results according to defined mathematical functions. In this paper, the authors discuss the development and description of an application module for the determination of modal parameters based on experimental modal analysis (EMA) and operational modal analysis (OMA) from data obtained by digital image correlation (DIC) measurements [8]. The relationship between the ISTRA4D software and Scilab is described in Fig. 1. Based on the visual representation of the measured object, it is possible to identify a point, curve or polygon on its surface that defines our region of interest on the analyzed object. From the ISTRA4D software, it is then possible to export displacements, relative deformations, point coordinates, and time characteristics. It is also possible to obtain the outputs of the analog channel, which represents, for example, the excitation signal. The export of this data is then processed by Scilab.

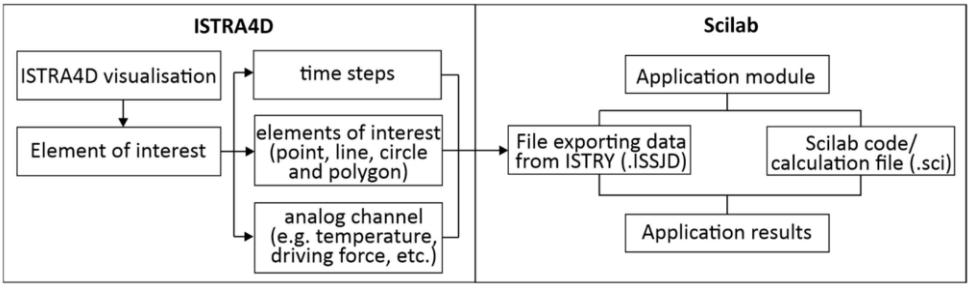


Fig. 1 ISTRA4D—Scilab relation.

The application module consists of two main files. They are text documents in the format:

a) .ISSJD, which defines the export of the required ISTRA4D measurement data to Scilab.

b) .SCE or .SCI, which works with the exported measurement data and transforms them on the basis of mathematical-physical relations into the required resulting parameters. It is the script itself created in Scilab.

The first interface of the modal analysis application module focuses on EMA, i.e., the estimation of modal parameters based on data from the frequency response function (FRF), which is influenced by the excitation frequency spectrum [8].

3 Measurement of input data using DIC for EMA solution

The application module was built based on a series of real sample measurements using the DIC method. For this purpose, a simple rectangular steel plate was chosen as the analyzed object, which had an analyzed area with the dimensions 156 × 152 mm and a thickness of 0.8 mm. The analyzed sample was made of DC03 steel. It is a deep-drawn steel, suitable for automotive body interior parts and other mouldings. The stochastic pattern was applied to the analyzed area in the form of pre-printed spots on a vinyl film. The steel sheet was woven at the bottom edge using two L-sections connected by screws (Fig. 2). Fig. 2 also shows the excitation location (EMA) where the impact hammer blow was located on the back side of the steel sheet [8].

Fig. 2 Laying of the analyzed steel plate and the excitation point.

The measurement was carried out with the Q-450 system from Dantec Dynamics using high-speed cameras. Calibration and correlation of the images were carried out using Istra4D software [8].

The measurement and evaluation conditions were as follows:

- Image acquisition time = 1 s;
- Frame rate = 5000 fps;
- Facet size = 21 pixels;
- Application grid spacing = 17 pixels.

In the first step of the solution, all measurement images were exported from the ISTRA4D software into hierarchically arranged. HDF5 files containing temporal information, frame counts and displacement matrices (Fig. 3) [9]. The images were exported in the form of HDF5 files, with 2004 frames out of 5000 exported in total to reduce the computation time required. For the needs of the application module we developed, only the displacement matrices in the directions of all three axes are sufficient. The displacement matrices are loaded directly by a command from Scilab (.SCE file), which operates on the data of one of them as needed [8].

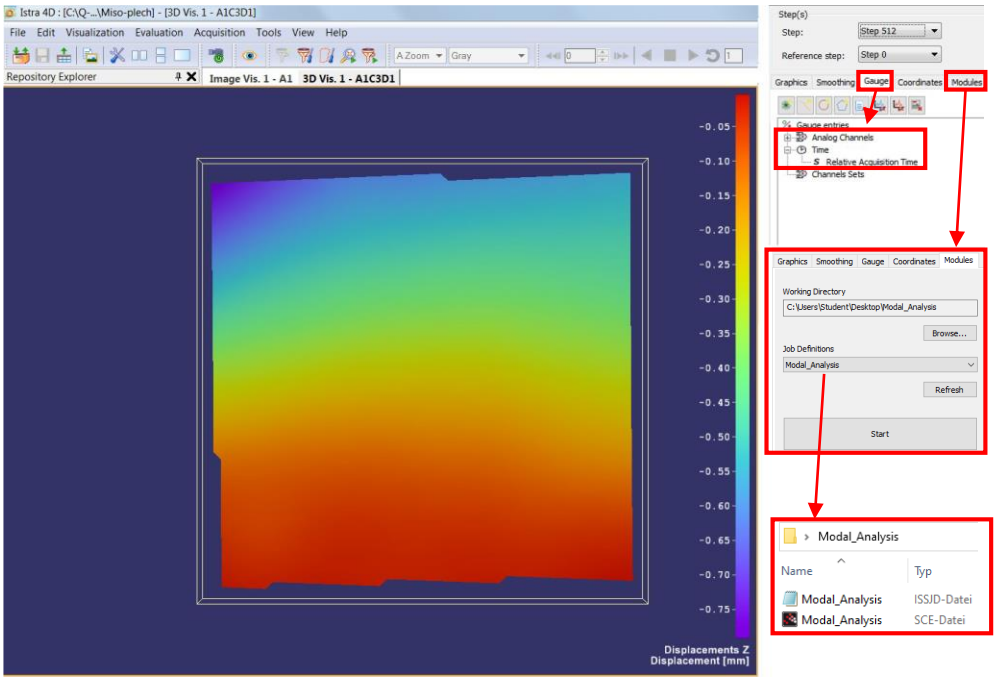


Fig. 3 ISTRA 4D working environment

The methodology for processing the measured and exported data using the Scilab mathematic-physical relationships is defined in Fig. 4. The .SCE file, which represents a computational script written in the Scilab programming language, must be placed in the same folder as the .ISSJD file that defines the export of the time increments.

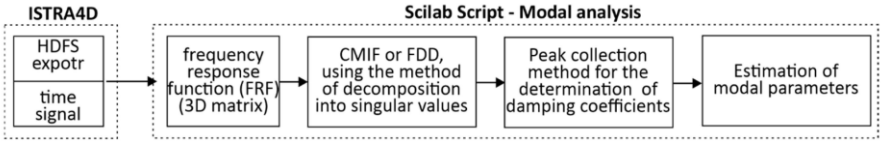


Fig. 4 Processing of exported measurement data in Scilab.

Starting the application displays a window (Fig. 5) that automatically defines the acquisition time based on the number of selected .HDF5 files.

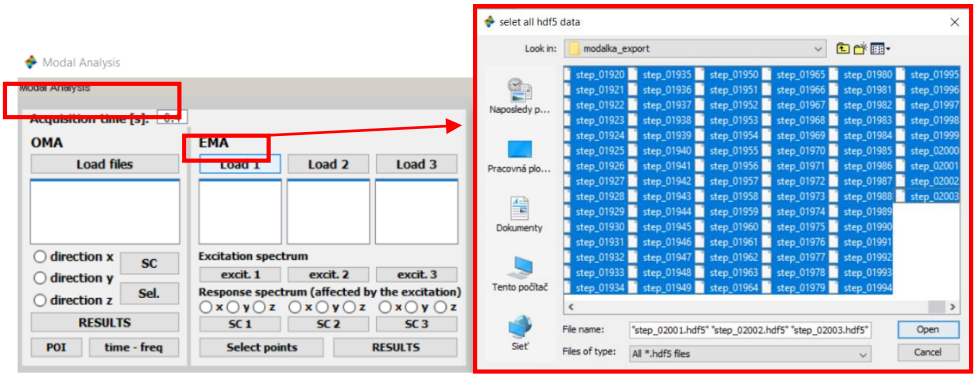


Fig. 5 Initial GUI window for determining modal parameters.

For the demonstration of the application module, only one measurement file was used, i.e., the system was built by an impact hammer strike at only one location; hence, we will only look for one singular curve.

The ISTRA4D software stores the temporal data from the measurements in terms of a column vector and orientates the spatial data into row vectors. Scilab command lines define the input parameter time, which is automatically calculated as the difference between the second and first time steps multiplied by the number of selected .HDF5 files. Since the acquisition time was 1 s at 5000 fps, when considering 2000 frames, the time is adjusted to 0.4 s.

The first function requires the user to select the path to the exported .HDF5 files from, the measurement to determine the number of time steps and the naming of the files to work with next. The quantities that are also required between functions are always defined as a global parameter.

For the analysis, the excitation signal from the analog output was adjusted to a timedomain waveform based on signal processing theory. Subsequently, it was transformed into a frequency spectrum by using the fast Fourier transform (FFT) (Fig. 6).

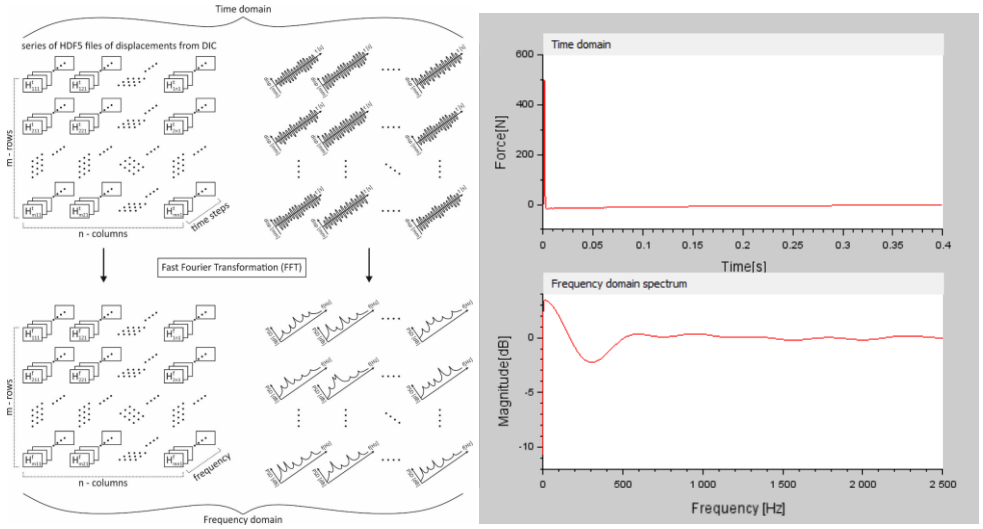


Fig. 6 Transformation of 3D displacement matrix from time domain to frequency spectrum and excitation signal in time domain and frequency spectrum.

A three-dimensional displacement matrix of the oscillating steel plate in the Z-axis direction, where the third dimension of the matrix represents time, was then used to determine the modal shapes. To describe each dimension of the matrix, a representation model was created, shown in Fig. 6. The product of the number of rows and columns gives the number of points (facets) at which displacements were measured. Each of these points has a specific characteristic function dependent on time t. These displacement functions of all points were transformed from the time domain to the frequency spectrum by a fast Fourier transform (FFT) (Fig. 6) [8,9].

Subsequently, the FRF matrix was determined based on the response in the form of the frequency spectrum of the displacement matrix, which is influenced by the frequency spectrum of the excitation signal and transformed into a vector notation (Fig. 7).

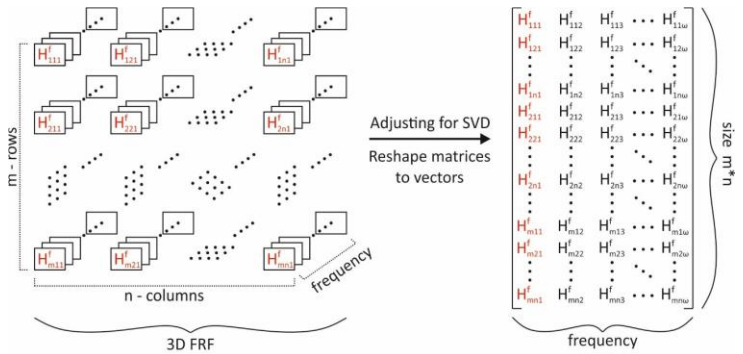


Fig. 7 Transformation of the FRF matrix into vector notation.

By plotting the singular values on the frequency axis, a singular waveform was obtained whose peaks indicated the presence of modes of oscillation (Fig. 8). The respective frequencies of the individual peaks are the natural or resonant frequencies. For each natural frequency, there is an intrinsic mode of oscillation. The eigenmodes of oscillation are exported for the selected frequency from the FRF matrix.

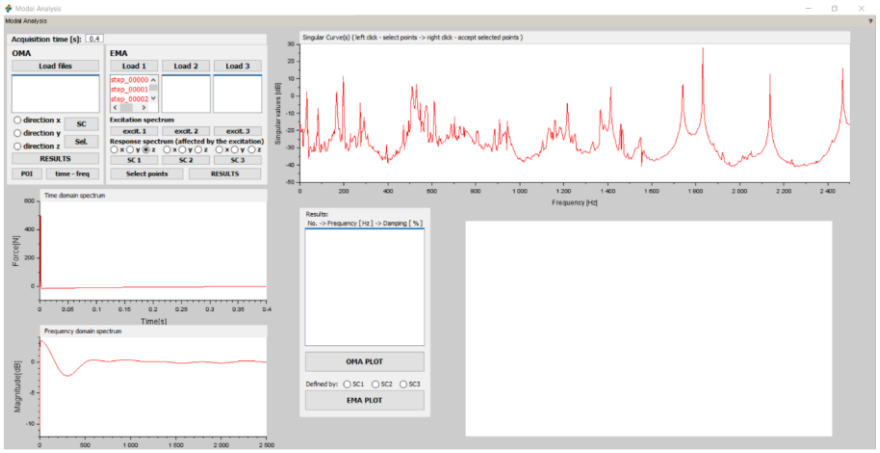


Fig. 8 Singular curve.

Another modal parameter that needs to be determined is the damping coefficient of the mode of oscillation. Since the Scilab software does not provide the ability to automatically select the half-power points based on the peaks, the nearby frequencies need to be manually selected.

Fig. 9 shows the natural frequencies and the corresponding mode damping coefficients determined from the Z-axis displacement matrix obtained by the DIC method (points 1–6). It is also possible to see peaks in the singular waveform that are not natural frequencies of the oscillating steel airfoil, but represent modes resulting from the fit (points X). In the same way, the modal parameters in the X and Y axis directions can be analyzed in the proposed application [8].

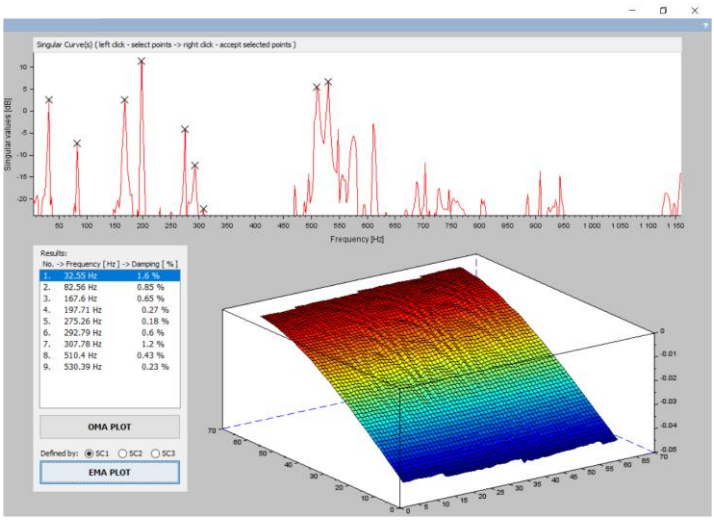


Fig. 9 Resulting modal parameters obtained with the application module.

Each natural frequency has its own specific mode of oscillation. The individual mode shapes with their respective frequencies are shown in Fig. 10.

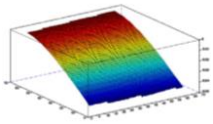
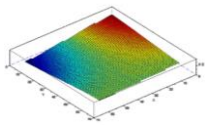
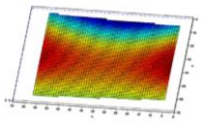
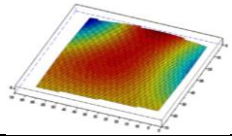
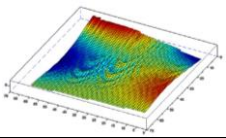
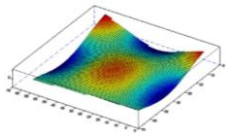
Mode 1 = 32.55 Hz	Mode 2 = 82.56 Hz
	
Mode 3 = 197.71 Hz	Mode 4 = 275.26 Hz
	
Mode 5 = 307.78 Hz	Mode 6 = 530.39 Hz
	

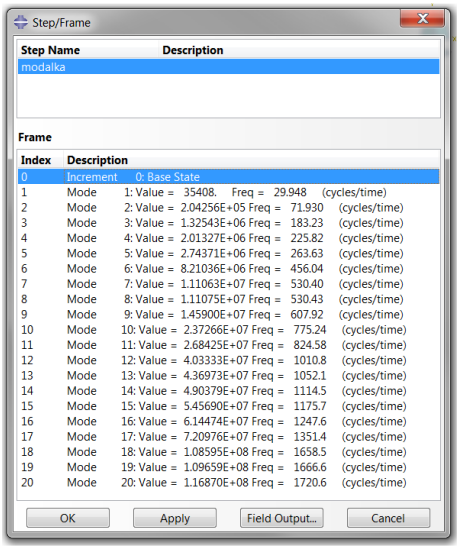
Fig. 10 Modal shapes and corresponding EMA eigenfrequencies.

4 FEM modal analysis

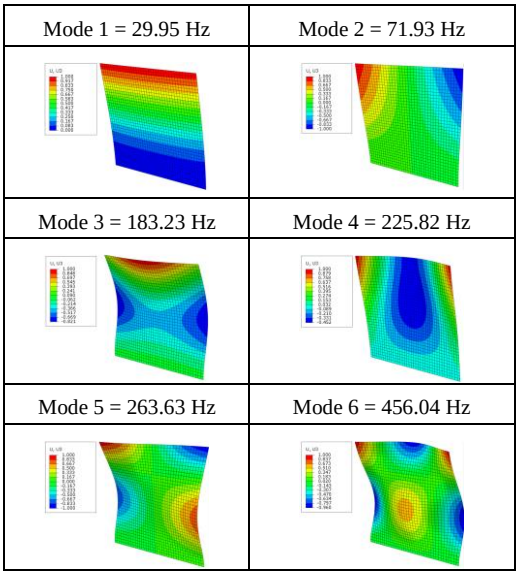
The correctness of the results obtained by experimental modal analysis based on DIC measurements was verified numerically. There are many numerical methods such as peridynamics [10], discrete element method [11], damage mechanics [12] and others, but due to the availability of software, we used the finite element method (FEM) to verify our solution. The numerical model was constructed in the form of a shell element to which the material properties of the steel and the dimensions of the real measured steel plate were assigned. The boundary condition of the fit was simulated by taking all degrees of freedom

of motion at the bottom edge of the numerical model. The mesh was meshed from quadrilateral elements.

The resulting frequencies obtained by the FEM method are shown in Fig. 11a. To compare the results of the natural frequencies and modal shapes, the first six modes were selected whose natural shapes are shown in Fig. 11b [8,13].



(a)



(b)

Fig. 11 (a) Eigenfrequencies of the modes determined by the FEM method, (b) Modal shapes (Z direction) and corresponding natural frequencies obtained by the FEM method.

5 Discussion

The resulting eigenfrequency values obtained experimentally by modal analysis using the DIC method and numerically by the FEM method are shown in Table 1. The differences of the natural frequencies in Hertz and percentage are also given in Table 1.

Table 1. Resulting eigenfrequency values obtained by EMA analysis and FEM analysis.

Mode	EMA Analysis [Hz]	FEM Analysis [Hz]	Difference [Hz]	Difference [%]
1	32.55	29.95	2.6	7.99
2	82.56	71.93	10.63	12.88
3	197.71	183.23	14.48	7.32
4	275.26	225.82	49.44	17.96
5	307.78	263.63	44.15	14.34
6	530.39	456.04	74.35	14.01

In Fig. 12, the deviations of the resulting natural frequencies of the different modes of oscillation obtained by numerical and experimental analysis are plotted [8].

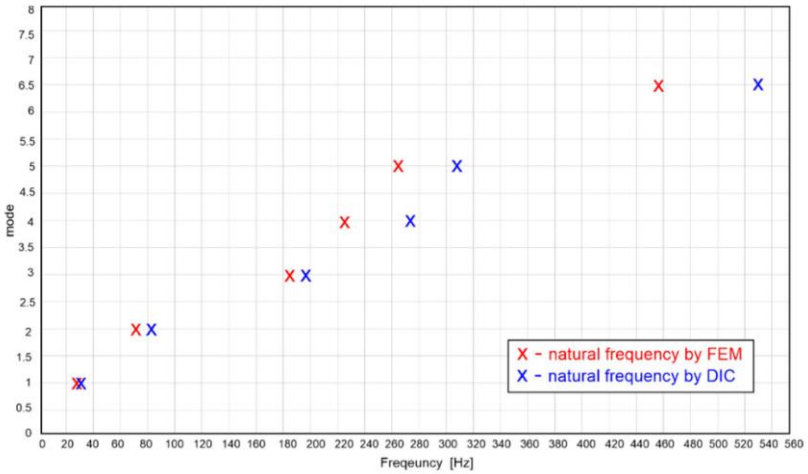


Fig. 12 Comparison of eigenfrequency results obtained by EMA analysis and FEM analysis.

The differences arising in the results are attributed to the choice of boundary conditions in the numerical FEM method. In the experiment, it is difficult to achieve a perfect fit of the analyzed sample.

Conclusion

Currently, either of the numerical modal analyses are used to estimate the modal parameters, or if a more accurate estimation corresponding to the real material properties and boundary conditions of the fit is required, an experimental solution is approached. The commonly used experimental modal analysis procedures are currently conditioned by the application of acceleration sensors on the object under analysis, by means of which the resulting frequency response function is obtained.

This paper describes a new approach to estimate the natural (resonant) frequencies, as well as the corresponding modal oscillation shapes and damping coefficients, based on the results of measurements obtained by the optical digital image correlation method. The resulting modal shapes are obtained based on a displacement matrix.

The main advantage of this new methodology is the fact that, by using high-speed measurements, a full-field analysis of the imaged area of the object is possible, which implies that responses at all points of the object can be obtained on the basis of a single measurement. Conventional methods, with their acceleration sensors, can negatively affect the modal parameters of the system by their mass for sensitive analyzed objects. The full-field analysis by the DIC method makes it possible to determine the responses of any point on the analyzed object without the additional mass of the sensors.

The application module which has been designed in Scilab is able to determine the modal parameters based on the measured displacement matrix by the DIC method.

The aforementioned application is able to determine natural frequencies, modal shapes, and damping coefficients based on experimental modal analysis when the FRF function takes into account the excitation of the system. It is also possible to estimate the modal parameters based on the principle of operational modal analysis when the excitation of the system is unknown.

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