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TEMPERATURE FIELD SOLUTION ANALYSIS OF CYLINDRICAL PARTICLES IN CO-CURRENT CONTACT WITH FLUID MEDIUM

ANALÝZA RIEŠENIA TEPLOTNÉHO POĽA VALCOVÝCH ČASTÍC PRI SÚPRÚDNOM KONTAKTOVANÍ S FLUDNÝM MÉDIOM

Abstract

Major characteristics of the heat exchange process are that the temperature and the stress field depend on time, what essentially affects the residual stresses, as well as the energy demands of the process. In technical literature there are numerous solutions of heating and cooling [1], [2] of a solid body using various heat exchangers published, but few of them are devoted to the study of the influence of the thermal capacitance ratio of contact phases. The peculiarity of the problem is that the heat transfer varies along the length of the heat exchanger. A solution of a spherical body temperature field in co-current contact with fluid medium can be found in the form of infinite series using the Fourier method [3]. Under the condition that the non-stationary temperature field is known, the thermal stress field of the spherical body can be determined during the heat exchange process [4]. The temperature field of cylindrical particles can be determined in the same way. Considering the geometry of the solid phase, the problem has to be solved in cylindrical coordinates. In order to take into account the boundary conditions further simplifications have to be introduced. The object of the this paper is to verify numerically the affordability of such simplifications.

Abstrakt

Základnou charakteristikou rôznych procesov výmeny tepla je časová závislosť teplotného poľa a poľa mechanických napätí, ktoré v rozhodujúcej miere ovplyvňujú energetickú náročnosť procesu, ale i pole zostatkových napätí. Odborná literatúra ponúka riešenie celého radu problémov ohrevu a ochladzovania pevných telies [1], [2], v rôznych typoch výmenníkov, ale málo sa venuje problematike vplyvu pomeru tepelných kapacít oboch kontaktovaných fáz. Zvláštnosťou týchto problémov je, že hnacia sila prestupu tepla sa mení po dĺžke výmenníka. Riešenie teplotného poľa pre telesá guľového tvaru v podmienkach súprúdneho kontaktovania s tekutinou je možné nájsť pomocou Fourierovej metódy v tvare nekonečného radu. Na základe známeho nestacionárneho teplotného poľa môžeme vyšetriť i teplotné napätia v priebehu procesu výmeny tepla [3]. Podobným spôsobom je možné riešiť i teplotné pole valcovitých častíc. Vzhľadom na geometriu častíc pevnej fáze takúto úlohu je potrebné riešiť vo valcovom súradnicovom systéme. Kvôli zavedeniu okrajových podmienok je však potrebné zaviesť dodatočné zjednodušujúce predpoklady. Numerické overenie opodstatnenosti zavedenia týchto predpokladov je predmetom tohto článku.

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1 NOMENCLATURE

a	coefficient of temperature diffusivity	$[m^2s^{-1}]$
L	particle length	[m]
M	mass flow	$[kgs^{-1}]$
R	outer radius of cylinder	[m]
t	time	[s]
T	temperature	[K]
c	specific heat	$[Jkg^{-1}K^{-1}]$
α	coefficient of heat transfer	$[Wm^{-2}K^{-2}]$
λ	heat conductivity	$[Wm^{-1}K^{-1}]$
ρ	density	$[kgm^{-3}]$
Θ	dimensionless temperature	[-]

Indices

f liquid phase

s solid phase

0 initial value

p surface value

c calorimetric

0 zero order

1 first order

2 PROBLEM FORMULATION

In a co-current heat exchanger the particles of ideal cylindrical shape, and the outer radius R are in direct contact with liquid. It is assumed that their material is isotropic. The particles at the beginning of the heat exchange process have the uniform temperature of T_{s0}. The heating/cooling liquid temperature is T_{f0}, and it is in direct contact with the particles. It is assumed that the co-current flow of the particles and the liquid phase can be considered a flow in a cylinder driven by a piston. Inside the particles no heat generation rate per unit volume is assumed. Within the solution the thermo-mechanical properties of the solid phase and the gas (such as specific heat, coefficient of thermal conductivity) are considered to be independent of the temperature and their values are constant. The coefficient of heat transfer α between the gas and the particles is also constant over their cylindrical surfaces with ideally insulated ends α=0. Under these assumptions one dimensional, rotationally symmetrical temperature field is generated. The temperature field outside the heat transfer affected area, further from the cylinder ends can be determined at any time as a temperature field of an infinite cylinder, on condition, that the thermal capacitance ratio of contact phases is kept unchanged and the heat conduction along the y axis is neglected [4].

3 MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

The temperature field of a cylindrical body is described by the Fourier-Kirchhoff heat equation, which along with the boundary and initial conditions [2, 4] after introducing some dimensionless variables such as

$$Bi = \frac{\alpha D}{\lambda}$$ the Biot number

$$Fo = \frac{at}{R^2}$$ the Fourier number

$m = \frac{M_s c_s}{M_f c_f}$ thermal capacitance ratio of contact phases

$\rho = \frac{r}{R}$ dimensionless radius

$\Theta_s = \frac{T_s - T_{s0}}{T_{f0} - T_{s0}}$ dimensionless temperature difference of solid phase

$\Theta_{sc} = \frac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$ dimensionless average temperature difference of solid phase

$\Theta_{sp} = \frac{T_{sp} - T_{s0}}{T_{f0} - T_{s0}}$ dimensionless temperature difference of solid phase on the cylindrical surface of particles

$\Theta_f = \frac{T_f - T_{s0}}{T_{f0} - T_{s0}}$ dimensionless temperature difference of liquid phase

can be expressed as

$$\left[\frac{\partial\Theta_s}{\partial Fo} = \frac{\partial^2\Theta_s}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Theta_s}{\partial\rho}\right] \;, \tag{1}$$

which is supplemented with the following boundary condition

$$1 - m\Theta_{sc} - \Theta_{sp} = -\frac{1}{Bi}\left[\frac{\partial\Theta_s}{\partial\rho}\right]_{\rho=1} \;. \tag{2}$$

The dimensionless average calorimetric temperature and the initial conditions can be formulated as

$$\Theta_{sc} = 2\int_0^1 \rho\Theta_s d\rho \;, \quad Fo=0 \;, \quad \Theta_{sp}=0 \;, \quad \Theta_{sc}=0 \;. \tag{3}$$

Equation (1) after considering the boundary and initial conditions (2) and (3) can be solved using the Fourier method. The temperature field of the body then can be expressed in the following form

$$\Theta_s = \frac{1}{1+m} + \sum_{i=1}^{\infty}\frac{-2k_i J_1(k_i)}{4mJ_1^2(k_i) + k_i^2\left[J_0^2(k_i) + J_1^2(k_i)\right]}\, e^{-k_i^2 Fo} J_0(k_i\rho) \;. \tag{4}$$

The dimensionless average temperature difference of the solid phase and the dimensionless temperature difference of the liquid phase can be derived as follows

$$\Theta_{sc} = \frac{1}{1+m} - \sum_{i=1}^{\infty}\frac{2J_1^2(k_i)}{4mJ_1^2(k_i) + k_i^2\left[J_0^2(k_i) + J_1^2(k_i)\right]}\, e^{-k_i^2 Fo} \;, \tag{5}$$

$$\Theta_f = \frac{1}{1+m} + m\sum_{i=1}^{\infty}\frac{4J_1^2(k_i)}{4mJ_1^2(k_i) + k_i^2\left[J_0^2(k_i) + J_1^2(k_i)\right]}\, e^{-k_i^2 Fo} \;, \tag{6}$$

where in equations (4), (5), (6) k_i stand for the roots of the following transcendental equation

$$\frac{k_i}{Bi} = \frac{2m}{k_i} + \frac{J_0(k_i)}{J_1(k_i)} \;. \tag{7}$$

In the solution of the heat exchange process we assumed that the particle ends are ideally insulated. In reality this condition is not met. To find out how much the heat transfer between particle ends and the liquid affects the temperature field of the body a numerical study has been carried out using limit value of the thermal capacitance ratio of the contact phases m=0. For the aforementioned case an

analytical solution can easily be arrived at utilising equation (11), which represents a non-stationary temperature field of a heated/cooled infinite cylinder with Fourier boundary conditions and constant environmental temperature.
The numerical simulation of the cooling of particles of a finite size was carried out using the ANSYS finite element package, where some of the entries (such as the particle geometry and thermo-mechanical properties of contact phases) were chosen in such a way, that Bi=10, L/2R=4 a m=0 and additionally heat transfer between particle ends and liquid was taken into consideration. In heat exchangers condition m=0 corresponds to a constant liquid phase temperature. The solution of the problem is quite straightforward. Figure 1 shows the numerical results at time Fo= 0,005. The surface temperature along the particle length is depicted in figure 2 for various times Fo .

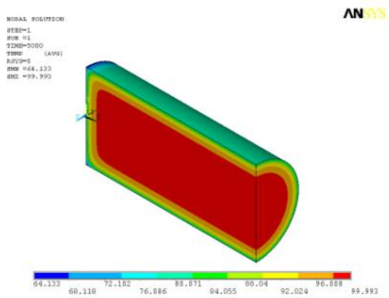

Fig. 1 Particle temperature at time Fo=0.005

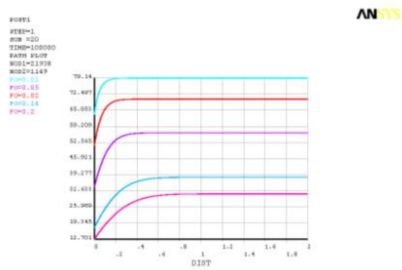

Fig. 2 Surface temperature for various times (Fo)

4 CONCLUSIONS

Comparing the numerical and analytical results of the presented model it can be stated, that the analytical temperature field coming from equation (1), which describe the co-current contact of solid particles and a liquid medium is sufficiently accurate if L/D>4.

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