

**EXPERIMENTAL VERIFICATION THE FRAMED STRUCTURE SHAKEDOWN THEORY**
**JÁN E V I N**

The paper deals with the experimental verification of "the shakedown theory" of the frame structure subjected to cross-bending stress due to elasto-plastic effect from the viewpoint of the complex system of the reliability conditions of the loading capacity limit state. The aim of the experimental investigation was to study the strain deformations of the most stressed section of the test model suffering cyclic changes of mechanical and thermal loadings with the purpose of obtaining the working structure diagram.

Materials are plastically deformed due to the stress applied above certain extent. Material behaviour in a plastic area is subjected to external agents. One of such possible factors are cyclic changes of mechanical and thermal loading of a material resulting in the rise of ultimate areas of plastic strains. Besides that it is important to define the change ranges of external forces and temperatures at which plastic strains occur only with the first loading cycles. After a certain number of loading cycles the increase of plastic strains is highly reduced and with further loading the structure is deformed almost elastically. Calculation of strength with the effect of repeated loads, especially with the cyclic effect of mechanical and thermal loading, may be done by "the theory of shakedown". The most suitable reliability and durability consideration of any structure type exposed to the thermal field can be realized by tests simulating operational conditions. The use of models for experimental study of failure processes due to the thermal effect is connected with difficulties. Therefore conditions similar to the real ones are simulated. Models utilization was justified in the cases where attained results were used not only for strength estimation of a given structure, but also for a partial verification of hypotheses used for calculation.

The static theorem called "the Melan's shakedown theorem" is usually expressed by the two statements:

1. The structure is shaken down to the cyclic load if its behaviour after one cycle or several first ones becomes flexible since it is possible to find such a time independent field of residual stresses  $\sigma_{ij}^*$  that their sum with stress in an ideal flexible structure  $\sigma_{ij}^0$  at any load changes within determined limits forms the so called safe stress state

$$\sigma_{ij}^* = \sigma_{ij}^e + \sigma_{ij}^r \quad (1)$$

lying inside the plasticity surface. The statement is based upon the fact that the shakedown will actually be realized if it may happen with the cyclic loadings within given limits.

2. The structure does not shakedown if there is no time-independent field of residual stresses, thus in the sum with flexible stress at any loading changes it constitutes a permitted stress state.

The main part of "the Koiter's theorem" is the equation which enables to determine the limit loading value at the given range of 1st change provided that its repeated effect with either arbitrary or fixed loading programme does not lead to the cyclic plastic strain (alternating plastic strain or unilateral increase of plastic strain).

$$\int_0^T \left\{ \int_V \dot{X}_i \dot{u}_{i0} dV + \int_{S_p} \dot{p}_i \dot{u}_{i0} dS \right\} dt = \int_0^T \int_V f(\dot{\epsilon}_{1j0}^p) dV \quad (2)$$

The equation (2) defines the equality between the external forces effect on residual displacements of plastic energy per cycle interval T.

In technical practice there are often cases when the body is suffers both external loading equal to one parameter and the cyclic changeable thermal field. The failure mechanism is thus defined on the base of the external load and corresponding stresses of additional loading in all points of the body in which increments of plastic strain over a cycle are distinct from zero. At the calculation such a loading acting upon a developing failure is considered to be constant and equal to its maximum value corresponding with the most unfavourable loading programme.

Kinematic theorem utilization can be widened for such cases when the structure is stressed by both mechanical loading and thermal cycles, and thus the equation is

$$\int_V \bar{X}_i \Delta u_{i0} dV + \int_{S_p} \bar{p}_i \Delta u_{i0} dS + \int_{V_d} \sigma_{ij}^* \Delta \epsilon_{1j0}^p dV = \int_V f(\Delta \epsilon_{1j0}^p) dV \quad (3)$$

in which  $\bar{X}_i$ ,  $\bar{p}_i$  are maximum values of volume and area forces, the stresses  $\sigma_{ij}^*$  are derived from thermal loads. The equation now expresses the decrease of the structure carrier ability corresponding with the thermal cycles effect.

Tensometric measuring was performed on the framework subjected to the cyclic mechanical and cyclic thermal loading in selected cross sections. The cyclic mechanical loading was realized by force screws, whereby one of them was inducing upwards loading while the other one was used for the downward direction. Upward loading was carried out by the roll. Thermal cycling loading was induced in the electric furnace. Heating speed growth was achieved by the two additional heating spirals and four electrical infra-red heaters.

The framework was built-in the base board made of the material, the temperature extensibility coefficient of which was lower than that of the steel. Further extensibility decrease of

the base board due to the temperature was attained by the thermal insulation of asbestos fabric. The configuration of the experimental equipment is shown in figure 1.

The proposed method of the experimental investigation applied verify this theoretical solution acquired the procedure by means of which it is possible to confirm or eliminate the correctness of the results and at the same time to eliminate systematic errors of the experiment. In the first phase of the experimental investigation the slow mechanical cyclic loading was solved. The loading was caused by force screws, whereby the loading value was induced by dynamometers. By means of the method mentioned above and with connected tensometers, there were, recorded only bending strain values. By the help of experimental method it was established that the loading change leads to the change of the bending strain sign.

In further procedure the problem of strains due to cyclic mechanical loading with different levels of thermal loading was being solved. Cyclic mechanical loading was established in the same way as it had been mentioned above. The temperature level in the course of these cyclic mechanical changes was changed in maximal degree within the range  $4^{\circ}\text{C}$ .

Relationship of the cyclic loading and temperature at the constant level of mechanical stress was further solved. The thermal growth was limited by the used tensometric sensors and applied binding agent. Owing to the tensometric sensors it was not necessary to use the curve of simulative strain caused by temperature.

Measurement was carried out by tensometric sensors distributed by the firms M-M MEASUREMENTS GROUP and HOTTINGER BALDWIN MESSTECHNIK. Adhesive M-Bond 610 and EP 250 by M-M MEASUREMENTS GROUP was applied to tensometric sensors WK-06-125BQ-175.

Mechanical loading was identified by dynamometers KMB M 5KN type and electric furnace STE 23 type made by Chirana was used to induce thermal loading as well as two resistant electrical spirals and four ceramic infra-red heaters 513 type made by ELEKTRO PRAGA Hlinsko. The temperature was measured by the contact electronic multimeter V-640 type. The measurement was performed by tensometric equipment M 200, made by MIKROTECHNA Prague.

Bending strain values were measured in five cross sections, both with the cyclic mechanical loadings and the cyclic thermal loadings. At the cyclic mechanical loadings it was established by both the calculation and experimentally that maximal strains are in the cross section of the loading place. With the test

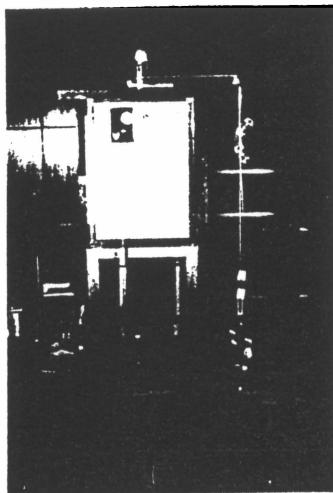


Fig.1

model 15 strain values are given in the place of tensometer T3 fig. 2. In other cross sections the measured strain values correspond with the calculated values.

### THE TEST MODEL 15

The working diagram

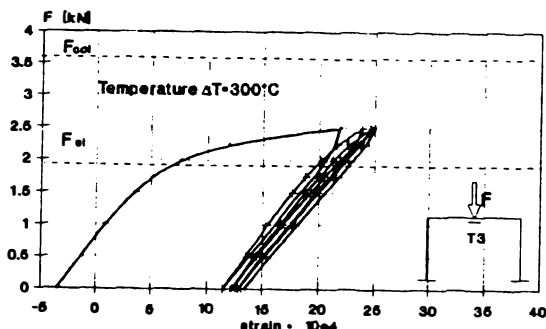


Fig. 2

The proposed method of the experimental investigation in the first phase with loading to a certain force value and at the gradual unloading to zero value, and repeated loading and unloading to the same bounds, proves that all further cycles starting with the third one are equal. It is not possible to draw conclusions for description of material behaviour with the loading cycling number lower than three. On the other hand, it is not possible to consider the structure material behaviour if the material was under any kind of loading before it had been experimentally investigated to demonstrate the structure shakedown.

### References

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