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Large Strain Analysis Based on Electrical Resistance Measurement

Analýza velkých deformací s využitím odporové tenzometrie

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Abstract:
The paper presents conceive summary of facts influencing outputs of metallic resistance strain gauges during measurement of large strains. Analysis of mistakes produced by non-linearity both gauge factor and output of Wheatstone quarter bridge is executed. The field of strain gauge application is complemented by new possibilities of potential drop method with sophisticated instrumentation.
Keywords: large strain, strain gauge, compensation of non-linear effects, potential drop method.

1.0 Introduction

Term "large strain" or "high elongation" in mechanics of continuum is rather relative and then determination of its range depends on the part of mechanics under interest. Lower limit of large strain should be determined by non-negligible value of deflection from the linear dependence of deformations on load (as consequence of geometric or material reasons), upper boundary is limited by mechanical properties of a material, esp. its ductility. Application of strain gauges brings another limitation of this boundary to the highest measuring range of gauges and cements which temporary reaches ≈20%. Although demand on measurement of such high strains is not too often, it is very urgent at post-yield stage measurements and during tests to destruction. The aim of strain measurement on defined or normalised specimens serves to verification of right description or to improvement of stress-strain field mathematical model at stress concentrator vicinity.

Respecting this the analysis of experimental data precision is necessary, especially with regards to two known sources of non-linearity in comparison to the traditional proportional relationship used for strain gauge measurement evaluation:
- dependence of gauge sensitivity (gauge factor) on magnitude of strain,
- non-linearity of Wheatstone bridge at high changes of resistance.

It is referred already in classical handbook [1] that these effects are mutually compensated but without quantitative evaluation of the result which have not been found in any available literature too.

The physical principle of strain gauges (piezoresistance) is applicable for direct measurement of large strain by proper application of potential-drop method. Final part of the paper presents conceive description of the realised experiment during tensile test of aluminium alloy specimen.

2.0 Application of strain gauges

2.1 *Non-linearity of metal strain gauge sensitivity*

The high accuracy of the method at most frequented strain range $\varepsilon \leq 1\%$ (10 000 μm/m) is partially decreased at range of applicability of a standard foil strain gauges $\varepsilon \leq 5\%$ and still not thoroughly analysed at the highest range $\varepsilon \approx 20\%$ for special gauges [2] and [3] . It is good to remind, that at such extreme strain resistance changes of 120 Ω gage reach nearly 50 Ω and value of Poisson's ratio cannot be assumed as constant 0,5. For full range of isovolumetric (plastic) deformation it must be determined as a function of strain ε from expression

$$\mu = \frac{1+\varepsilon-\sqrt{1+\varepsilon}}{\varepsilon(1+\varepsilon)}, \tag{1}$$

for $\varepsilon = 0{,}2$ is then $\mu = 0{,}435$ and $\mu = 0{,}5$ is the limit of this expression for $\varepsilon \to 0$.

As main problem seems to be a gaining of general sensitivity - strain dependence of measuring element that is created as a meander in foil manufactured from constantan prepared by special mechanical and thermal treatment. A theoretical description of the phenomenon is thoroughly presented in works e.g. [2] or [4], where both specific resistance and shape changes due to uni-axial stress state are considered at elastic strain range (for used constantan below $\approx 0{,}5\%$) while in plastic range isovolumetric shape deformation and their contribution to resistance changes are only taken in account.

Basic physical law for resistance of prismatic conductor is

$$R = \rho \frac{\ell}{A} \tag{2}$$

where ρ is specific resistance, ℓ is length of the conductor and A is area of its cross section. After uni-axial plastic deformation the original values will change to values with apostrophe and eq. (2) will change to

$$R' = \rho \frac{\ell'}{A'} \tag{3}$$

Assuming the condition of constant volume expressed by

$$\ell' A' = \ell A \tag{4}$$

we can express ratio of final to original resistance as

$$\frac{R'}{R} = \frac{\ell' A}{\ell A'} = \left(\frac{\ell'}{\ell}\right)^2 = (1+\varepsilon)^2 = 1 + 2\varepsilon + \varepsilon^2 \tag{5}$$

where ε is Lagrangian strain. After introduction change of resistance $\Delta R = R' - R$ it is possible make a modification

$$\frac{R+\Delta R}{R} = 1 + 2\varepsilon + \varepsilon^2 \Rightarrow \frac{\Delta R}{R} = 2\varepsilon + \varepsilon^2 \tag{6}$$

This relation can be assumed as the basic expression for relation of resistance change of metal element exposed to large strain and serves to derivation of strain sensitivity noted for this case $k^A_{(\varepsilon)}$

$$\frac{\Delta R}{R} = k^A_{(\varepsilon)} \cdot \varepsilon \Rightarrow k^A_{(\varepsilon)} = 2 + \varepsilon \tag{7}$$

which is presented e.g. both in handbook [2] and in monograph [3] with notice, that there is not full agreement with experiments that have gone on.

Moreover, at book [1] is presented apparently different expression of this relation having form

$$\frac{\Delta R}{R} = k(\varepsilon + \varepsilon^2) \Rightarrow k^B_{(\varepsilon)} = k(1+\varepsilon) \tag{8}$$

where k is well known gauge factor for small strain.

A difference between expressions (7) and (8) is evident and it can be quantified for k = 2 for meaningful interval of ε (see table 1). Values comparable with strain can be for both cases obtained by calculation of

$\frac{1}{2}\frac{\Delta R}{R}$, which is plotted for interval $-0.2 \le \varepsilon \le 0{,}2$ in fig.1 (upper two curves).

Both curves have character of a convex function i.e. values are increased in comparison to linear dotted line with the distance from origin of co-ordinates.

2.2 *Non-linearity of Wheatstone bridge*

Connection of single strain gauge with resistance R at one arm of symmetric Wheatstone bridge ordinary used for small strain measurements need analysis with complete formulas derived from Kirchhoff's laws of electric circuits, where values of ΔR compared to R cannot be considered as negligible. The solution will be carried out for two basic types of bridge excitation:

1) Constant voltage U = const

Output signal of the bridge given by ratio of measuring diagonal voltage ΔU to excitation voltage U is

$$\frac{\Delta U}{U} = \frac{\Delta R}{2(2R + \Delta R)} \tag{9}$$

2. Constant current I = const

Introduction of initial voltage $U_0 = IR$ enables create similar but different expression of output signal

$$\frac{\Delta U}{U} = \frac{\Delta R}{4R + \Delta R} \tag{10}$$

The difference between outputs in this two cases is quantified for meaningful interval of ε in table 2 and plotted again in form of values comparable with strain

$2\frac{\Delta U}{U}$ in interval $-0.2 \le \varepsilon \le 0{,}2$ in fig.1 (lower two curves).

Both curves have character of a concave function i.e. values are decreased in comparison to the linear dotted line with distance from the origin of co-ordinates.

2.3 *Superimposed deviation from linearity.*

Resulting deviation from linear law valuable for small strain can be evaluated for all four combinations

1) type of gauge with sensitivity $k^{A}_{(\varepsilon)}$ connected into circuit U = const ,
2) type of gauge with sensitivity $k^{B}_{(\varepsilon)}$ connected into circuit U = const ,
3) type of gauge with sensitivity $k^{A}_{(\varepsilon)}$ connected into circuit I = const ,
4) type of gauge with sensitivity $k^{B}_{(\varepsilon)}$ connected into circuit I = const

after introduction relations (7) and (8) into (9) and (10). Final values expressed again in the form $\frac{1}{2}\frac{\Delta U}{U}$ are for formerly used intervals presented in table 3, deviation of the same values from ideal linear law is presented in table 4 and fig.2 (in percentage).

3.0 **Electric Potential Method**

3.1 *Principle and applicability*

The method called usually potential drop is based on measurement of potential fields on electrically conductive bodies via properly placed electrodes (ordinary welded). Despite of fact that its application in experimental mechanics has long history, it is considered as material testing method, esp. for crack growth monitoring (e.g. in [2] is not mentioned). This situation seems to have been changed in the last few years after considerable accuracy and sensibility improvements of measuring device (synchronous detection, low frequency constant current feeding, statistical computer evaluation etc.) created by Dr. L. Korec. They open new application fields in measurement of post-yield strain, temperature creep, observation of wall thickness changes (corrosion and erosion) and of course crack initiation and growth with all types of loading.

Application of the method for large strain measurement during tensile test is based on the same principle as metallic strain gauge [5]. The only substantial difference lies in substitution of strain gauge grid by the body of a specimen so that for isovolumetric deformation the principal relationship (5) must be true again. Because measured part of specimen has constant current I, any two distanced points equipped by measuring electrodes indicate potentials U (initial) and U′(strained) proportional to R and R′. The following relation analogous to (5) then can be written

$$\frac{U'}{U} = 1 + 2\varepsilon + \varepsilon^2 \tag{11}$$

It is quadratic equation with the only one physically real root

$$\varepsilon = -1 + \sqrt{\frac{U'}{U}} \tag{12}$$

which is valid for both signs of the strain.

3.2 *Results of large strain measurement at aluminium alloy.*

One of the first experiments was executed by Dr. Korec in co-operation with ÚTAM AVÈR as a part of grant GAÈR 106/99/1467 headed by Dr. J. Zemánková.

Hard aluminium alloy cylindrical specimen by supplying (outer) and measuring (inner) electrodes was equipped. The reference values of strain were obtained from edge extensometer based on inductive principle and magnitude of force was taken from MTS load cell. Loading by constant velocity was produced by mechanical testing machine Testatron.

One result from testing is presented in fig.3 where ideally linear plot of strain is represented by straight line. Values obtained from potential method directly by evaluation (12) are denoted ◊pot2. They clearly illustrate the basic character of metal piezoresistance, when after individually defined output in elastic area comes course very closed to isovolumetric resistance change predicted by shape change theory accordingly to (6). After shifting the graph (noted Δpot1) to remove the elastic effect ($\varepsilon_{el} \cong 0{,}5\%$ for this alloy) the character of non-linearity similar to the gauge sensitivity increasing with tension strain in fig.1 can be observed.

This change of piezoresistance output is typical and was observed in all other experiments and during high temperature creep (up to 600°C) too.

4.0 **Conclusions**

1. Executed calculations of mutual influence of strain gauge sensitivity and Wheatstone bridge non-linearity prove positive compensating effect of their superposition both in formerly mentioned case of constant voltage excitation and in newly derived case of constant current. Results contained in tables and graphs are influenced by an uncertainty of real sensitivity relation on large strain because test of fit verification of formulas (7) and (8) is not available. For the only commercially produced type of resistance strain gauges EP from Vishay M.G. covering the range $-20\% \le \varepsilon \le 20\%$ it should be worthwhile to build it experimentally. Nevertheless it can be recommended for large strain measurement to use connection of a gauge as quarter bridge when under its constant voltage excitation relative mistakes from ideal linear output are lesser then 10% and under its constant current excitation they are below 7% in tension and 13% in compression (tab 4).
   When the type of sensitivity-strain dependence according to (7) will be confirmed the constant current excitation will be advantageous because deviation from linearity in the range $-20\% \le \varepsilon \le 20\%$ will not overcome 1.1%. In all cases of strain gauge measurement in this large strain interval attention must be paid to sufficient range of measuring instrument input that must be more than 120 mV/V.
2. Application of direct specimen electrical resistance measurement by improved potential drop principle is perspective quantitative method for all-volume research of strain. For prismatic specimen of any cross section the evaluation based on formula (12) has sufficient accuracy. Cases of specimens with complicated shape (notched, CT specimens etc.) need individual optimisation of electrode localisation and calibrating test.

Literature

[1] K. Hoffmann: An Introduction to Measurements Using Strain Gauges (Hottinger Baldwin Messtechnik GmbH, 1989)

[2] A. S. Kobayashi: Handbook on Experimental Stress Analysis, (Prentice Hall Press, 1987)

[3] Measurement Group: TECH-TIP TT-605, (Measuring Group Inc., 1983)

[4] G. S. Holister: Experimental Stress Analysis. (Cambridge University Press, 1967)

[5] L. Korec: Potential method for plastic and creep deformation measurements (TECHLAB s.r.o., Praha, 2000)

Tables of various non-linearity accompanied large strain measurement

ε (%)	-50	-20	-10	-5	-1	0	1	5	10	20	50	100
ε	-0,5000	-0,2000	-0,1000	-0,0500	-0,0100	0,0000	0,0100	0,0500	0,1000	0,2000	0,5000	1,0000
Table1: Dependence of two types of strain gauge sensitivity from eq. (7) and (8)												
k^A	-0,7500	-0,3600	-0,1900	-0,0975	-0,0199	0,0000	0,0201	0,1025	0,2100	0,4400	1,2500	3,0000
k^B	-0,5000	-0,3200	-0,1800	-0,0950	-0,0198	0,0000	0,0202	0,1050	0,2200	0,4800	1,5000	4,0000
Table 2: Relative outputs from quarter Wheatstone bridges with various excitation from eq. (9) and (10)												
U=const	-0,50000	-0,12500	-0,05556	-0,02632	-0,00505	0,00000	0,00495	0,02381	0,04545	0,08333	0,16667	0,25000
I=const	-0,33333	-0,11111	-0,05263	-0,02564	-0,00503	0,00000	0,00498	0,02439	0,04762	0,09091	0,20000	0,33333
Table 3: Resulting non-linearity after compensating effect by superposition or eq. (7), (8) and (9), (10)												
k^A withU=c	-0,30000	-0,10976	-0,05249	-0,02562	-0,00502	0,00000	0,00498	0,02438	0,04751	0,09016	0,19231	0,30000
k^B withU=c	-0,16667	-0,09524	-0,04945	-0,02493	-0,00500	0,00000	0,00500	0,02494	0,04955	0,09677	0,21429	0,33333
k^A with I=c	-0,23077	-0,09890	-0,04987	-0,02498	-0,00500	0,00000	0,00500	0,02498	0,04988	0,09910	0,23810	0,42857
k^B with I=c	-0,14286	-0,08696	-0,04712	-0,02433	-0,00497	0,00000	0,00502	0,02558	0,05213	0,10714	0,27273	0,50000
Table 4: Final relative deviation of results in tab. 3 from ideal linear solution, fig. 2												
k^A ; U (%)	-20,0	-9,8	-5,0	-2,5	-0,5	0,0	-0,5	-2,5	-5,0	-9,8	-23,1	-40,0
k^B ; U (%)	33,3	4,8	1,1	0,3	0,0	0,0	0,0	-0,2	-0,9	-3,2	-14,3	-33,3
k^A ; I (%)	7,7	1,1	0,3	0,1	0,0	0,0	0,0	-0,1	-0,2	-0,9	-4,8	-14,3
k^B ; I (%)	42,9	13,0	5,8	2,7	0,5	0,0	0,5	2,3	4,3	7,1	9,1	0,0

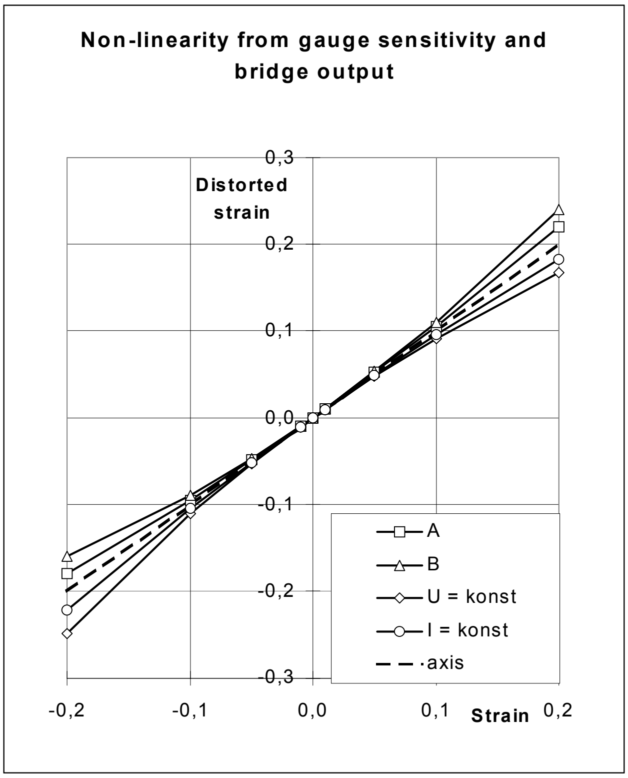


Fig. 1

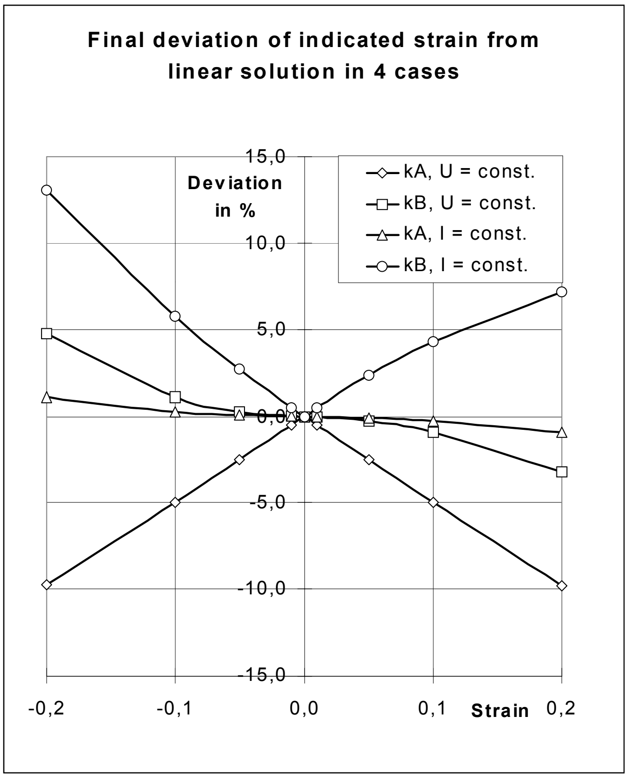
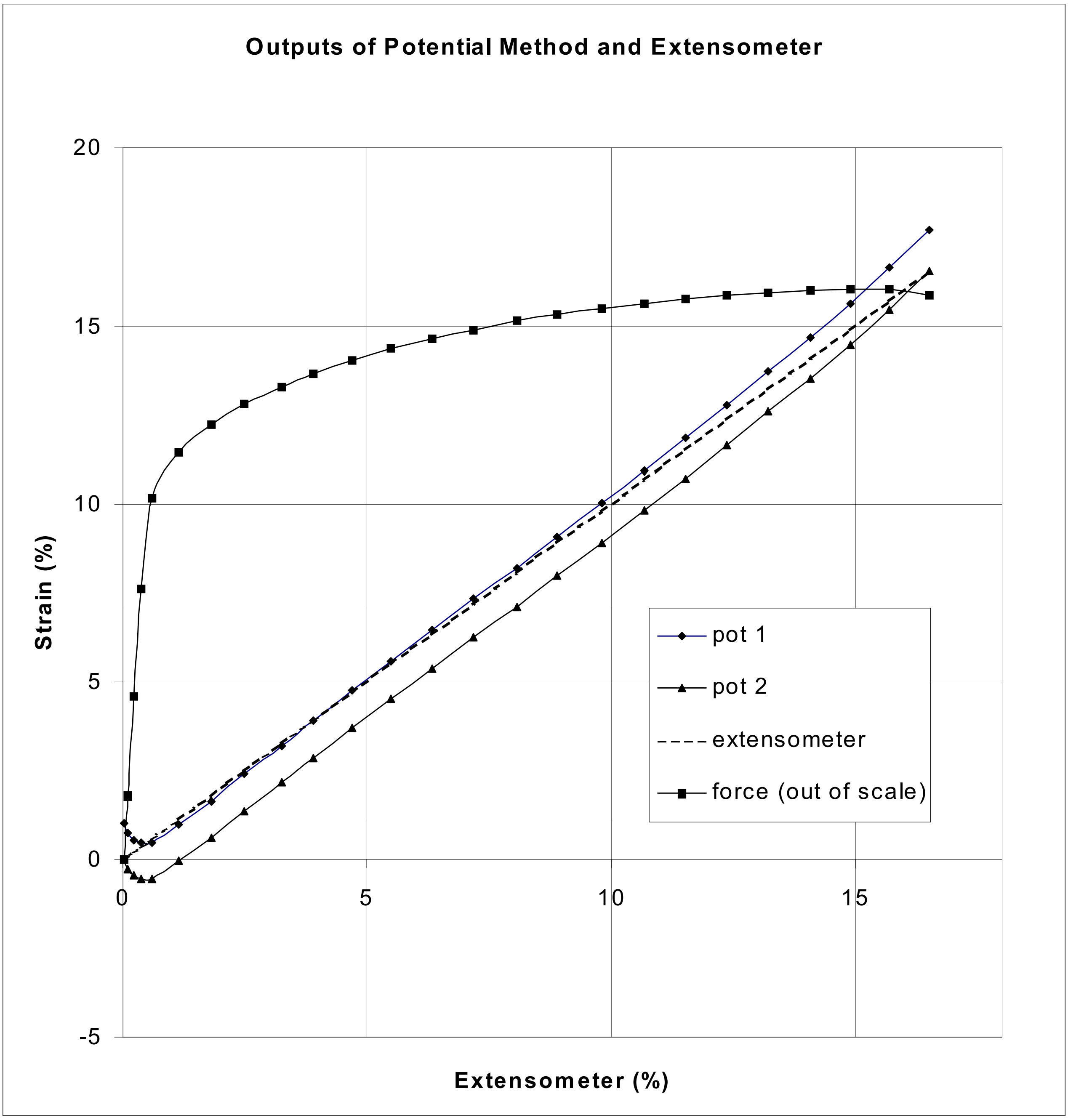


Fig. 2



Outputs of Potential Method and Extensometer
Strain (%)
Extensometer (%)
20
15
10
5
0
-5
0
5
10
15
pot 1
pot 2
extensometer
force (out of scale)

Fig. 3