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Ladislav ÉCSI*, Pavel ÉLESZTŐS**

CONSTITUTIVE EQUATION WITH INTERNAL DAMPING FOR MATERIALS UNDER DYNAMIC AND CYCLIC LOADINGS

KONŠTITUTÍVNA ROVNICA S VNÚTORNÝM TLMENÍM PRE DYNAMICKY A CYKLICKY ZAŤAŽENÉ MATERIÁLY

Abstract

In this paper a universal constitutive equation with internal damping is presented for materials under dynamic and cyclic loadings. The model adapts the idea of a spring dashpot system connected in parallel for continuum, utilizing appropriate deformation measures, which are independent of rigid body motion and thus it enables more precise numerical simulation of real material behaviour resulted from internal damping. In the presented work the model application is shown using cyclic loading and infinitesimal strain/deformation formulation based finite element analysis with extended NoIHKH material model for cyclic plasticity of metals.

Abstrakt

Autori v prekladanom článku prezentujú univerzálnu konštitutívnu rovnicu s vnútorným tlmením pre dynamicky a cyklicky zaťažené materiály. Model adoptuje myšlienku paralelne zapojenej pružiny a tlmiča pre kontinuum využitím vhodných mierok deformácie, ktoré sú nezávislé na tuhom pohybe telesa a preto umožňuje presnejšiu numerickú simuláciu správania sa skutočného materiálu zapríčineného vnútorným tlmením. V prezentovanej práci autori ukážu použitie modelu pri cyklickom zaťažení metódou konečných prvkov s opisom pomocou nekonečne malých pretvorení a rozšíreným NoIHKH materiálovým modelom pre cyklickú plasticitu kovov.

1 INTRODUCTION

An engineering construction during its operation has to withstand various loadings; the significant part of which can be characterized as dynamic or cyclic. To model properly the behaviour of such a construction internal damping of its material has to be taken into account. According to the authors' best knowledge in the contemporary literature there is no universal mathematical model proposed to model the internal damping of a construction material. The lack of such a model can lead to spurious numerical behaviour under dynamic and cyclic loadings.

2 THEORY BACGROUND

Considering the analogy between continuum and a spring dashpot system connected in parallel, where the spring force and the damping force depend on the relative displacement and relative velocity of the spring ends, the Cauchy stress tensor of a material with internal damping can be expressed using elastic-plastic associative plasticity, incremental infinitesimal strain/deformation formulation [1],[4] and the extended NoIHKH material model [2] as

* MSc., Institute of Applied Mechanics and Mechatronics, Faculty of Mechanical Engineering, SjF. STU Bratislava, Námestie slobody 17, 812 31 Bratislava, tel. (+421) 902 305 737, e-mail ladislav.ecsi@stuba.sk

** Assoc. Prof., MSc., PhD., Institute of Applied Mechanics and Mechatronics, Faculty of Mechanical Engineering, SjF. STU Bratislava, Námestie slobody 17, 812 31 Bratislava, tel. (+421) 905 719 103, e-mail pavel.elesztos@stuba.sk

$$\boldsymbol{\sigma}^{n+1} = {}^{elast}\boldsymbol{\sigma}^{n+1} + {}^{damp}\boldsymbol{\sigma}^{n+1} \tag{1}$$

where: $${}^{elast}\boldsymbol{\sigma}^{n+1} = \mathrm{C} : \left(\boldsymbol{\varepsilon}^{n+1} - {}^{p}\boldsymbol{\varepsilon}^{n}\right), \qquad \text{in elastic loading/unloading} \tag{2}$$

$${}^{elast}\boldsymbol{\sigma}^{n+1} = \mathrm{C} : \left(\boldsymbol{\varepsilon}^{n+1} - {}^{p}\boldsymbol{\varepsilon}^{n+1}\right) = \mathrm{C} : \left[\boldsymbol{\varepsilon}^{n+1} - \left({}^{p}\boldsymbol{\varepsilon}^{n} + \Delta\lambda \frac{\partial f^{\,n+1}}{\partial \boldsymbol{\sigma}^{n+1}}\right)\right], \qquad \text{in plastic loading} \tag{3}$$

$${}^{damp}\boldsymbol{\sigma}^{n+1} = \mathrm{C}_{damp} : \mathbf{d}^{n+\frac{1}{2}}, \tag{4}$$

$$f^{\,n+1} = \sqrt{\frac{3}{2}(\boldsymbol{\Sigma}^{n+1} - \mathbf{X}^{n+1}) : (\boldsymbol{\Sigma}^{n+1} - \mathbf{X}^{n+1})} - \sigma_y - R^{n+1} \leq 0\,, \tag{5}$$

$$R^{n+1} = Q\left(1 - e^{-b\,{}^{p}\varepsilon^{n+1}}\right), \tag{6}$$

$$\dot{\mathbf{X}}^{n+1} = \mathrm{C}_{cycl} : {}^{p}\mathbf{d}^{n+\frac{1}{2}} - \left[\gamma_\infty - \left(\gamma_\infty - \gamma_0\right) e^{-\omega\,{}^{p}\varepsilon^{n+1}}\right]\mathbf{X}^{n+1}\,{}^{p}\dot{\varepsilon}^{n+1}\,. \tag{7}$$

Equations (2) and (3) represent the elastic part of the Cauchy stress tensor (1) in elastic loading/unloading conditions and in plastic loading conditions. Equation (4) denotes the damping part of the Cauchy stress tensor. Equations (5)-(7) define the yield surface and the evolution equations of the extended NoIHKH material model [2], namely the NoIH rule [3] for isotropic material hardening (6) and the NoKH rule [3] for kinematic material hardening (7). $\boldsymbol{\varepsilon}, {}^{p}\boldsymbol{\varepsilon}, \mathbf{d}, {}^{p}\mathbf{d}$ denote the infinitesimal strain tensor, the plastic part of the infinitesimal strain tensor, the strain rate tensor and the plastic part of the strain rate tensor. $\boldsymbol{\Sigma}, \mathbf{X}, {}^{p}\varepsilon, {}^{p}\dot{\varepsilon}, \Delta\lambda$ stand for the deviatoric stress tensor, the back stress tensor, the accumulated plastic strain, the accumulated plastic strain rate and the plastic multiplier value. The remaining symbols denote constant material parameters. The fourth order cyclic material tensor C_{cycl} and the damping tensor C_{damp} were formally constructed in the same way as the elastic material tensor using two independent variables ν_{cycl}, E_{cycl} and ν_{damp}, E_{damp}, which ensures isotropy

$$\mathrm{C}_i = 2G_i\mathrm{I} + \lambda_i\left(\mathbf{1}\otimes\mathbf{1}\right), \quad G_i = \frac{E_i}{2\left(1+\nu_i\right)}, \quad \lambda_i = \frac{\nu_i E_i}{\left(1+\nu_i\right)\left(1-2\nu_i\right)} \quad \text{for} \quad i = cycl, damp\,. \tag{8}$$

3 NUMERICAL EXAMPLE

As a numerical example a solid bar, size $1\ \mathrm{m} \times 1\ \mathrm{m} \times 3\ \mathrm{m}$ was studied applying cyclic tension. One end of the bar was fixed and the second end underwent a prescribed axial deformation determined by a sine function and amplitude 2.5/3.5mm corresponding to elastic/plastic loading, while it was guided in the remaining two directions. In the numerical experiment one loading cycle was realized using 15 degree angular increments in each time step. Cases with and without internal damping were studied, using $0.04\,Hz$, $4.16\,Hz$ and $41.66\,Hz$ loading frequency corresponding to $1.0\ s$, $0.01\ s$ and $0.001\ s$ time step values. As a simplification all material parameters were considered to be constant. Table 1 contains the used material properties. Since no experimental tests were carried out, some of the used plastic material properties are considered to be informative.

Tab. 1 Material properties

E [Pa]	E_{cycl} [Pa]	E_{damp} [$\mathrm{Pa\cdot s}$]	$\nu = \nu_{cycl} = \nu_{damp}$ [-]	σ_y [Pa]
$2.1\cdot10^{11}$	$2.1\cdot10^{5}$	0.0 / $2.1\cdot10^{8}$	0.3	$2.0\cdot10^{8}$
Q [Pa]	b [-]	γ_∞ [-]	γ_0 [-]	ω [-]
$5.0\cdot10^{7}$	3.0	20.0	10.0	10.0

4 NUMERICAL RESULTS

Figures 1-2 show the axial deformation, Von Mises stress and the accumulated plastic strain distribution in one longitudinal cross section of the bar at maximum tension corresponding to the plastic loading case with internal damping using $0.001\ \mathrm{s}$ time step.

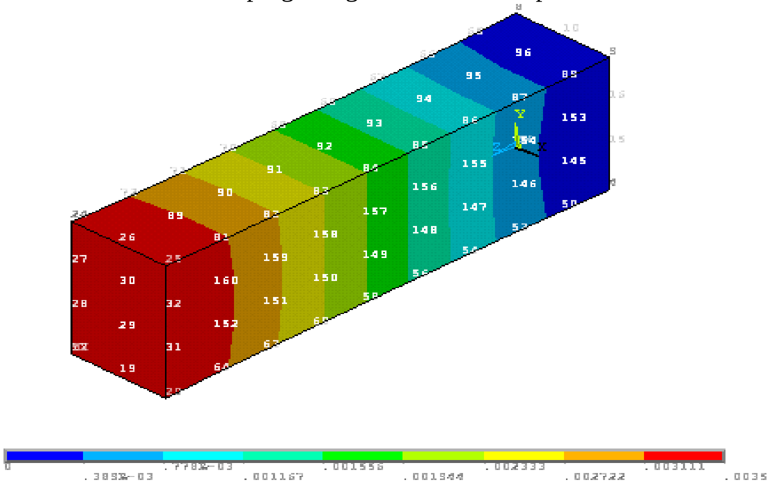
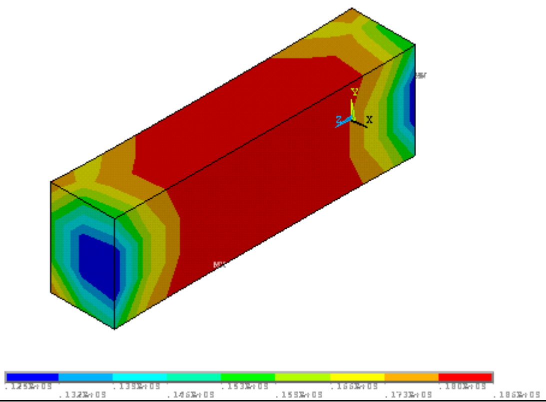
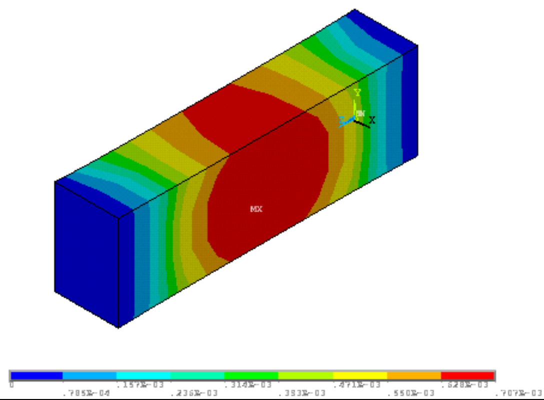


Fig. 1 Maximum axial deformation in one longitudinal cross section of the bar



Von Mises stress distribution



Accumulated plastic strain distribution

Fig. 2 Von Mises stress and accumulated plastic strain in one longitudinal cross section of the bar

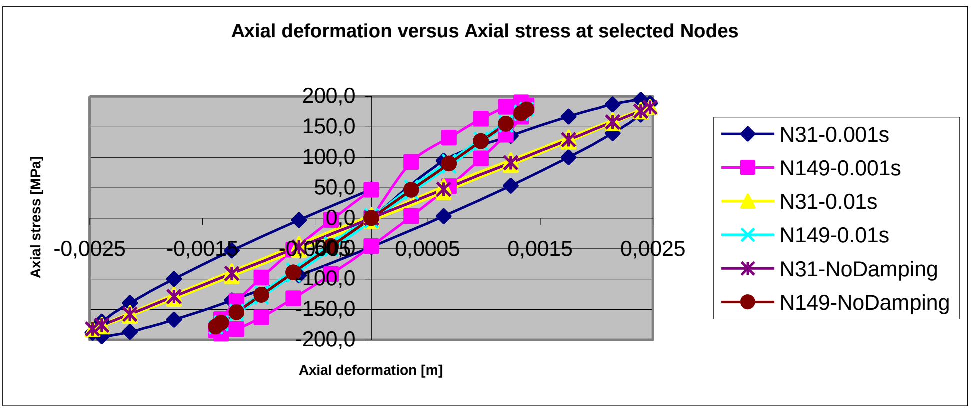


Fig. 3 Hysteresis loops at nodes N31, N149 in elastic loading conditions corresponding to various time steps

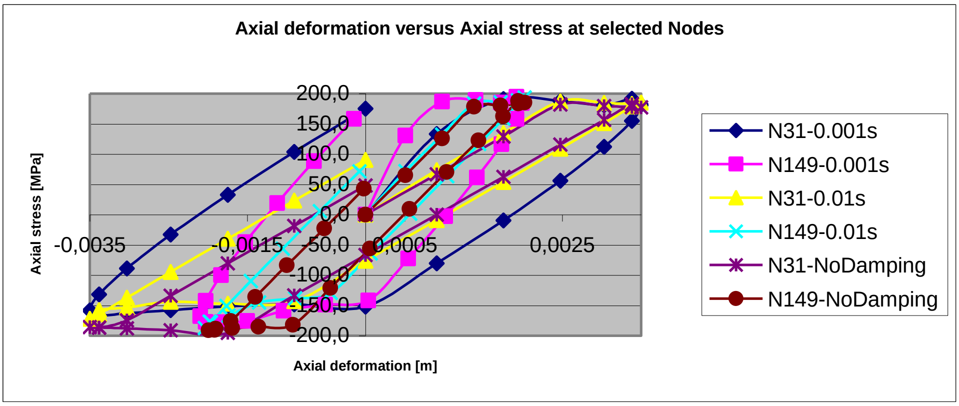


Fig. 4 Hysteresis loops at nodes N31, N149 in plastic loading conditions corresponding to various time steps

Figures 3-4 show the axial deformation versus axial stress curves at selected nodes at the bar end N30 and in its middle part at node N149. As can be seen in figure 3, there is no energy dissipation in elastic loading without damping, the system is conservative and the axial deflection versus axial stress curve is linear. Applying internal damping hysteresis loops were created and the material curve no longer could be described as linear. Figures 3 and 4 imply that the area of the hysteresis loop is proportional to the deformation rate, i.e. the higher the deformation rate the greater the loop area as well as the amount of the dissipated energy and the resistance against deformation. Also, in limiting state, as the deformation rate approaches zero, the effect of internal damping vanishes.

5 CONCLUSIONS

In this paper a universal constitutive equation with internal damping was presented for materials under dynamic and cyclic loadings. The model was capable to dissipate energy and damp the system in both, elastic and plastic loading conditions. Since the damping is independent of rigid body motion, not only in dynamic and cyclic analyses will the presented model result more accurate numerical simulations, but also in coupled-thermal structural calculations.

6 ACKNOWLEDGEMENT

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REFERENCES

[1] CRISFIELD, M. A. *Non-linear finite element analysis of solids and structures, Essentials*, Vol. 1, Chichester, John Wiley & Sons LTD, 2000, 345 pp. ISBN 0-471-97059-X.

[2] ÉCSI, L. Extended NoIHKH model usage for cyclic plasticity of metals, *Engineering Mechanics.* 2006, Vol. 13, Nr. 2, pp. 83-92, ISSN 1210-2717.

[3] LEMAITRE, J. *Handbook of material behaviour models, Deformations of materials*, Vol. 1, London, Academic press, 2001, 350 pp. ISBN 0-12-443342-1.

[4] SIMO, J. C., HUGHES, T. J. R. *Computational inelasticity*, NY, Springer, 1998, 392 pp. ISBN 0389-97520-9.

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