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Numerical And Experimental Analysis Of Plastic Deformation In The Vicinity Of The Crack Tip

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Abstract:

In the first section, the rate of strain energy under loading of a crack specimen will be discussed, followed by a discussion of numerical analysis of plastic fields in the vicinity of the crack tip, and finally a comparison with an experiment carried out by an optical method of deformed circle. Since the results (e.g. force vs. displacement) obtained from the experiment and analysis did not match well, a correction of the stress-strain curve was carried out. The original stress-strain curve was derived from a smooth round bar tensile test but in the vicinity of the crack tip occurs different state of stress, than that in round bar. Thus, the values of calculated and measured strains in selected places in the vicinity of the crack tip had been observed and the stress-strain curve had been modified until a certain allowed difference was achieved. The force vs. displacement dependence was then again calculated for the modified stress-strain curve.

Key Words:

Non-linear Fracture Mechanics, Plastic Strain, FEA, Method of Deformed Circle.

1. Introduction

This work concerns the field of non-linear fracture mechanics when high plastic strain occurs in the vicinity of the crack tip, resulting in ductile fracture. In these analyses, a notable decline of stress at the crack tip is observed in some cases before fracture, which is experimentally proved. An analysis of energy behaviour in the specimen is discussed in the first section. The critical force value, at which the initiation of crack growth begins, was derived using the strain energy density criterion. The comparison of the plastic field obtained from Finite Element Analysis (FEA) and from the experiment is discussed next. The stress-strain curve (true stress vs. logarithm of strain) derived from the tensile test was used as the constitutive relation. In comparison of the results, discrepancies were found between experimental and numerical analyses in the values of strain and also in the loading force F versus displacement u_A of the point A dependence. The reason for this showed to be the constitutive relation, which was used in the numerical analysis. For this reason, a modification of the stress-strain dependence had been carried out, with the intention to bring the numerical results to the experimental ones.

The FEA was performed using MARC K7.3 system.

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2. Description of the Method of Deformed Circle

The method is based on the optical observation of deformation of a hexagonal grid of points deposited on the test specimen surface. The shape and the size of the plastic zone on the specimen surface is determined from total strain in the front of the crack.

The method is derived from the axiom of the transformation of a sphere or circle into an ellipsoid or an ellipse respectively, as used for example in the Ziebel method [1]. Assuming homogeneous deformations of a homogeneous and isotropic material, it is possible to consider that the circle transforms into an ellipse, which can be assumed always on a small region. (This assumption is the base of all experimental methods for strain field determination.) The values of principal strains (1) in the plane of the specimen surface can then be easily found from the parameters of the ellipse interpolating a set of six points of the grid ('one cell').

Thus, the measurement is not based on the relative displacements of particular points, which is usual for similar methods, but it is based on the parameters of the ellipse.

$$\varepsilon_{11} = \frac{a-r}{r}, \quad \varepsilon_{22} = \frac{b-r}{r} \tag{1}$$

In local coordinate system of the ellipse, the principal strains are measured directly, i.e. the maximum deformation in current place. All strain components in the global coordinate system can be calculated afterwards when needed, including the shear component. Higher accuracy for the same sensitivity of the recording device can be achieved when compared to the use of an orthogonal grid. This concerns mainly the case of heterogeneous strain fields when the orientation of the grid does not correspond with the directions of the principal strains.

The technical performance of the experiment is relatively easy. A hexagonal grid is deposited on the polished specimen surface using the photoresist method. The grid is scanned during the experiment with a CCD camera and the data obtained are recorded with SVHS VCR or saved directly onto a computer disc. No special laboratory conditions are requested for sensitivity of tenths of percent. During the performed experiments, the obtained sensitivity was under 0.5 percent of relative strain for length of 20 pixels (i.e. the diameter of the original circle). Obviously, a higher accuracy was achieved for higher diameters.

Requested pictures are then selected and processed with a program created by the author in Matlab 4.2c.1 with additional Image Processing Toolbox 1.0b.

3. Analysis of the Central Notched Specimen

The crack specimen observed in this analysis was 2CCIIb with dimensions $h = 40$ mm, $w = 51$ mm, $t = 6$ mm, and crack length $2a$, where $a = 4.8$ mm. The specimen was made of aluminium alloy characterised by material constants: Young's modulus $E = 69122$ MPa, Poisson's ratio $\nu = 0.315$, and stress-strain dependence shown in Figure 2.

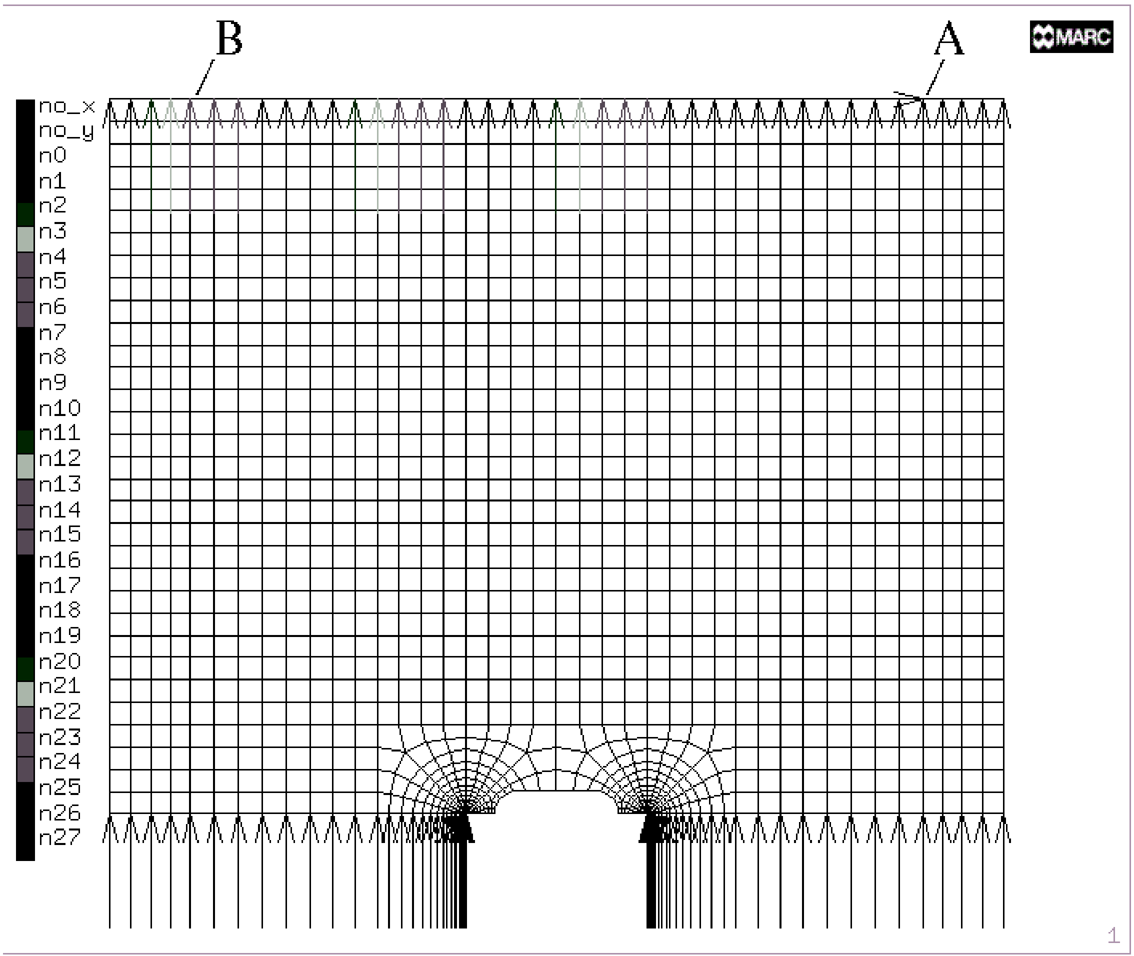

Figure 1. FEM mesh of the 2CCIIb specimen

The model for FEA was created in pre-processor MENTAT 3.3 of MARC K7.3 system. The mesh of this model together with the applied boundary conditions is shown in Figure 1. A 2D plane stress analysis was performed. Despite the fact that the geometry allowed analysis of one quarter of the specimen, a half of the specimen had to be considered, because of non-symmetrical loading (the values used were obtained from the experiment). A 3D analysis was also carried out, but due to the small thickness of the specimen, the results obtained were almost the same as those from the plane stress case. The radial length of the smallest elements, situated at the crack tip, was $r = 0.05$ mm, and this length increased with $k = 1.2$ ratio. The angle between the long sides of these elements was 15°. Four and eight-node quad elements were tested throughout the analyses. Since the problem considered was elastic-plastic, the four-node elements proved sufficient. The model was loaded both in the displacement control as well as in the force control.

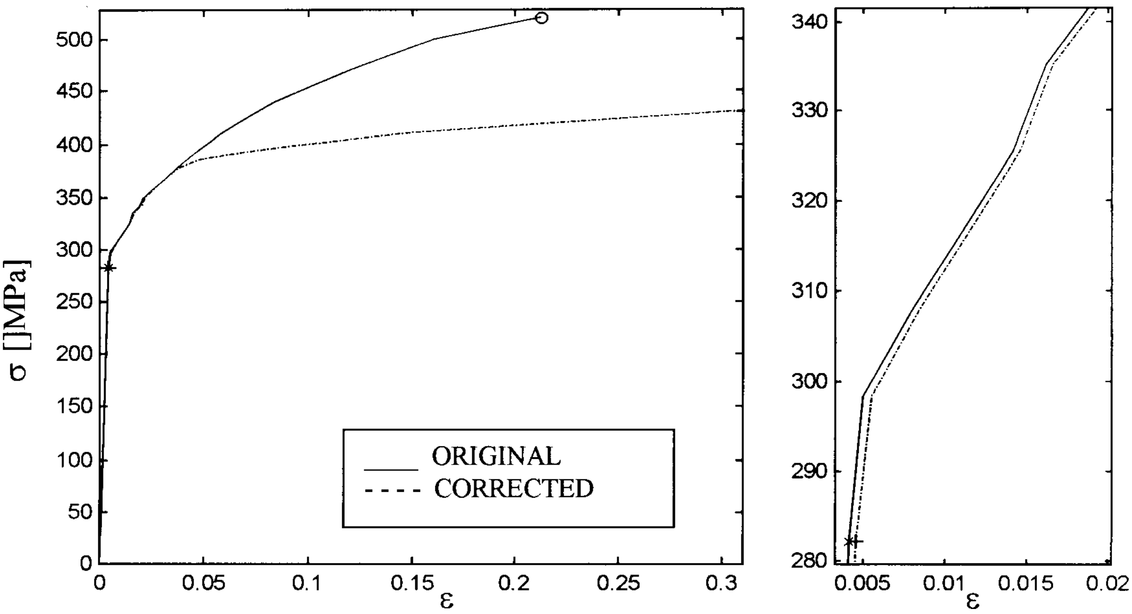

Figure 2. Stress-strain curves (detail of the yield stress region on the right)

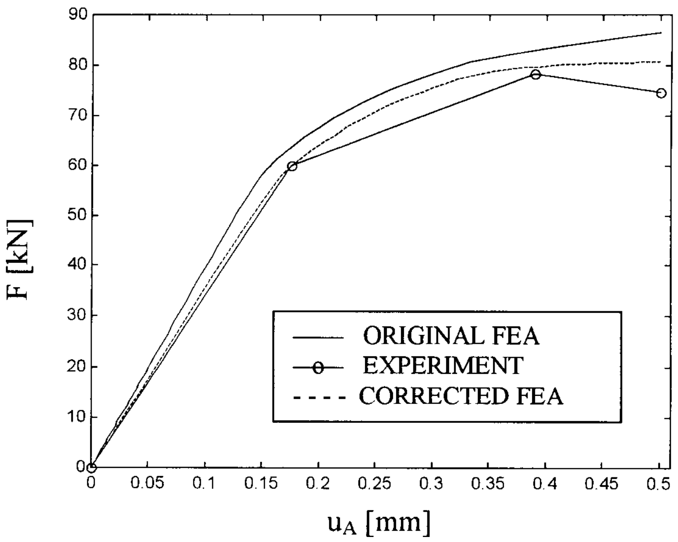

Figure 3. F vs. u_A (displacement of the point A) dependence

At first, the calculated and experimental dependencies F vs. u_A (loading force vs. displacement of the point A, see Figure 1) were compared. It can be seen from Figure 3 that the FEA model was more rigid than the real specimen, yet they compared well in some characteristics. A noticeable change in behaviour of the force occurred at its value of $F = 60$ kN. For the model, this occurred when the plastic field spread across the whole cross-section in the crack plane. In the last phase, $0.4 < u_A < 0.5$, the true force decreased with very dynamic trend during which the displacements of the points A and B increased rapidly. The calculated force did not, which could have been predicted from the stress-strain dependence σ - ε (see Figure 3) used in FEA.

Other quantities to be observed during the loading were strain energy, its elastic and plastic components, and their derivatives. These quantities should reveal some behaviour of the specimen too. A noticeable increase in plastic strain energy can be seen in Figure 4 for displacement value of $u_A = 0.14$ mm corresponding to force $F = 55$ kN. The derivatives of strain energies in Figure 5 yet more clearly show the moment of change of this increase, and on the other side, the decrease of the elastic component.

Among the most important is the information about the value of force at the moment of initiation of the crack growth. Several theories can be used for the determination of this moment. In this case, the criterion of strain energy density was used [7]. It states that the initiation of the crack growth occurs when so-called local relative minimum of the strain energy density, derived from calculated stress-strain deformation at the crack tip, reaches critical value. This criterion can then be expressed as follows:

$$\left(\frac{dU}{dV}\right)_{\min} = \left(\frac{dU}{dV}\right)_c \tag{2}$$

From the material stress-strain curve, the initiation for this model was determined as a moment in which the force and displacement of point A reach values of $F = 59{,}7$ kN, and $u_A = 0{,}15$ mm respectively. In Figure 5, it is the region of high increase in plastic and decrease in elastic strain energy components.

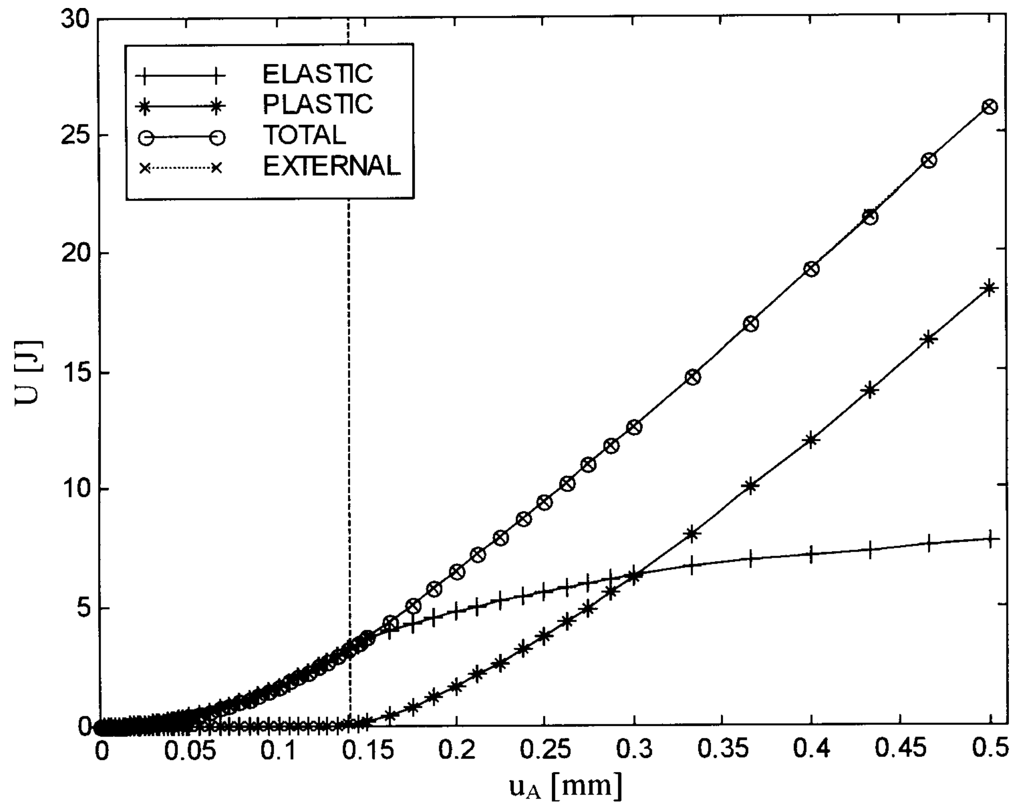

Figure 4. Strain energy and its components vs. displacement in point A

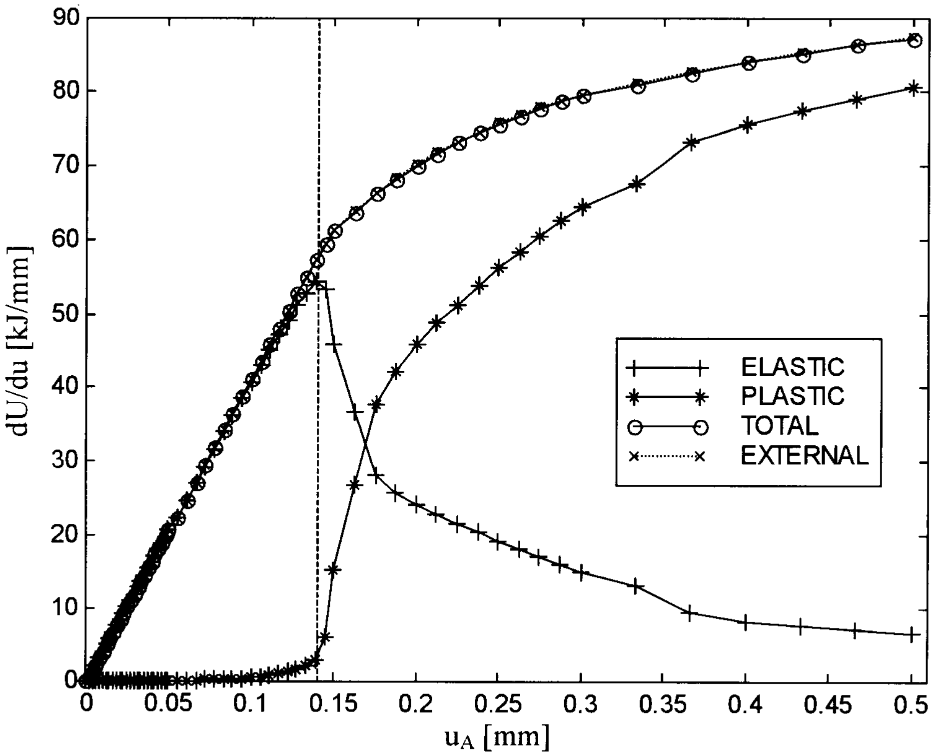

Figure 5. Derivatives of strain energies vs. displacement in point A

4. Comparison of the FEA and Experiment Results

Using the deformed circle method (see Chapter 2 and [2]), the strain field at the crack tip was determined. A discrepancy was found between this field and the field calculated by FEM. The reason for this is most likely the incorrect stress-strain curve (constitutive relation), which was previously suggested from the force behaviour (see Figure 3). This discrepancy led to intention of revaluating the data used in the FEA model. For this reason, a modification of the stress-strain curve was carried out. Points in the vicinity of the crack tip were chosen and their strain values were observed and compared with experimentally obtained values during the loading process. Figure 6 shows the course of the equivalent plastic strain obtained from experiment at load corresponding to the displacement of $u_A = 0.5$ mm. The peak of the graph corresponds to a place closest to the crack tip. The discrepancies between values obtained by both methods were removed by modifying the stress-strain curve during the loading process until the following condition was satisfied:

$$\left| \varepsilon_{num} - \varepsilon_{\exp} \right| \leq \Delta \ , \tag{3}$$

where ε_{num} and ε_{exp} are the numerically and experimentally obtained values of the equivalent plastic strain a Δ is assessed allowed error.

The modified stress-strain curve is in Figure 2. It can be seen that this diagram, derived from the values in the immediate vicinity of the crack tip, has lower hardening slope than that of the experimental one. It can also be seen (in Figure 3) that the recently calculated F vs. u_A dependence compares better with the experimental data.

With the modified stress-strain curve in the analysis, conformity of the calculated and measured strain fields was found except for the points in the immediate vicinity of the crack tip. The values in this region calculated with the FEA were higher than those from the experiment. This can be explained by the fact that in ductile materials microvoids arise in regions with high concentration of stress. These microvoids grow and coalescence occurs resulting in a less rigid material. The FEA model mentioned above probably cannot describe this behaviour, since it is based on Von Mises yield condition:

$$\tfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] = \sigma_k^2 \,, \qquad (4)$$

where σ_1, σ_2 and σ_3 are the principal stresses and σ_k is the material yield stress.

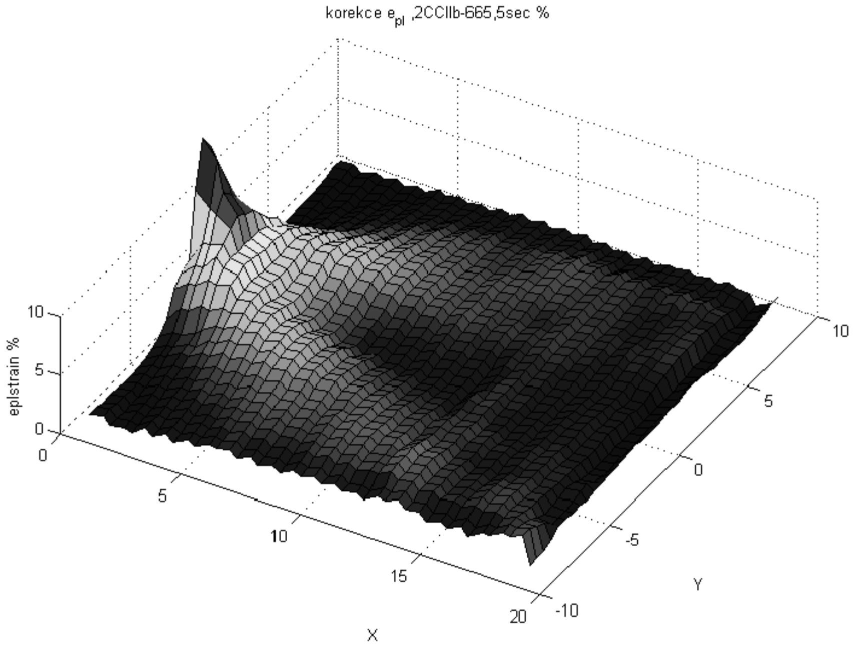

Figure 6. The course of experimentally obtained equivalent plastic strain

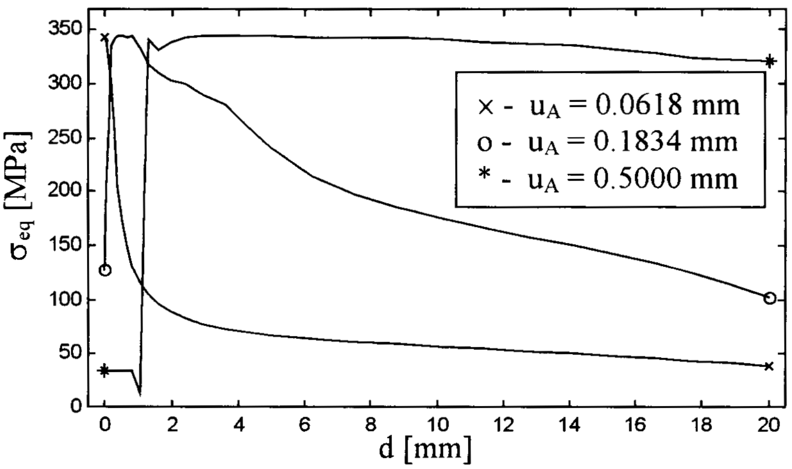

Figure 7. Equivalent Von Mises stress along the front of the crack

A better correspondence between the experiment and numerical solution could be achieved if a different theory for the numerical model was applied. It should be a theory that includes the influence of hydrostatic stress which occurs at the crack tip. For example, the modified Gurson model seems acceptable [8].

5. Conclusion

This work concerns the field of non-linear fracture mechanics, stress and strain fields in ductile materials, where ductile fracture occurs. The experiment was performed using modified method of grids consisting of circles regularly placed across the crack front of the specimen. This method shows to be convenient for analyses concerning plastic strain. The numerical analysis was performed using the MARC system. The incremental theory of plasticity and the Newton-Raphson iterative procedure was applied in the analysis.

The changes in energy of external forces and in the components of strain energy during increasing loading F were also observed. More information about the specimen behaviour was derived from the energy derivatives versus the displacement dependence. It showed clearly that the moment of the most remarkable changes occurs when the initiation of crack growth begins; according to the theory of strain energy density.

One of the main tasks of this work was the creation of a numerical model of a real crack specimen capable of high plastic strain in the vicinity of the crack tip.

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