KMITANIE KONŠTRUKCIÍ S IMPERFEKCIAMI

NATURAL VIBRATION OF IMPERFECT STRUCTURES

Ján RAVINGER, Martin PSOTNÝ[1]

Abstrakt

Pre postavenie problému kmitania konštrukcií s imperfekciami je nutné akceptovať geometricky nelineárnu teóriu. Vyhodnotením prírastkovej tuhostnej matice môžeme akceptovať zvyškové (zvarové) napätia ako i začiatočné deformácie. Stĺpy, rámy i tenkostenné konštrukcie sú citlivé na začiatočné nedokonalosti (imperfekcie). Teória ako i získané výsledky sú základom pre rozpracovanie nedeštruktívnej metódy pre určovanie napätosti konštrukcií

Kľúčové slová: Nedeštruktívne metódy, stabilita, vzpernosť, dynamická stabilita, kmitanie, geometricky nelineárna teória, začiatočné nedokonalosti (imperfekcie), tenkostenné konštrukcie.

Abstract

Using the geometric non-linear theory (The Total Lagrange Description) in dynamics we can establish the problem of the natural vibration of the structure including the effects of the structural and geometrical imperfections. The incremental stiffness matrix can take into account the residual stresses (structural imperfections) and the geometrical initial displacements (geometrical imperfections) as well. The behaviour of columns, frames and thin-walled structures is sensitive to imperfections. This theory and results can be used as a base for the non-destructive method for the evaluation of the level of the load and the imperfections**.**

Keywords: Non-destructive methods, stability, buckling, dynamic stability, vibration, geometric non-linear theory, initial imperfections, thin-walled structures.

INTRODUCTION

The problem of the combination of the linear stability and the vibration of structures was solved a long time ago. By the linear stability we mean the assumption of an ideal structure. Differences, between theoretical results and the reality, forced researchers to search for more accurate models. The slender web as a main part of the thin-walled structure has significant post-buckling reserves and for a description of them it is necessary to accept a geometric non-linear theory. Burgeen (1951) formulated the problem of vibration of an imperfect column. The problem of the vibration of the slender web as a non-linear system was formulated by Bolotin (1956). The vibration of the rectangular slender web taking into account the geometrical initial imperfections have been investigated by many researchers. (Wedel-Heinen, J. 1991, Hui, D. 1984, Ilanko, S. and Dickinson, S. M. 1991, Yamaki, V. at al. 1983). Ravinger (1994) presented the general theory for the dynamic post-buckling behaviour of a thin-walled panel (a slender web with flanges) taking into account the geometrical initial imperfections and the residual stresses as well. This paper presents a short summary of this theory. The presented results show the peculiarities for the differences of the support conditions for static behaviour and for the vibration of the columns, frames and a slender web as a main constructional element of a thin-walled structure.

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THEORY

Von Kármán theory has been used for the description of the post-buckling behaviour of a thin-walled panel with geometrical imperfections and residual stresses. By including inertia forces the problem is extended into dynamics. A direct formulation of Hamilton's principle for the dynamic post-buckling behaviour of a slender web leads to a system of conditional equation describing this non-linear dynamic process. In the case of a static problem, linearized assumptions are accepted for the evaluation of the elastic critical load. Analogously, in the case of a dynamic problem, Hamilton's principle is arranged in incremental form for the evaluation of the free vibration.

A system of conditional equations in the incremental formulation of the post-buckling behaviour of the slender web or a thin-walled structure can be written as

$$\boldsymbol{K}_{\boldsymbol{MP}}\Delta\ddot{\alpha}_P + \boldsymbol{K}_{\boldsymbol{INCP}}\Delta\alpha_D + \boldsymbol{K}^T_{\boldsymbol{INCWP}}\Delta\alpha_W + \boldsymbol{F}_{\boldsymbol{INTP}} - \boldsymbol{F}_{\boldsymbol{EXTP}} - \Delta\boldsymbol{F}_{\boldsymbol{EXTP}} = \boldsymbol{0} \qquad (1a)$$

$$\boldsymbol{K}_{\boldsymbol{INCW}}\Delta\alpha_W + \boldsymbol{K}_{\boldsymbol{INCWP}}\Delta\alpha_P + \boldsymbol{F}_{\boldsymbol{INTW}} - \boldsymbol{F}_{\boldsymbol{EXTW}} - \Delta\boldsymbol{F}_{\boldsymbol{EXTW}} = \boldsymbol{0} \qquad (1b)$$

where indexes mean $_M$ - mass, $_{INC}$ - incremental, $_{INT}$ – internal, $_{EXT}$ – external, $_P$ – plate, $_W$ - web. The incremental stiffness matrix of the web $\boldsymbol{K}_{\boldsymbol{INCW}}$ is linear and, consequently, we can proceed to the elimination of Eqn.(1b) and thus obtain the system of equations

$$\boldsymbol{K}_{\boldsymbol{MP}}\Delta\ddot{\alpha}_P + (\boldsymbol{K}_{\boldsymbol{INCP}} - \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCW}}\boldsymbol{K}_{\boldsymbol{INCWP}})\Delta\alpha_P + \boldsymbol{F}_{\boldsymbol{INTP}} - \boldsymbol{F}_{\boldsymbol{EXTP}} -$$
$$\boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCW}}\boldsymbol{F}_{\boldsymbol{INTW}} + \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCWP}}\boldsymbol{F}_{\boldsymbol{EXTW}} - \Delta\boldsymbol{F}_{\boldsymbol{EXTP}} + \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCW}}\Delta\boldsymbol{F}_{\boldsymbol{EXTW}} = \boldsymbol{0} \qquad (2)$$

Taking out the inertia forces from Eqn.(2) we have conditional equations describing the static post-buckling behaviour of a slender web or large displacements of plates. The Newton-Raphson iteration can be used for the solution of these equations. From a number of rules whose are valid in numerical models of static non-linear problems we use one which can help us in establishing the free vibration problem.

$$\boldsymbol{K}_{\boldsymbol{INC}} \equiv \boldsymbol{J} \quad \Rightarrow \quad \boldsymbol{K}_{\boldsymbol{INC}} = \boldsymbol{K}_{\boldsymbol{INCP}} - \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCw}}\boldsymbol{K}_{\boldsymbol{INCWP}} \qquad (3)$$

This means that the incremental stiffness matrix is equal to the Jacobian of the Newton-Raphson iteration of the system of non-linear algebraic equations.

The problem of the free vibration including the effects of initial imperfections (initial displacement and residual stresses) can be obtained in the following way. We suppose a system Eqn.(2) in equilibrium; then:

$$\boldsymbol{F}_{\boldsymbol{INTP}} - \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCWP}}\boldsymbol{F}_{\boldsymbol{EXTW}} - \Delta\boldsymbol{F}_{\boldsymbol{EXTP}} + \boldsymbol{K}^T_{\boldsymbol{INCWP}}\boldsymbol{K}^{-1}_{\boldsymbol{INCW}}\Delta\boldsymbol{F}_{\boldsymbol{EXTW}} = \boldsymbol{0} \qquad (4)$$

We suppose a zero increments in the external load:

$$\Delta\boldsymbol{F}_{\boldsymbol{EXTP}} = \Delta\boldsymbol{F}_{\boldsymbol{EXTW}} = \boldsymbol{0} \qquad (5)$$

The increments of the plate displacements are assumed as:

$$\Delta\alpha_P = \Delta\overline{\alpha}_P\, sin(\omega t)\,, \qquad (6)$$

where ω is the circular frequency of the free vibration.

Inserting this into Eqn.(2) we have a problem of eigenvalues and eigenvectors:

$$\left|\boldsymbol{K}_{\boldsymbol{INC}} - \omega^2\boldsymbol{K}_{\boldsymbol{MP}}\right|_{det} = 0 \qquad (7)$$

Eigenvalues are the circular frequencies and eigenvectors represent the modes of the vibrations. The incremental stiffness matrix includes the level of the load and the initial imperfections as well.

EXAMPLES

The vibration of simple supported column

Simple supported column loaded in compression is the simplest example for the explanation of the presented theory. In this case it is very important how we suppose the edge conditions in the point of the action of the external load. In the case of moving support we have obtained a trivial result. To take in consideration the effects of initial displacements, the support must be fixed during the vibration process. (Fig. 1).

Vibration of the column with the second mode of the initial displacement

Fig. 2 shows the solution of the vibration of the column where the initial displacement is the combination of the first and the second mode of the buckling. The second mode is dominated. Even the mode of initial displacement is mode 2 (the second mode), the mode of the vibration is the mode 1 (the first mode). We can say, that the initial displacements cause the deviation of the frequency to compare to frequency of the ideal column (column without initial displacements - imperfections) or to the column with moveable support in axis direction. In the case of the second mode of the initial displacement this deviation of the frequency is much smaller.

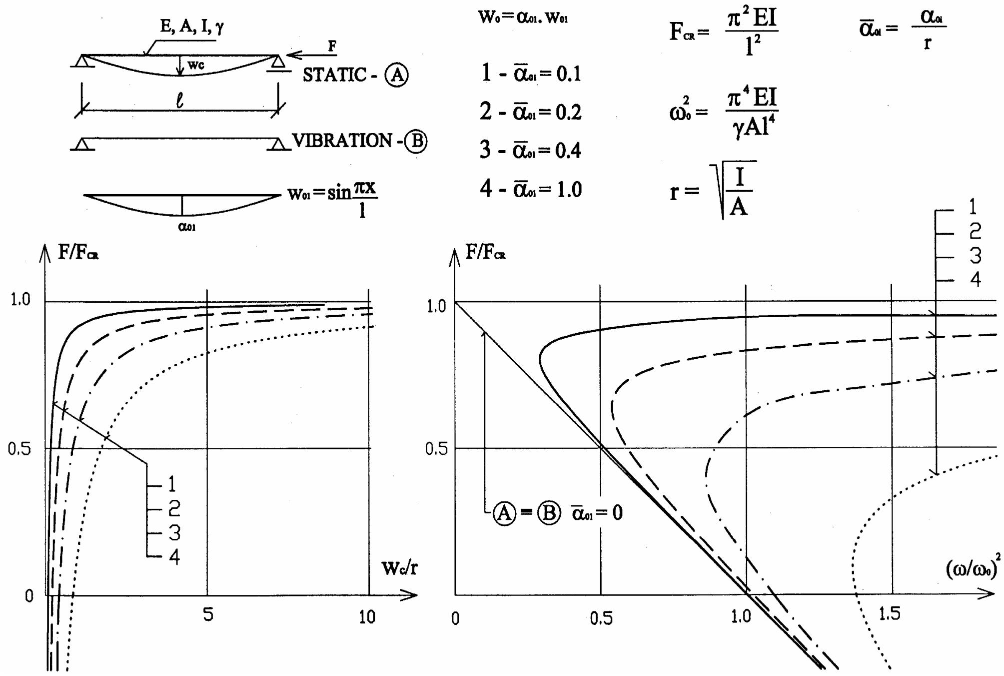

Fig.1 The vibration of simple supported column

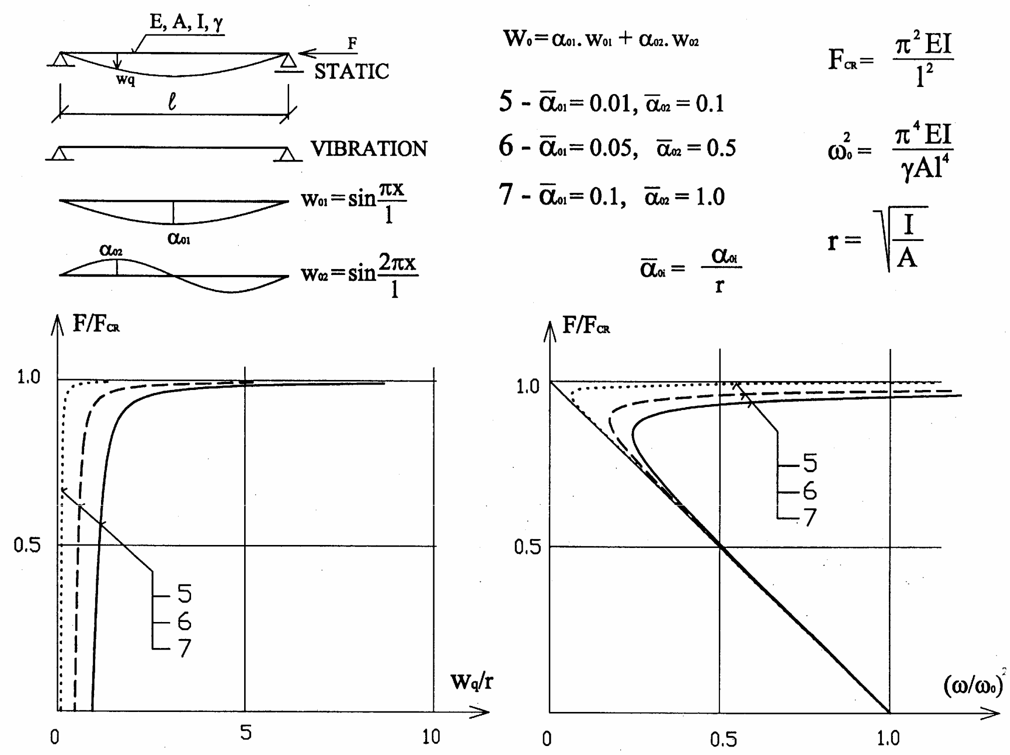

Fig.2 Vibration of the column with the second mode of the initial displacement

Vibration of the frame

We have arranged the load condition to get the mode of the buckling different to the mode of vibration. (Fig.3). In that case the relationship between the load and the square of the circular frequency is non-linear. Analogously as in the case of the column we will suppose a different edge condition for static load and for the vibration. If the point of application of the load is fixed during the vibration process we can take in consideration the effects of initial displacements.

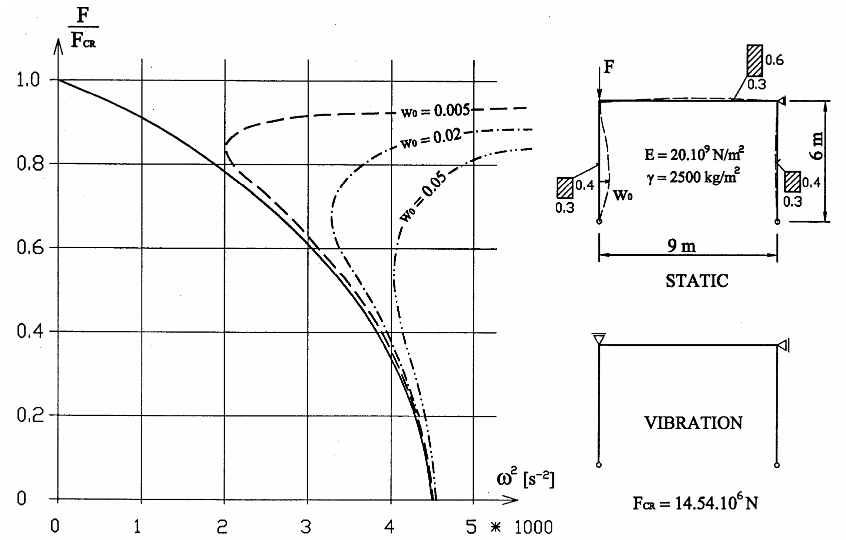

Fig.3 Vibration of the frame

Slender web loaded in compression.

In the case of the slender web loaded in compression the in-plane edge conditions play a crucial role as well. If the mode of the initial displacement is the same as second buckling mode the static post-buckling behaviour could be in this second mode. This initial displacement mode has an influence to the circular frequency, but the mode of vibration is the first buckling mode. (Fig.4)

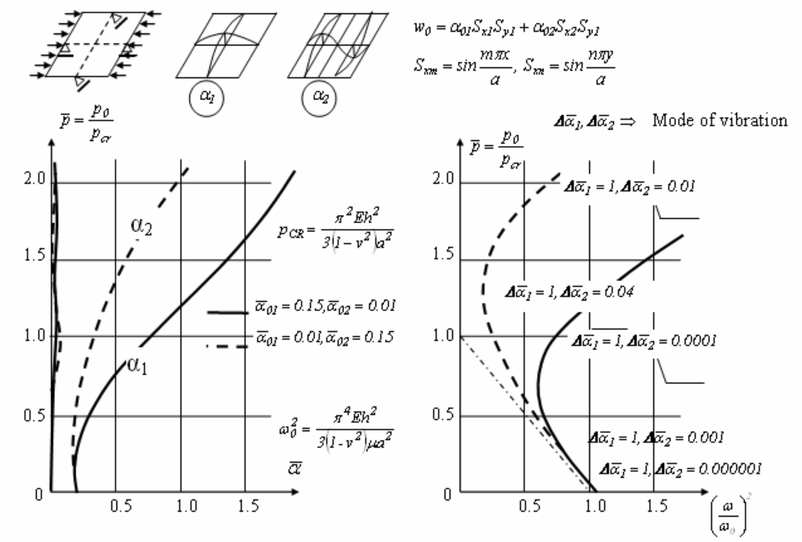

Fig.4 Influence of the mode of the initial displacement for the vibration of slender web loaded in compression

EXPERIMENTAL INVESTIGATION

The experimental investigation of the dynamic post buckling behaviour of the slender web (or a thin-walled panel or a thin-walled structure) needs high quality equipments and experiences. The presented results have been arranged at The Institute of Construction and Architectures of The Slovak Academy of Sciences.

Figs.5 to 7 show the test of the steel panel loaded in compression. (The square panel, width 400mm, flanges 60x8 mm connected by a welding). We used to suppose the distribution of the welding (residual) stresses with the positive yield stresses near the edge of the web. In this case the compression loading-unloading process should not produce the redistribution of the residual stresses. To explain the results from Figs.5,6 needs to accept the "bending" residual stresses into the consideration. The panel of the thickness of the web $h=1.53$ mm produces a snap-though effect. (Fig.7). The upper and lower load levels of the jump can be established. The transformation via eigen modes has been used for the theoretical evaluation of this example (Ravinger 1992).

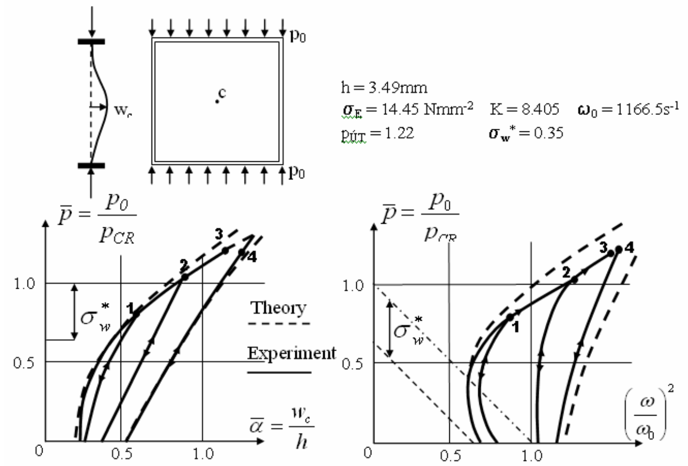

Fig.5 Comparison of the theoretical and experimental results for the panel with the thickness of the web 3.49 mm

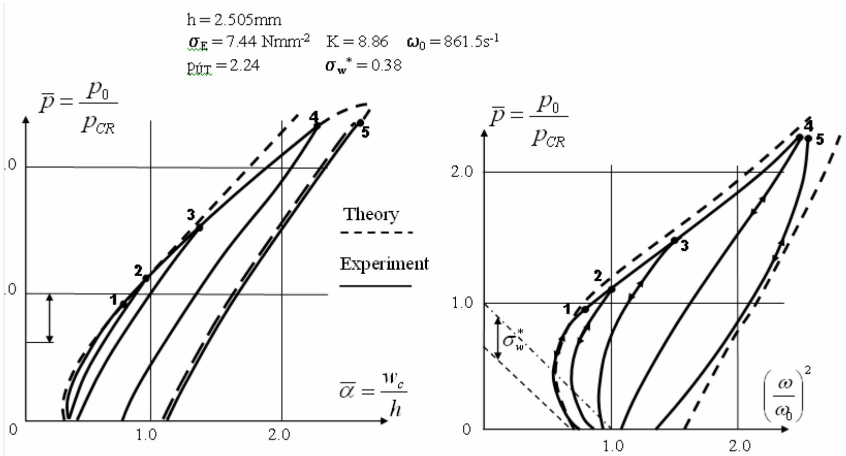

Fig.6 Comparison of the theoretical and experimental results for the panel with the thickness of the web 2.505 mm

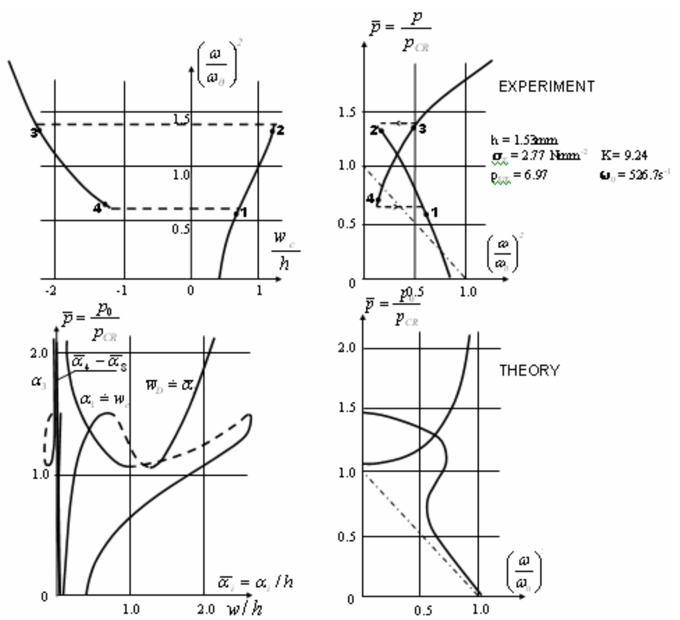

Fig.7 Vibration of slender web during the snap-trough

CONCLUSION

The theory and the experiments have proved the sensitivity of the circular frequency of free vibration to the level of the load and different types of initial imperfections. This knowledge can be used as an inverse idea. Measuring of the natural frequencies can give us a picture about the stresses and imperfections in the structure. This idea represents a base for a non-destructive method for the evaluation of the properties of the frame structure or the thin-walled structure at all. The natural frequency is sensitive even to residual stresses and it is very important detail.

Presented results show the peculiarities in the edge conditions. We must distinguish the edge conditions for behaviour of the structure during the application of the load (static or dynamic) and the edge conditions for the free vibration process.

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