40.EAN
40. MEZINÁRODNÍ KONFERENCE EXPERIMENTÁLNÍ ANALÝZY NAPĚTÍ
40th INTERNATIONAL CONFERENCE EXPERIMENTAL STRESS ANALYSIS
3. – 6. VI. 2002, PRAHA/PRAGUE, CZECH REPUBLIC

UNCERTAINTY OF MEASUREMENT RESULTS

Radovan Novotný – Josef Bradík[1]

Development of technical equipment and new physical discoveries provide new possibilities of measurement. The article deals with problems of expression of uncertainty in quantitative measurements as part of correct laboratory and technical practice. The article summarizes the requirements for expression of uncertainty and the basic reasons for its evaluations. There is defined the term uncertainty and described the general methodology for evaluation of uncertainty. This methodology is fully according to current international requirements for expression of uncertainty in measurement.

Keywords: **quality, metrology, measurement, result of measurement, uncertainty of measurement, coverage factor**

1 Overview

Current condition of production and competition involve the implementing of quality control. Development of technical equipment and new physical discoveries provide new possibilities of measurement and new possibilities in quality control. The requirements to quantity and quality of information grow up in all spheres of technical practice.

An uncertainty of measurement is the basic parameter that characterises quality of result of measurement. General, the basic requirements to expression of uncertainty in measurement are given by normative documents and standards. International standard ČSN EN ISO/IEC 17 025 “General requirements for the competence of testing and calibration laboratories” in paragraph 5.10.3 requires to test report includes “statement on the estimated uncertainty of measurement (where applicable)” and in paragraph 5.4.6 simultaneously say that calibration and testing laboratories shall have and shall apply procedures for estimating uncertainty of measurement.

In certain cases the nature of the test method may preclude rigorous, metrologically and statistically valid, calculation of uncertainty of measurement. In these cases the laboratory shall at least attempt to identify all the components of uncertainty and make a reasonable estimation, and ensure that the form of reporting of the result does not give a wrong impression of the uncertainty. Reasonable estimation shall be based on knowledge of the performance of the method and on the measurement scope.

Expression of uncertainty of measurement is directed also by requirements of European Accreditation of Laboratories (EAL). The basic documents are EAL-R2 “Methodology of expression of uncertainty in calibration laboratories” (for calibration laboratories) and EAL-G23 “Expression of uncertainty in quantitative testing” (for calibration and testing laboratories).

[1] *Ing. Radovan Novotný*
Brno University of Technology
Faculty of Electrical Engineering and Communication
Department of Microelectronics
Údolní 53, 602 00 Brno
Email: novotnyr@feec.vutbr.cz

Ing. Josef Bradík
Brno University of Technology
Faculty of Electrical Engineering and Communication
Department of Power Electrical and Electronic Engineering
Technická 8, 616 00 Brno
Email: bradik@feec.vutbr.cz

2 Uncertainty of measurement

The formal definition of the term "**uncertainty of measurement**" is as follows:

- Parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably attributed to the measurand.

The basic types of uncertainty:

Type A: Type A evaluation of uncertainty is based on method of evaluation uncertainty by the statistical analysis of series of observation.

Type B: Type B evaluation of uncertainty is based on method of evaluation of uncertainty by means other then the statistical analysis of series of observations.

Combined standard uncertainty is standard uncertainty of the result of measurement when the result is obtained from the values of a number of other quantities. It is the estimated standard deviation associated with the result and is equal to the positive square root of the combined variance obtained from all variance and covariance components. Combine standard uncertainty is denoted $u_c(y)$.

Expanded uncertainty is quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. Expanded uncertainty is denoted U(y).

Uncertainty of measurement is inseparable part of measurement results and its importance is obvious from figure 1 where only result signposted A is correct, because this estimate of measurand and its uncertainty are fully in required zone. All other results (signposted B, C, D) are not satisfied.

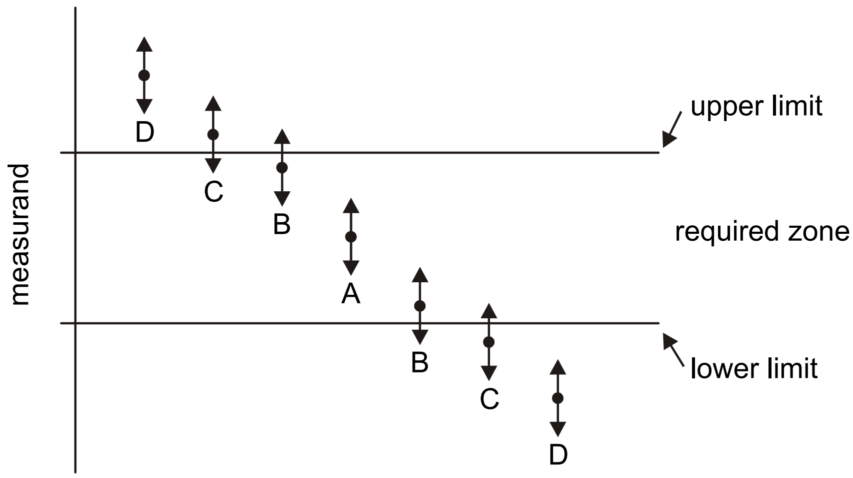

Figure 1: Importance of uncertainty of measurement

3 Methodology for evaluation of uncertainty of measurement

Overall uncertainty of measurement is expressed by specific mathematic methodology from single components of uncertainty. Evaluating of single sources of uncertainty in single partial steps is example of modular access to evaluation of uncertainty components. It is suitable to proceed from modular access in design of way for their evaluation and expression.

In particular measurement, it is necessary to proceed from actual methods of measurement for evaluation of uncertainties, because the single sources of uncertainty that participate on overall uncertainty result from this methodology. The mathematical methodology for evaluating and expressing of uncertainty is universal and in practice it must be particularised for current technical condition to conform to actual necessities of particular assignment. This methodology can be generally summarize to following steps:

Step 1: Analysis of input conditions

The first step of measurement predicates in choosing appropriate method of measurement and appropriate measurement devices. It should be done by analysis of input conditions and input requirements. It is seasonable to approximately estimate possible sources of uncertainty because it could be important factor for choosing of measurement method. Measurement can be modelled mathematically to the degree imposed by the required accuracy of the measurement.

Step 2: Mathematical model of measurement

Usually, a measurand Y is not measured directly, but is determined from N other quantities X_i through a functional relationship f by following equation:

$$Y = f(X_1, X_2, ..., X_N)$$

The functional relationship f is given by chosen measurement method and expressed the way by which the measurand is obtained from input quantities X_i. The mathematical model of the measurement that transform the set of repeated observations into the measurement result is of critical importance because, in addition to the observation, it generally includes various influence quantities that are inexactly known. This lack of knowledge contributes to the uncertainty of the measurement result, as do the variations of the repeated observations and any uncertainty associated with the mathematical model itself.

Because the mathematical model may be incomplete, all relevant quantities should be varied to the fullest practicable extent so that the evaluation of uncertainty can be based as much as possible on observed data. The mathematical model should always be revised when observed data, including the result of independent determination of the same measurand, demonstrate that the model is incomplete. A well-designed experiment can greatly facilitate reliable evaluations of uncertainty and is an important part of the art of measurement.

Step 3: Identification of all sources of uncertainty

When estimating the uncertainty of measurement, all uncertainty components, which are importance in the given situation, shall be taken into account using appropriate methods of analysis. In practice, there are many possible sources of uncertainty in a measurement, for example following:

- incomplete definition of the measurand
- imperfect realization of the definition of the measurand
- nonrepresentative sampling
- inadequate knowledge of the effects of environmental conditions on the measurand
- imperfect measurement of environmental conditions
- personal bias in reading
- finite instrument resolution
- inexact values of measurement standards or reference materials

The possible sources of uncertainty result from mathematical model of measurement. It is necessary to identify all significant sources of uncertainty and it is also necessary not to "double-count" uncertainty components. Practically, the sources are not necessarily independent, and so some of sources may contribute to other sources. Good way for identification of sources of uncertainty and finding possible correlations is using Ishikawa charts.

As example, there is shown Ishikawa chart of sources of uncertainty in electromagnetic non-destructive testing in figure 2. It may be seen that any quantities can be components of uncertainty in more sources. Of course, in practise these possible sources would be reduced for a few really important sources by using method and other uncertainty components could be neglected.

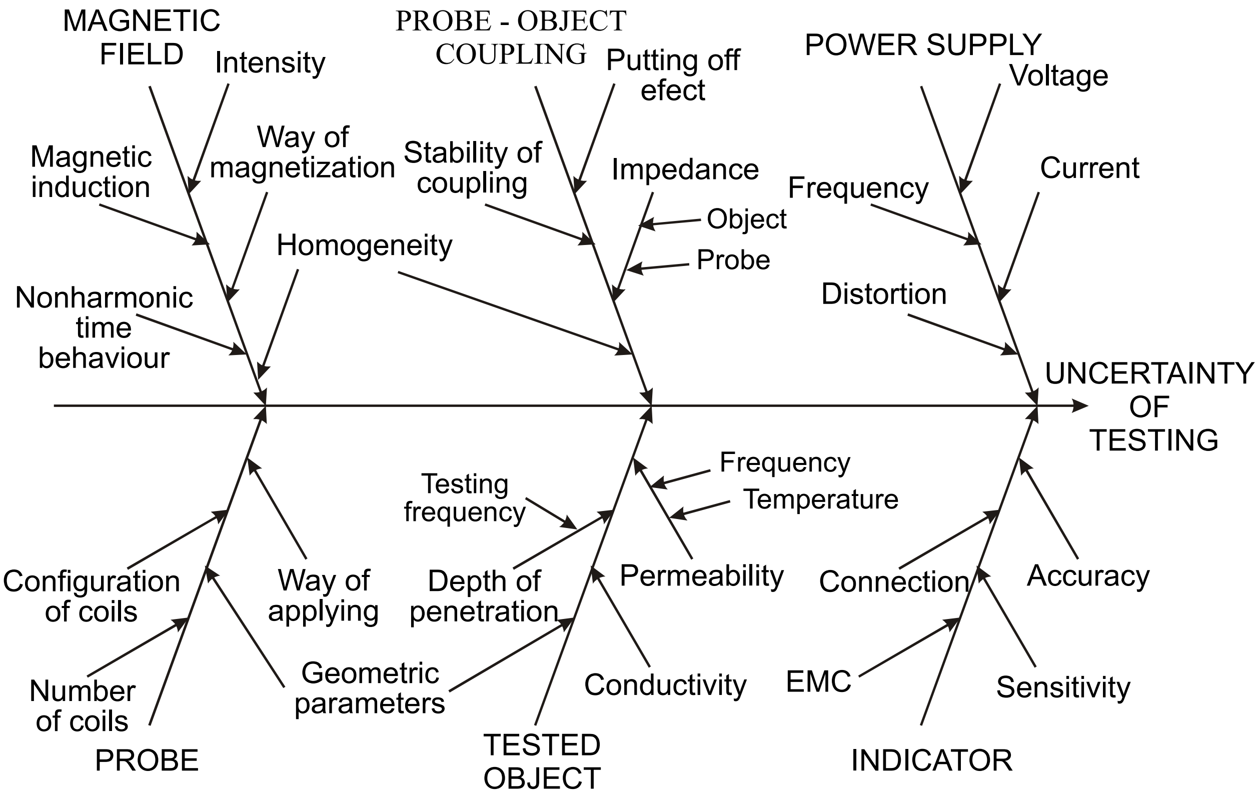

Figure 2: Ishikawa chart of sources of uncertainty in electromagnetic non-destructive testing

The uncertainties from all identified important sources of uncertainty are grouped into two categories (Type A and Type B) according to the way of estimation as is defined in chapter 2.

Step 4: Evaluation of input standard uncertainties

As it was written, input uncertainties are grouped into two categories (Type A and Type B evaluation). Both types of evaluation are based on probability distribution and the uncertainty components resulting from either type are quantified by variances or standard deviations. While Type A standard uncertainty is obtained from a probability density function derived from an observed frequency distribution, Type B standard uncertainty is obtained from an assumed probability density function based on the degree of belief.

When an input quantity X_i is estimated from n independent repeated observations (Type A), standard uncertainty $u(\bar{x})$ associated with arithmetic mean $\bar{x}$ is given by experimental standard deviation of the mean, thus:

$$u(\bar{x}) = \sqrt{\frac{1}{n(n-1)} \cdot \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}$$

When the input quantity X_i is not estimated from repeated observations, the associated estimated variance or the standard deviation is evaluated by scientific judgement based on all of the available information on the possible variability of X_i. In these cases, the standard uncertainty $u(x_i)$ is given by square root of variance of appropriated probability distribution to input quantity X_i. It often may be possible to estimate only bound (upper and lower limits) for X_i without knowledge about probability distribution. Then it may be assumed trapezoidal probability distribution with half-width a and top of width $2a\beta$ or its extreme cases – rectangular or triangular probability distribution. Then the standard uncertainty is given by:

$$u(x_i) = \sqrt{\frac{a^2(1+\beta^2)}{6}}$$

For β = 1 the trapezoidal probability distribution passes to rectangular probability distribution, for β = 0 the trapezoidal probability distribution passes to triangular probability distribution.

In practice, the input standard uncertainties must be evaluated for all significant sources of uncertainty by one of this way of evaluation. Then it is possible to express overall uncertainty of result of measurement.

Step 5: Expressing of overall uncertainty

The final step in expressing of uncertainty is given by combining the standard uncertainties to one parameter that characterizes overall uncertainty of result of measurement. This parameter is known as combined standard uncertainty and is denoted $u_c(y)$. The combined standard uncertainty is given by:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\cdot\frac{\partial f}{\partial x_j}\cdot u(x_i)\cdot u(x_j)\cdot r(x_i,x_j)}$$

where $r(x_i, x_j)$ is correlation coefficient that characterizes the degree of correlation between x_i and x_j.

When input quantities are uncorrelated, equation is reduced to:

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)}$$

These equations are termed as "law of propagation of uncertainty". The partial derivatives, often call sensitivity coefficients, describe how the output estimate y varies with changes in the values of the input estimates x_i.. By expressing of sensitive coefficients and single members of law of propagation of uncertainty it is possible to analyse single uncertainty components, find the dominant uncertainty component and (when it is required or useful) try to eliminate or reduced it. It is helpful to use any statistical methods, for example Pareto analysis as is shown on figure 3. There is performed Pareto analysis of uncertainty components in practical non-destructive testing of material properties realised by measurement of higher harmonics component of voltage induced in coil to that the controlled material is inserted.

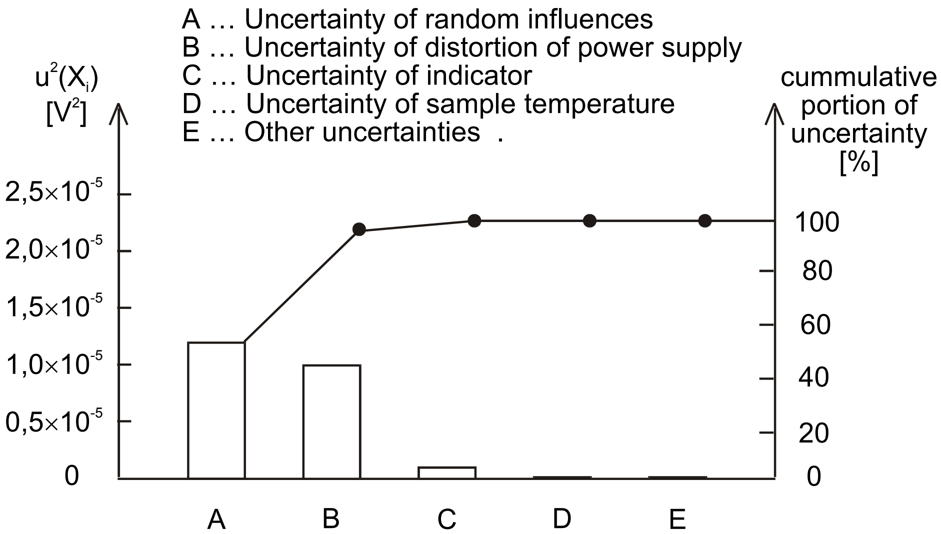

Figure 3: Pareto analysis of uncertainty components

The importance of expressing of uncertainties in measurement was unambiguously supported by experiments namely not only in a relation to results of measurements, but also in a relation to choose of measurement methodology. Quantification of components of entry uncertainty enables to delimit exactly the percent part of particular share in total uncertainty of measurements. On the basis of this knowledge it is possible to option the most acceptable method and to optimise the methodology of testing.

Although the combined standard uncertainty can be used to express the uncertainty of a measurement result, in some cases it is often necessary to give a measure of uncertainty that defines an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. This additional measure of uncertainty is termed expanded uncertainty. The expanded uncertainty, denoted by symbol U(y), is obtained by multiplying $u_c(y)$ by a coverage factor, denoted by symbol k:

$$U(y) = k \cdot u_c(y)$$

The result of a measurement is then conveniently expressed as Y = y ± U(y), which is interpreted to mean that the best estimate of the value attributable to the measurand Y is y, and that y - Y to y + U is an interval that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to Y.

The value of the coverage factor is chosen on the basis of the required level of confidence to be associated with the interval defined by U(y). Typically, k is in the range 2 to 3. When the normal distribution applies and $u_c(y)$ has negligible uncertainty, $U(y) = 2 \cdot u_c(y)$ defines an interval having a level of confidence of approximately 95 percent, and $U(y) = 3 \cdot u_c(y)$ defines an interval having a level of confidence greater than 99 percent.

4 Conclusion

When reporting the result of measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurements results cannot be compared, either among themselves or with reference values given in a specification or standard. Such parameter of quality of result of measurement is uncertainty of measurement.

The article describes the basic mathematically methodology for expressing of uncertainty of measurement in simple steps. This methodology is in the whole harmony with universal rules and procedures, which are required by legislature and prescriptive documents. The principle of solving these problems can be based on the modular model, which enables to combine single sources of uncertainties according to elective method of testing or measurement and ensures the more extensive usability of evaluated methodology. Generalization of experience from particular branches will bring the complement of general theory, which the laboratories consider as their know-how in the own cases in this time.

5 References

[1] Guide to the Expression of Uncertainty in Measurement. International Organization for Standardization 1995

[2] ČSN EN ISO/IEC 17 025: General requirements for the competence of testing and calibration laboratories. ČSNI, 2001

[3] Guide EAL: EAL-R2: Methodology of expression of uncertainty in calibration laboratories. EAL (ČIA/ČSNI), 1996

[4] Guide EAL: EAL-G23: Expression of uncertainty in quantitative testing. EAL (ČIA/ČSNI), 1996

Acknowledgement

This research has been supported by the Czech Ministry of Education in the frame of Research Plan MSM 262200022 *MIKROSYT Microelectronic Systems and Technologies*.