Analysing the Influence of the Inner Pressure on the Modal Parameters of the Vessel

Martin Hagara[1,a], Pavol Lengvarský[1,b], Róbert Huňady[1,c], Miroslav Pástor[1,d] and Peter Palička[1,e]

[1]*Technical University of Košice, Faculty of Mechanical Engineering, Department of Applied Mechanics and Mechanical Engineering, Letná 1/9 Košice-Sever 042 00, Slovakia;*

[a]*martin.hagara@tuke.sk,* [b]*pavol.lengvarsky@tuke.sk,* [c]*robert.hunady@tuke.sk,* [d]*miroslav.pastor@tuke.sk,* [e]*peter.palicka@tuke.sk*

Abstract: In dynamic analysis of structures, modal parameters estimation plays an important role. The paper describes the experimental assessment of the modal parameters, i.e. mode shapes, natural frequencies and damping ratios, of the pressure vessel filled with helium under two states, i.e. by its inner pressure equal to 80 bar and the analysis of the empty vessel, whereby the inner pressure is expected to be 0 bar. Moreover, it provides a description and application of the simple and effective method for the determination of Rayleigh coefficients α and β, which are commonly used in the finite element analysis as an approximation to viscous damping. The obtained results prove that the inner pressure has significant influence on the analysed modal parameters. As the analyses of the vessel were realised under free-free boundary conditions, the results obtained can be simply used to correlate finite element analytical model.

Keywords: Experimental Modal Analysis; Modal parameters; Rayleigh damping model; Inner pressure; Vessel

1 Introduction

Damping plays an important role when analysing the dynamic behaviour of structures. Viscous and hysteretic damping models are the most common damping models used in civil, mechanical, material, and automotive engineering, respectively. The viscous damping model assumes that the damping force is proportional to the velocity, and thus the damping ratio and natural frequency are independent of the excitation level. On the other hand, the hysteretic damping model refers to a situation where the damping force or coefficient depends on the displacement or strain history of the system [1].

One of the most common damping specified in time history response analysis is Rayleigh damping, a form of viscous damping proportional to a linear combination of mass **M** and stiffness **K** matrices according to

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}, \tag{1}$$

where coefficients α and β are not generally known, but calculated from the modal damping ratios [2,3]. Traditionally, two modes of vibration (i^{th}- and j^{th}-order) are selected and their damping ratios ζ_i, ζ_j and their natural damped frequencies ω_i, ω_j obtained through experimental modal analysis (EMA) are used to determine α and β [4,5] using the following relation

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\omega_i\omega_j}{\omega_j^2 - \omega_i^2} \begin{pmatrix} \omega_j & -\omega_i \\ -\frac{1}{\omega_j} & \frac{1}{\omega_i} \end{pmatrix} \begin{Bmatrix} \zeta_i \\ \zeta_j \end{Bmatrix}. \tag{2}$$

The determination of the damping ratio ζ_j as a function of frequency is then realized according to

$$\zeta_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2}. \tag{3}$$

Typical dependence of damping ratio on the frequency by considering the Rayleigh damping model is shown in Fig. 1.

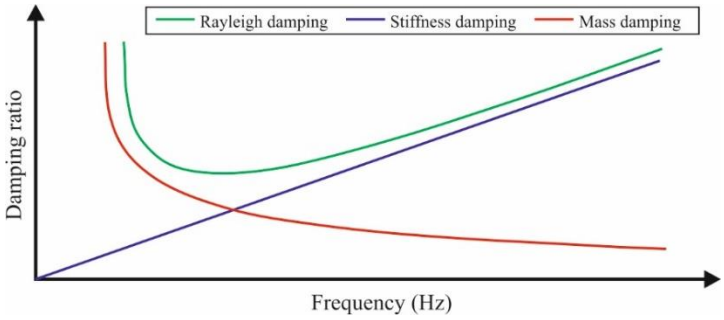

Fig. 1 Typical dependence of damping ratio on the frequency.

2 Measurement and methods

The estimation of the modal parameters, including Rayleigh damping coefficients, was carried out at two states of a pressure vessel (Fig. 2). Firstly, when it was filled with helium at 80 bar and secondly, when the vessel was empty, and thus the inner pressure was 0 bar (or low value approaching 0 bar caused by the remaining unreleased helium). The basic dimensions of the pressure vessel are the length 260 mm and outer diameter 103 mm. The mass of the filled vessel is approximately 1770 g. According to [6], the vessel is formed by welding the central cylindrical part to the bottom or cap of the vessel. A glycerine manometer GS04/20 was used to determine the values of the inner pressure.

Fig. 2 Analysed pressure vessel.

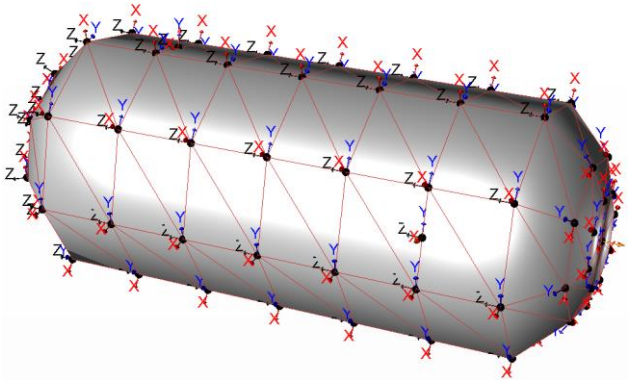

Fig. 3 The geometry of the pressure vessel, created in MTC hammer software, used to perform the experimental modal analysis.

2.1 Experimental modal analysis of the pressure vessel

Experimental data of frequency response functions (FRFs) form was acquired using the Brüel&Kjær Pulse® system with the LAN-XI 3050 module. The Pulse MTC Hammer software was used to perform the measurements, in which a simplified pressure vessel

geometry was created (Fig. 3). Soft rubber cords were used to suspend the vessel from a rigid frame (Fig. 4).

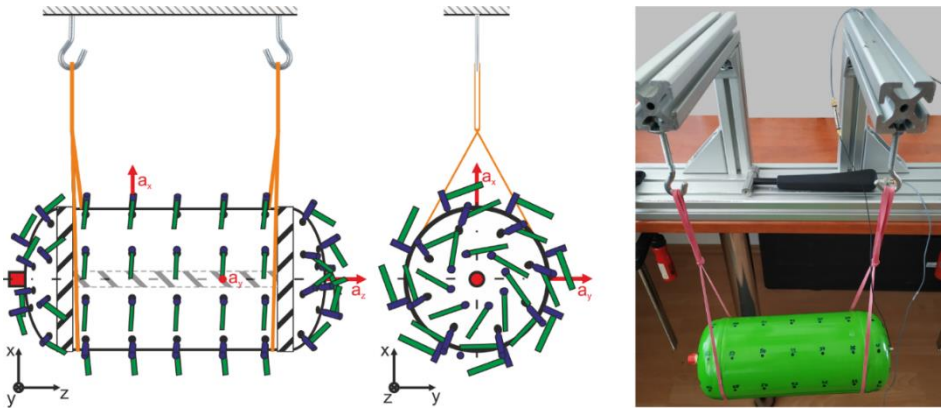

Fig. 4 Representation of reference points, accelerometer locations, and free-free boundary conditions.

Measurement in the form of the fixed embedded accelerometer and roving hammer-impact method was carried out using a Brüel&Kjær 8206 modal hammer with a plastic tip. At the same time, the structure's response in the three mutually perpendicular X, Y, and Z directions was recorded successively at three measurement points by a Brüel&Kjær 4374 single-axis miniature acceleration sensor. The resulting signals were obtained by averaging using the linear peak hold average method of the fast Fourier transform (FFT) signals, which increased the accuracy of the FRFs. The complex mode indicator functions (CMIFs) obtained from the measurement are depicted in Fig. 5.

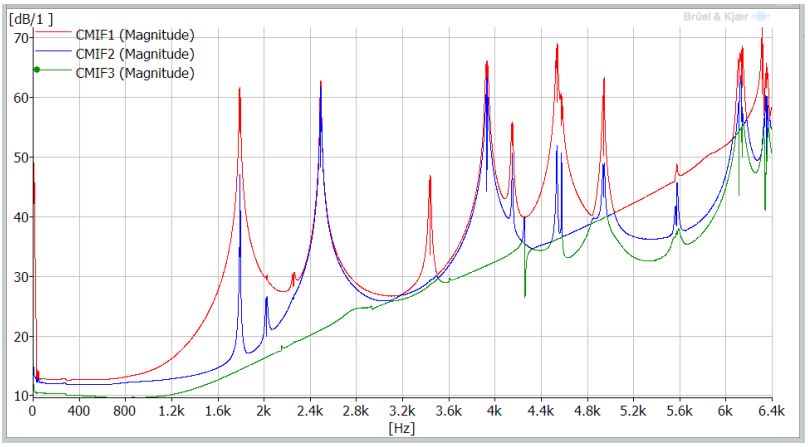

Fig. 5 Complex modes indicator functions (CMIFs) obtained by experimental modal analysis of the filled pressure vessel.

The obtained frequency response functions were evaluated in Pulse Reflex®, providing information on the natural frequencies, damping ratios, and individual mode shapes of the vessel vibration in the frequency range from 0 Hz to 6400 Hz. Stability diagrams resulting from the Complex Mode Indicator Function (CMIF) were used to estimate the vibration modes. The stability diagrams were plotted sequentially for the selected frequency bands, while the Rational Fraction Polynomial-Z method was used to identify the stability poles. Based on the most stable poles, the possible modes of vibration of the vessel were determined over the entire analysed frequency range. Since, in some cases, modes were estimated that

were not directly related to the vibration of the pressure vessel but only of its valve, these modes were removed from the estimated modes of vibration. In the case of a vessel loaded with an internal pressure of 80 bar, these were the modes with the following frequencies 2021 Hz, 3436 Hz, and 3440 Hz. The selected possible modes were analysed using Auto Modal Assurance Criterion (Auto MAC) method based on orthogonality checking. The natural frequencies and damping ratios of the vessel vibration were obtained from the experimental modal analyses of the filled and empty pressure vessel, as shown in Table 1.

Table 1. The natural frequencies and damping ratios obtained for the filled and empty vessel.

	80 bar		**0 bar**	
	Frequency (Hz)	**Damping ratio (%)**	**Frequency (Hz)**	**Damping ratio (%)**
Mode 1	1788	0.1161	1665 (↓6.9%)	0.0579 (↓50.1%)
	1794	0.1201	1673 (↓6.8%)	0.0593 (↓50.6%)
Mode 2	2489	0.0941	2209 (↓11.3%)	0.0499 (↓47.0%)
	2490	0.0948	2211 (↓11.3%)	0.0503 (↓47.0%)
Mode 3	3923	0.0877	3754 (↓4.3%)	0.0453 (↓48.3%)
	3935	0.0881	3765 (↓4.3%)	0.0469 (↓46.8%)
Mode 4	4146	0.0847	3830 (↓7.6%)	0.0420 (↓50.4%)
	not found		not found	
Mode 5	4533	0.0773	4501 (↓0.7%)	0.0458 (↓40.8%)
	4536	0.0785	4504 (↓0.7%)	0.0436 (↓44.5%)
Mode 6	4945	0.0759	4677 (↓5.4%)	0.0442 (↓41.8%)
	4948	0.0757	4685 (↓5.3%)	0.0448 (↓40.8%)
Mode 7	5560	0.0622	5593 (↑0.6%)	0.0410 (↓34.1%)
	5571	0.0609	5617 (↑0.8%)	0.0403 (↓33.8%)
Mode 8	6113	0.0872	6009 (↓1.7%)	0.0465 (↓46.7%)
	6141	0.0877	6028 (↓1.9%)	0.0474 (↓45.9%)
Mode 9	6347	0.0771	6033 (↓4.9%)	0.0466 (↓39.5%)
	6356	0.0792	6044 (↓4.9%)	0.0469 (↓40.8%)
Mode 10	not found		6222	0.0439
			6248	0.0446

In both analysed cases, the so-called multiple modes of vibration typical for axisymmetric structures were found. In the case of an empty pressure vessel, the first ten vibration modes were estimated in the frequency range from 0 Hz to 6400 Hz. The tenth mode was not identified for the filled vessel because its corresponding frequency exceeds the analyzed frequency range. A decrease of up to 11.3% can be seen for almost all natural frequencies. Only in the case of mode seven, this change did not occur, even though the natural frequency increased by 0.6-0.8%. However, this mode differed in character from the others (significant vibration in the valve region visible from the FEM analysis not included in this paper). The estimated mode shapes of the vessel obtained from the experimental modal analysis are shown in Fig. 6. The individual vessel mode shapes were identical for both analysed cases.

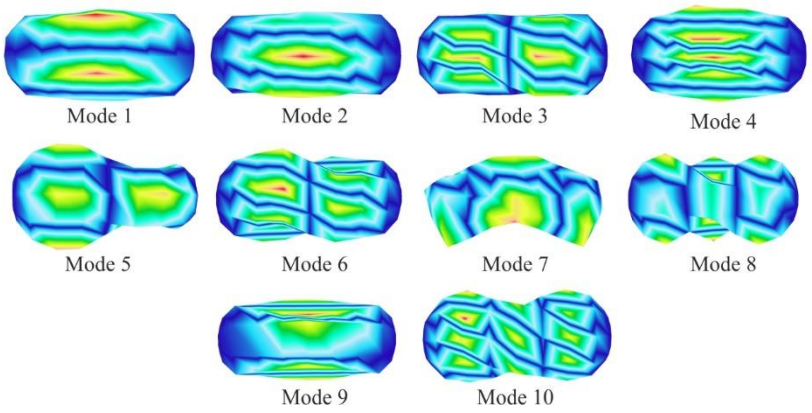

Fig. 6 Mode shapes of the analysed vessel obtained by experimental modal analysis.

2.2 Estimation of Rayleigh damping coefficients

The coefficients α and β for estimating the Rayleigh damping according to relation (1) were calculated from the experimentally determined values of the natural frequencies and damping ratios of the individual modes for both states of the pressure vessel. Estimates of the above coefficients were obtained sequentially for the individual frequency bands defined by the n^{th} and first vibration modes but only for the first from the multiple modes. The obtained results are presented in Table 2 and Table 3.

In both analysed states of the vessel, a very low value of the β coefficient approaching zero can be observed, i.e., the damping of the vessel is proportional to its mass. This case is characterized by very low damping at higher structure modes. Comparing the coefficient α, it is possible to observe a decrease of more than 50% in the case of an empty vessel.

Table 2. Rayleigh damping coefficients for the analysed vessel filled with helium at 80 bar.

	Frequency (Hz)	**Damping ratio (%)**	**β**	**α**
Mode 1	1788	0.1161		
Mode 2	2489	0.0941	2.836E-08	22.509
Mode 3	3923	0.0877	3.560E-08	21.595
Mode 4	4146	0.0847	3.269E-08	21.963
Mode 5	4533	0.0773	2.617E-08	22.785
Mode 6	4945	0.0759	2.509E-08	22.922
Mode 7	5560	0.0622	1.588E-08	24.084
Mode 8	6113	0.0872	3.031E-08	22.263
Mode 9	6347	0.0771	2.416E-08	23.038

Table 3. Rayleigh damping coefficients for the empty analysed vessel (0 bar pressure).

	Frequency (Hz)	Damping ratio (%)	β	α
Mode 1	1665	0.0579		
Mode 2	2209	0.0499	2.076E-08	9.845
Mode 3	3754	0.0453	2.067E-08	9.854
Mode 4	3830	0.042	1.720E-08	10.234
Mode 5	4501	0.0458	1.998E-08	9.929
Mode 6	4677	0.0442	1.836E-08	10.107
Mode 7	5593	0.041	1.482E-08	10.495
Mode 8	6009	0.0465	1.747E-08	10.205
Mode 9	6033	0.0466	1.750E-08	10.201
Mode 10	6222	0.0439	1.565E-08	10.404

The minor differences in the values of the parameter α obtained in both measurements for different defined frequency bands declare that the Rayleigh damping model estimated for the whole analysed frequency range (between modes 1 and 9 for a vessel with 80 bar gas pressure, respectively 1 and 10 for an empty vessel) should be sufficiently accurate also in the case of the vibration analysis of the lower modes.

A comparison of the dependence of the damping ratio on frequency obtained for different defined frequency bands in the case of the two analysed vessel states can be seen in Fig. 7-9.

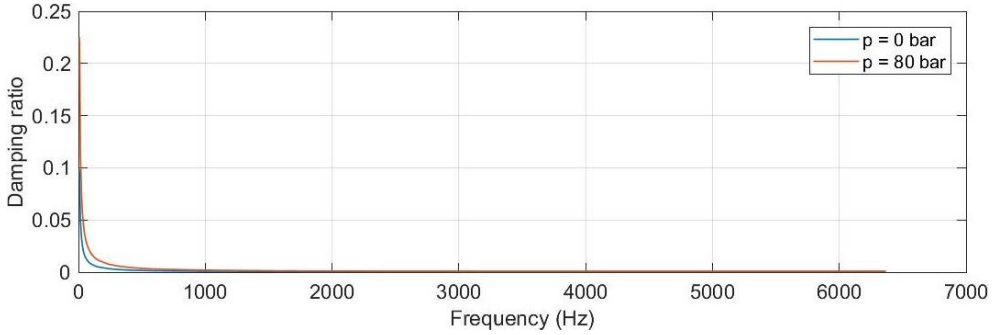

Fig. 7 Frequency dependence of Rayleigh damping ratio obtained for the frequency band ω_2-ω_1.

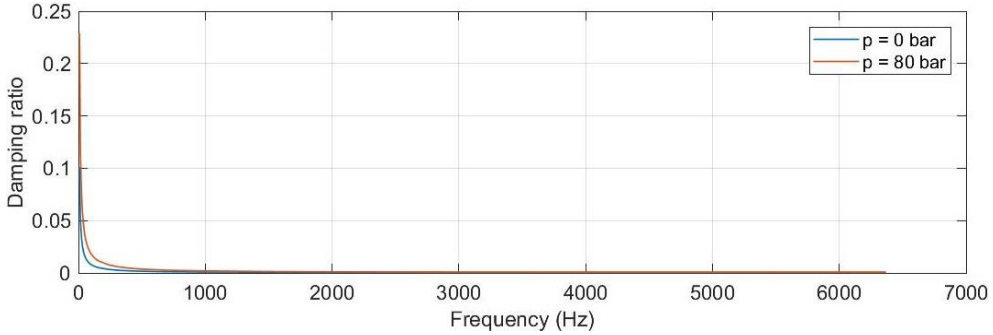

Fig. 8 Frequency dependence of Rayleigh damping ratio obtained for the frequency band ω_6-ω_1.

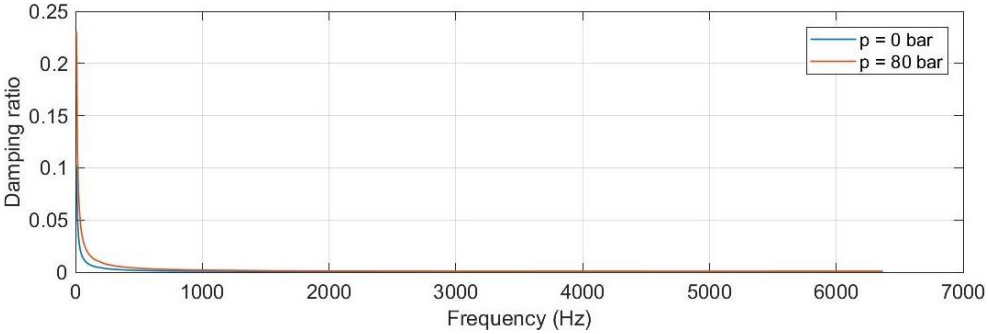

Fig. 9 Frequency dependence of Rayleigh damping ratio obtained for the frequency band ω_9-ω_1.

Conclusion

The paper describes the methodology for the assessment of the modal parameters of the pressure vessel. The analysis of the mode shapes, natural frequencies, and damping ratios was realized for two different states of the vessel, i.e., the vessel filled with helium at a pressure of 80 bar and an empty vessel, whereby the inner pressure of the gas was expected to be 0 bar. The obtained results confirm that the inner pressure influences the analysed quantities, i.e., the higher the pressure, the higher the natural frequencies and damping ratios. On the other hand, there is no difference in the vibration mode shapes.

The second part of the paper deals with estimating the vessel Rayleigh damping model, a form of viscous damping proportional to the linear combination of mass and stiffness matrices often used in finite element analysis. According to Fig. 1, it was obtained in both cases that the damping of the vessel is proportional to its mass. Moreover, it can be concluded that the inner pressure also influences the Rayleigh damping coefficients, i.e., α and β, which decrease with decreasing inner pressure. As the experimental modal analyses of the vessel were realized using soft rubber cords to minimize the effect of bond stiffness, the authors aim to use the obtained results in the vessel numerical model verification process.

Acknowledgements

This work was supported by the Ministry of Education, Science, Research and Sport of the Slovak Republic under the project VEGA 1/0516/22.

References

[1] Liu, M. and Corman, D.G. "Formulation of Rayleighdamping and its extensions," *Computers and Structures*, **57**(2), pp. 277-285 (1995). ISSN 1879-2243.

[2] Geraschenko, V.S., Grishin, A. S. and Gartung, N. I. "Approaches for the calculation of Rayleigh damping Coefficients for a time-history analysis," *Structures under Shock and Impact*, **XV**, pp. 227-237 (2018). ISSN 1743-3509.

[3] Xu, J., Huang, Y and Qu, Z. "The high-frequency oscillation in sstems with Rayleigh damping model," in *Proceedings of the 12th Canadian Conference on Earthquake Engineering*, Quebec City, 2019, ed. (QC, Canada, June 17-20), pp. 1-8.

[4] Song, Z. and Su, C. "Computation of Rayleigh Damping Coefficients for the Seismic Analysis of a Hydro-Powerhouse," *Shock and Vibration*, **2017**, pp. 1-11 (2017). ISSN 1875-9203.

[5] Rahul, B., Dharani, J. and Balaji, R. "Optimal Method for Determination of Rayleigh Damping Coefficients for Different Materials using Modal Analysis," *International Journal of Vehicle Structures and Systems*, **13**(1), pp. 102-111 (2021). ISSN 0975-3540.

[6] EN 12205:2001 Transportable gas cylinders - Non-refillable metallic gas cylinders.