

# NUMERICAL SIMULATION OF DUCTILE FRACTURE BY MEANS OF COMPLETE GURSON MODEL AND COMPARISON WITH EXPERIMENT NUMERICKÁ SIMULACE TVÁRNÉHO LOMU POMOCÍ ÚPLNÉHO GURSONOVA MODELU A SROVNÁNÍ S EXPERIMENTEM

Vladislav Laš<sup>1</sup>, Luboš Řehounek<sup>2</sup>, Petr Jaroš<sup>3</sup>

This paper is focused on numerical simulation of ductile fracture by utilizing the complete Gurson model. The comparison of numerical and experimental results was carried out on a pre-cracked CT sample made of a highly ductile steel 15CH2MFA used for the production of reactor vessels. As the main macro-criterion for agreement with experiment served tensile force – crack opening diagram. As the micro-indicators of the capability of the complete Gurson model the initiation of the crack growth, as well as crack shape were chosen. Numerical simulations by means of the complete Gurson model were conducted in software ABAQUS and in case of the modified Gurson model in software MARC.

Příspěvek je věnován numerické simulaci tvárného lomu s využitím úplného Gursonova modelu. Jako příklad pro porovnání numerické simulace a experimentu byl použit CT vzorek s nakmitanou trhlinou vyrobený z oceli 15CH2MFA, z které byla vyrobena nádoba reaktoru. Jedná se o vysoce tvárnou ocel. Při porovnání výpočtu a experimentu byla sledována závislost síla a rozevření vzorku (COD), dále počátek šíření a tvar trhliny. Numerická simulace byla provedena pomocí úplného Gursonova modelu byla provedena v programu ABAQUS, v případě modifikovaného Gursonova modelu v programu MARC.

Keywords Ductile fracture, complete Gurson model, CT specimen, numerical simulation, experiment

Klíčová slova Tvárný lom, úplný Gursonův model, CT vzorek, numerická simulace, experiment

## Introduction

This work extends the scope of the work [4], where the modified Gurson (GTN) model was exploited for the numerical simulation of ductile fracture. Several disadvantages of this model are eliminated in the complete Gurson (CGM) model. Mainly, the CGM model introduced better implementation of the process of void coalescence, which is in GTN model described only by a material constant – the critical void volume fraction. It was shown in numerous observations that this coefficient is strongly influenced by the stress triaxiality (defined as the ratio of the mean normal stress and the conventional von Mises equivalent stress) which depends on the shape of the specimen. In other words, the process of void coalescence can be initiated for same material at different values of the critical void volume fraction. Numerical simulations using the CGM model were carried out in software ABAQUS. A free copy of the user subrouotine implementing the CGM model was obtained from the author of [8]. The goal of this paper was to present the result obtained by using GTN and CGM models compared to the experimental data.

<sup>&</sup>lt;sup>1</sup> Doc. Ing. Vladislav Laš, CSc.: Západočeská univerzita v Plzni, Fakulta aplikovaných věd, Katedra mechaniky, Univerzitní 22, 306 14 Plzeň, tel.: +420 377 632 326, e-mail: las@kme.zcu.cz

<sup>&</sup>lt;sup>2</sup> Ing. Luboš Řehounek: Západočeská univerzita v Plzni, Fakulta aplikovaných věd, Katedra mechaniky, Univerzitní 22, 306 14 Plzeň, e-mail: rehoun3@kme.zcu.cz

<sup>&</sup>lt;sup>3</sup> Ing. Petr Jaroš: Techlab s.r.o., Sokolovská 207, 190 00 Praha 9, tel.: +420 283 890 583, e-mail: techlab@czn.cz

#### **Complete Gurson model**

Material models combining plasticity and damage capable of managing ductile fracture have been developing for several decades. The Gurson model (GM) [2], which did not assume the phase of void coalescence, was generalized and improved. Tvergaard a Needleman [5] introduced the capability of the

model to describe the phase of void coalescence by means of the modified void volume fraction  $f^*$ .

By idealizing the true void distribution with spherical voids surrounded by matrix material the following yield function can be obtained

$$\phi = \frac{\sigma_e^2}{\sigma_M^2} + 2f^* \cos h \left(\frac{q_2}{2} \frac{\sigma_k^k}{\sigma_M}\right) - 1 + (q_1 f^*)^2 = 0, \qquad (1)$$

where  $\sigma_e$  is the conventional von Mises equivalent stress,  $\sigma_M$  is the flow stress of the matrix material,  $\sigma_k^k$  is the mean normal stress and  $f^*$  is the modified void volume fraction defined in eq. (5). The constants  $q_1$  and  $q_2$  were determined by numerical studies conducted by Tvergaard.

The process of increasing the void volume fraction in the material is described by two main effects – nucleation of new voids and the growth of existing voids. Mathematically this can be expressed in terms of the corresponding void volume fraction rates

$$\dot{f} = \dot{f}_{nucl} + \dot{f}_{grw}$$
 with initial condition  $f(t_0) = f_0$ , (2)

where  $\dot{f}_{nucl}$  is the rate of void nucleation and  $\dot{f}_{grw}$  is the growth rate of existing voids.

The strain controlled nucleation law was proposed as

$$\dot{f}_{nucl} = \frac{f_N}{s\sqrt{\pi}} \exp\left[-\frac{1}{2} \left(\frac{\varepsilon_M^p - \varepsilon_N}{s}\right)^2\right] \dot{\varepsilon}_M^p, \qquad (3)$$

where  $f_N$  is the void volume fraction of particles available for void nucleation,  $\varepsilon_N$  is the mean nucleation burst strain,  $\varepsilon_M^p$  is the equivalent plastic deformation of the matrix material and *s* is the corresponding standard deviation.

The growth of the existing voids under plastic straining can be written as

$$\hat{f}_{grw} = (1 - f) \mathcal{E}^p : \boldsymbol{I},$$
(4)

where  $\dot{\varepsilon}^{p}$  is the plastic strain tensor and **I** is the second-order unit tensor.

The process of void coalescence corresponding to sudden drop of material load carrying capacity is described by introduction of the modified void volume fraction  $f^*$  which expresses an artificially accelerated void growth

$$f^{*} = f \quad \text{for} \quad f \leq f_{C},$$

$$f^{*} = f_{C} + \frac{f_{u}^{*} - f_{C}}{f_{F} - f_{C}} (f - f_{C}) \quad \text{for} \quad f > f_{C},$$
(5)

where  $f_C$  is the critical void volume fraction,  $f_F$  is the void volume fraction at final failure and the constant  $f_u^*$  is defined as  $f_u^* = \frac{1}{a_i}$ .

To sum up, for the description of the modified Gurson model the parameters  $q_1$ ,  $q_2$ ,  $f_0$ ,  $f_C$ ,  $f_F$ ,  $\varepsilon_N$ , s,  $f_N$  are needed. The recommended values of  $q_1$ ,  $q_2$  are  $q_1 = 1.5$ ,  $q_2 = 1$ , as suggested by Tvergaard after conducting a series of numerical studies. The remaining parameters must be determined by fitting numerical simulations onto experiments.

However, it is questionable to assume  $f_C$  is a material constant. Numerical studies clearly demonstrated strong dependence of  $f_C$  on  $f_0$ , as well as on the shape of the analyzed part. Hence,  $f_C$  is not a material constant, but becomes a field quality.

So far, only homogeneous modes have been considered in deriving the Gurson model. As argued by Thomason in [6], the localized deformation mode of void coalescence should be treated differently. He suggested that the localized deformation mode can be described by the so-called plastic limit load model (PLL). The stage of void coalescence depends on the competition between the two deformation modes. In the early stage of deformation, the voids are small and it is easier for the material to follow the homogeneous deformation node. With the advance of the plastic deformation and the increase of the void volume fraction, the stress required for localized deformation decreases. When the stress for localized deformation is equal to the stress for homogeneous deformation, the void coalescence will occur. The condition for void coalescence by internal necking of the intervoid matrix can be written as

$$\boldsymbol{\sigma}_{1}^{H} = \boldsymbol{\sigma}_{1}^{L}, \tag{6}$$

where  $\sigma_1^H$  is the homogeneous maximum principal stress and  $\sigma_1^L$  is the maximum principal stress of the localized area which represents the micro-capacity of a voided material to resist the localized deformation. The Thomason PPL criterion for void coalescence in 3D cases [8] can be written as

$$\frac{\sigma_{1}^{L}}{\sigma_{M}} < \left[ \alpha \left( \frac{1}{r} - 1 \right)^{2} + \frac{\beta}{\sqrt{r}} \right] \left( 1 - \pi r^{2} \right) \quad \text{no coalescence and} \quad f^{*} = f$$

$$\frac{\sigma_{1}^{L}}{\sigma_{M}} = \left[ \alpha \left( \frac{1}{r} - 1 \right)^{2} + \frac{\beta}{\sqrt{r}} \right] \left( 1 - \pi r^{2} \right) \quad \text{coalescence starts and set} \quad f_{C} = f ,$$
(7)

where  $\sigma_M$  is yield stress of the matrix material, r is the void space ratio defined as  $r = \sqrt[3]{\frac{3f}{4\pi}} e^{\varepsilon_1 + \varepsilon_2 + \varepsilon_3} / \left(\sqrt{e^{\varepsilon_2 + \varepsilon_3}/2}\right),$ 

where  $\varepsilon_{1,2,3}$  are principal strains and  $\alpha = 0.1$ ,  $\beta = 1.2$  are constants fitted by Thomason.

It is important to note that plastic load limit is strongly dependent on the sample geometry. For a material without voids, the plastic load is infinite. At the beginning of plastic deformation of a voidcontaining material, the void dimensions are very small and the corresponding plastic limit load is very large. When a void starts to grow, the plastic limit load decreases, which indicates that the possibility of plastic localization increases. The homogeneous deformation mode will be terminated once the localized mode of deformation becomes possible. It was reported after systematic verifications that the complete Gurson model is very accurate, in particular, for small initial void volume fractions.

## Determination of material parameters for GTN and CGM models

In the previous section the parameters of GTN model, i.e.  $q_1, q_2, f_0, f_c, f_F, \varepsilon_N, s, f_N$ , were introduced. In case of CGM model, the process of void coalescence is controlled by Thomason plastic limit load model. Hence, the knowledge of the critical void volume fraction  $f_c$  is not required as it arises from equations (7). Also, the value of  $f_F$  is not required either. For most materials, the value of the void volume fraction at final fracture  $f_F$  should not be less than 0.15. Based on the observations, the following approximate equation can be introduced,

$$f_F = 0.15 + 2f_0.$$
(8)

As the parameters  $q_1$  and  $q_2$  are often set to  $q_1 = 1,5$  and  $q_2 = 1$ , there are only four parameters  $f_0, \mathcal{E}_N, \mathcal{S}, f_N$  left to determine. One of the possibilities is to fit the results of numerical simulation of the fracture in round notched bars onto experimental data. The advantage of this procedure is that the numerical simulation can be carried out on an axisymmetric FE model. So, the whole fitting process is far less time-consuming than in case of full 3D FE models.

In this work the round notched bars with dimension from Fig. 1 were used for fitting material parameters. The material was steel 15CH2MFA, which is widely used for the production of reactor vessels. The comparison of the experimental and numerical  $F - \Delta d$  curves is shown in Fig. 2. This satisfactory agreement was achieved with the parameters presented in Table1.



Fig. 1: Dimensions of the notched bars

Notched bars	$q_1$	$q_2$	$f_0$	$f_{c}$	$f_F$	${\cal E}_N$	S	$f_N$
shallow, sharp	1,5	1,0	0,0001	0,11	0,25	0,1	0,1	0,01

Table 1: Parameters of the GTN model



Fig. 2: Tensile force – diameter change  $(F - \Delta d)$  curves

## Numerical simulation of ductile fracture of CT sample

To simulate ductile fracture of a CT sample the complete Gurson model was employed. The dimensions of the pre-cracked specimen with fatigue crack of the length a = 18 mm are shown in Fig. 3. The material of the specimen was high-strength steel 15CH2MFA with yield strength  $\sigma_K = 660 \text{ MPa}$ . This steel exhibits extensive ductility. The finite element model of the CT sample is depicted in Fig. 4. Because of the presence of two planes of symmetry, only a quarter of the sample was numerically solved (see Fig. 5). Material parameters were entered from Table 1.



The shape of the propagated crack in one quarter of the CT sample is shown in Fig. 5. It can be seen that the crack propagated rather deep into the sample but has not reached the boundary of the specimen.



Fig. 5: Visualization of the crack shape at COD = 3.5 mm



a) beginning of the crack propagation ( $f_c = 0.02$ ) b) propagated crack at COD=3.5mm ( $f_c = 0.08$ ) Fig. 6: Comparison of the value of  $f_c$  during crack propagation

From the conducted simulations it is evident that the value of the coefficient  $f_c$  changes not only with the shape of the analyzed part but also during the process of crack propagation. It reflects the actual state of triaxiality at the crack tip, as can be seen in Fig. 6. Fig. 6a shows the beginning of the crack propagation when  $f_c = 0.02$ . As the crack grows in the longitudinal direction, the value of  $f_c$  increases, too (see Fig. 6b). This effect can be explained by weakening of the "constraint effect", by which void growth is enabled. In Fig. 7 can be seen the tensile force – crack opening (F-COD) curves obtained experimentally and numerically by using both CGM and GTM models. It can be noticed that in case of using CGM the numerical results correspond with the experiment very well. It is believed that this correspondence is enabled by the capability of CGM model to determine  $f_c$  automatically by utilizing the Thomason plastic limit load model.

To compare the capability of both models,  $f_c$  was first determined by fitting the material constants on experimentally measured F- $\Delta d$  curves for notched bars. Regardless the geometry of the notched samples, the best results were obtained for the value of  $f_c$  equal to 0.11. This value was then used for the numerical simulation of crack opening of CT sample (see Fig. 7). However, better agreement with the experiment was reached for the value of  $f_c$  set to 0.06. This finding can be explained by analyzing the results of CGM model. Here, the value of  $f_c$  changed between 0.02 and 0.08 (see Fig. 6).



Fig.7: Tensile force - crack opening (F-COD) curves

## Conclusion

The aim of this work was to carry out a numerical analysis of ductile fracture which would show a good correspondence with reality. With the aid of CGM it turned out that the value of the coefficient  $f_c$  changes not only with the shape of the analyzed part but also during the process of crack propagation. This fact influences the shape of the fracture area, the stress state and distortion during the crack propagation. From the above-mentioned it is evident that to obtain realistic results of numerical simulation of ductile fracture it is necessary to use the CGM model.

The experience with the numerical simulation of crack propagation for ductile fracture shell be applied for numerical determination of J-R curves. This means that instead of complicated and expensive series of experiments it should be sufficient to conduct only few experiments (to obtain the stress-strain characteristic of the material and several F -  $\Delta d$  diagrams) which are necessary for the determination of the parameters of the CGM model. The J-R curves could be determined numerically afterwards.

#### Acknowledgement

*This paper was elaborated in support of the Grant GA ČR 101/03/0731 and within the scope of the research programme MSM 235200003.* 

## References

- Gubeljak, N., Zerbst, U., Kocak, M.: Prediction of the maximum load of pre-cracked welded bars using charpy data according to the default level of the European SINTAP procedure. International Journal of Pressure Vessels and Piping 79, pp. 89-98, 2002.
- [2] Gurson, A. L.: *Continuum Theory of Ductile Rupture by Void Nuclear and Growth.*J. Eng. Mat. Tech., Paper No. 76-Mat-CC, 1977.
- [3] Laš, V. Řehounek, L.: Aspects of modelling ductile fracture. Proceedings of University of West Bohenia Pilsen, vol. 5/2001, pp. 115-126, 2001.
- [4] Laš, V. Vacek, V. Řehounek, L.: Void model numerical simulation and comparison with experiment. 41<sup>st</sup> International Conference Experimental stress Analysis 2003, Milovy 2003.
- [5] Needleman, A. Tvergaard, V.: An Analysis of Ductile Repture in Notched Bars. J. Mech. Phys. Solids, Vol. 32, No. 6, Pergamon Press, 1984.
- [6] Thomason, P. F.: Ductile fracture of metals. Pergamon Press: Oxford 1990.
- [7] Thomason, P. F.: A three dimensional model for ductile fracture by the drowth and coalescence of *micro-void*. Acta Metallurgica, pp. 1087-1095, 1985.
- [8] Zhang, Z. L.: *A complete Gurson model, Nonlinear fracture and damage mechanics*. Edited by M.H. Aliabadi, WIT Press Sonthampton, Boston, pp. 223-248, 2001.