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STRESS STATE IDENTIFICATION BY NUMERICAL SIMULATION

OF THE HOLE DRILLING PRINCIPLE – PART A

IDENTIFIKACE STAVU NAPJATOSTI NUMERICKOU SIMULACÍ

ODVRTÁVACÍHO PRINCIPU – ČÁST A

Abstract

The hole drilling method used for the stress state identification is currently standardized by the E 837 international standard. The standartized hole drilling method theory is based on two constants adjusted for particular designs of drilling rosettes. The idea of the presented numerical simulation technique corresponds to the E 837 standard concepts but is more universal. It transforms the strains, arising during the hole drilling experiment, in a way similar to that of the E 837 standard but, unlike the E 837 standard, it executes the transformation completely. This new theory enables the drilling method to be applied both for a wider spectrum of materials, and for further measuring appliances. Moreover, the hole drilling process do not have to be extremely precise, which the whole procedure simplifies, since the new method principle includes an objective stress state identification, when evaluating drilling experiments, with respect to the drilled hole eccentricity.

Abstrakt

V současnosti je odvrtávací metoda pro identifikaci napjatosti podporována mezinárodní normou E 837 a v této aplikaci je teorie této metody založena na dvou konstantách, normovaných pro konkrétní konstrukce odvrtávacích růžic. Pojetí předkládané numerické simulační metodiky je v duchu mezinárodní normy E 837 univerzálnější, neboť na základě analogie transformuje poměrné deformace pro využití v odvrtávacím experimentu podobnou formou jako metodika normy E 837, ale na rozdíl od ní kompletně. Nová metoda otevírá využití odvrtávacího principu pro širší spektrum materiálů a dalších měřících prostředků, neboť to umožňuje její teorie. Odvrtávání otvorů nemusí být tak přesné, protože princip metody zahrnuje objektivní identifikaci napjatosti při vyhodnocení odvrtávacích experimentů s ohledem na excentricitu vrtaných otvorů a to vlastní odvrtávání zjednodušuje.

1 INTRODUCTION

The semi-destructive hole drilling principle is based on impairing of the inner force equilibrium of a strained structure by drilling of a relatively small circular hole perpendicularly to the surface. The hole induces a change of a strain state in its close vicinity. These changes can be adjusted to defined strains and thus used later for an identification of the original strain state after the strains relieved by the drilled hole are measured.

The relative radius $r = R/R_0 \geq 1$ is defined in r, α polar coordinates. If the hole of the radius R_0 has not been drilled yet, the thin plate depicted in Fig. 1, which is loaded uniaxially by principal stress σ_x, is loaded by stresses σ'_r, σ'_Θ, $\tau'_{r\Theta}$ in planes defined by r and α polar coordinates and

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marked by indices of their normal lines r, Θ. The stresses are determined in Eq. 1 from an elementary equilibrium on the planes ($A \Rightarrow A / sin\alpha$) or ($A \Rightarrow A / cos\alpha$).

$$\begin{Bmatrix} \sigma'_r = \dfrac{\sigma_x}{2}(1+\cos 2\alpha) \\ \sigma'_\Theta = \dfrac{\sigma_x}{2}(1-\cos 2\alpha) \\ \tau'_{r\Theta} = \dfrac{\sigma_x}{2}\sin 2\alpha \end{Bmatrix} \tag{1}$$

The theory of the hole drilling principle is based on the analytical Kirsch's stress-state solution of a plate with a hole drilled through perpendicularly and loaded on its x-borders by principal stress σ_x [1]. The Kirsch's equations (Eq. 2) describe the state of plane strain in the vicinity of the hole of radius R_0 (Fig. 1).

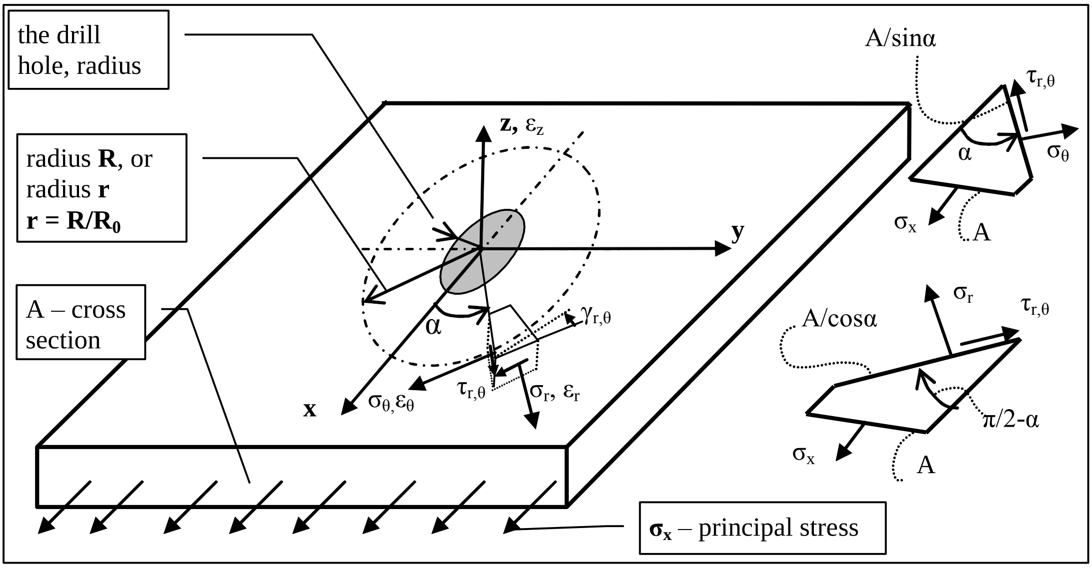

Fig.1 State of stress and strains around the drill hole

$$\begin{Bmatrix} \sigma''_r = \dfrac{\sigma_x}{2}(1-\dfrac{1}{r^2}) + \dfrac{\sigma_x}{2}(1+\dfrac{3}{r^4}-\dfrac{4}{r^2})\cos 2\alpha \\ \sigma''_\Theta = \dfrac{\sigma_x}{2}(1+\dfrac{1}{r^2}) - \dfrac{\sigma_x}{2}(1+\dfrac{3}{r^4})\cos 2\alpha \\ \tau''_{r\Theta} = \dfrac{\sigma_x}{2}(1-\dfrac{3}{r^4}+\dfrac{2}{r^2})\sin 2\alpha \end{Bmatrix} \tag{2}$$

The change of straining induced by the hole drilling in comparison to the original state is defined by the difference of corresponding components of Eqs. (1) and (2) in Eq. (3).

In comparison with Eqs. (1), the Eqs. (2) include terms dependant on the drilled hole, which are left in the Eqs. (3) that are otherwise of a character similar to Eqs. (1) and (2). If E stands for Young's modulus and ν for Poisson's ratio, the changes of plane stresses σ_r, σ_Θ, $\tau_{r\Theta}$ can be used for any isotropic material for a calculation of changes related to strains ε_r, ε_Θ, $\gamma_{r\Theta}$ and ε_z (see Fig. 1) in a point on the plate using the Hooke's law (Eq. (4)) and further modified to Eq. (5).

$$\left\{\begin{aligned}
\sigma_r &= \sigma_r'' - \sigma_r' = \frac{\sigma_x}{2}\left(-\frac{1}{r^2}\right) + \frac{\sigma_x}{2}\left(\frac{3}{r^4} - \frac{4}{r^2}\right)\cos 2\alpha \\
\sigma_\Theta &= \sigma_\Theta'' - \sigma_\Theta' = \frac{\sigma_x}{2}\left(\frac{1}{r^2}\right) - \frac{\sigma_x}{2}\left(\frac{3}{r^4}\right)\cos 2\alpha \\
\tau_{r\Theta} &= \tau_{r\Theta}'' - \tau_{r\Theta}' = \frac{\sigma_x}{2}\left(-\frac{3}{r^4} + \frac{2}{r^2}\right)\sin 2\alpha
\end{aligned}\right\} \tag{3}$$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\Theta} \end{bmatrix} = \sigma_x \begin{bmatrix} \left[\frac{-(1+\nu)}{2E}\right]\cdot\left[\frac{1}{r^2} - \frac{3\cdot\cos 2\alpha}{r^4} + \frac{4\cdot\cos 2\alpha}{r^2(1+\nu)}\right] \\ \left[\frac{-(1+\nu)}{2E}\right]\cdot\left[-\frac{1}{r^2} + \frac{3\cdot\cos 2\alpha}{r^4} - \frac{4\nu\cdot\cos 2\alpha}{r^2(1+\nu)}\right] \\ \left[\frac{2\cdot(1+\nu)}{2E}\right]\cdot\left[-\frac{3}{r^4} + \frac{2}{r^2}\right]\cdot\sin 2\alpha \\ \left[\frac{4\nu}{2Er^2}\right]\cdot\cos 2\alpha \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\frac{1}{r^2} - \frac{1}{r^2}\nu\right] + \left[\frac{1}{r^4}3 + \frac{1}{r^4}3\nu - \frac{1}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\frac{1}{r^2} + \frac{1}{r^2}\nu\right] - \left[\frac{1}{r^4}3 + \frac{1}{r^4}3\nu - \frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\frac{1}{r^4}3 + \frac{1}{r^2}2 - \frac{1}{r^4}3\nu + \frac{1}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \tag{5}$$

The hole drilling method for the stress state identification is based on the assumption, that the free surface is one of the principal planes. The stress state in the surface layer thus can be only a uniaxial or plane one. As such it should be identifiable by a measurement of strains relieved on the free surface of the pre-strained structure during the drilling of the hole perpendicular to the surface. The use of the hole drilling method for identification of residual stresses [2] is supported in E 837 standard [3]. It is valid for isotropic Hooke's materials with a known strain response to the drilling of the hole. The response is measured by strain gauges assembled to a drilling rosette. The response function is similar to radial ε_r or tangential ε_Θ strains identified in the Kirsch's solution of the thin plate with a hole as described in Eqs. (4) and (5).

$$\begin{aligned} \bar{\varepsilon}_r &= \sigma_x(\bar{A} + \bar{B}\cos 2\alpha) + \sigma_y(\bar{A} + \bar{B}\cos(2\alpha + \pi)) = \\ &= \sigma_x(\bar{A} + \bar{B}\cos 2\alpha) + \sigma_y(\bar{A} - \bar{B}\cos 2\alpha) \end{aligned} \tag{6}$$

A simplification of the goniometric function Eq. (5) describing a response of an ideal strain gauge placed with a deviation of angle α from a direction related to the principal stress σ_x is used (see Eq. (6)). Standard constant variables $\bar{A} = -\bar{a}(1+\nu)/2E$ and $\bar{B} = -\bar{b}/2E$ related to the particular design of the drilling rosette are used within the superposition of wanted principal stresses σ_x and σ_y. The $\bar{a}$ constant is objectively independent from the material to which it is drilled, while the $\bar{b}$ constant is simplified here, because it is mildly dependant on the Poisson's ratio ν of the Hooke's material (see Eqs. (4) and (5)). Both constants $\bar{a}, \bar{b}$ are tabulated in E 837 standard for particular types of drilling rosettes, the ratio of diameters $1/r = 2R_0/2R$ given and the relative depth of the drilled hole $z/2R$, where z is the depth of the hole.

Strain gauges of drilling rosettes have size of the winding comparable to diameters of drilled holes $2R_0$ or middle radii R, at which the strain gauges of the rosettes are placed. The central line of a typical strain gauge, in relation to which its local coordinate system s, g is defined, is oriented by g axis into an ideal center of the rosette, which is marked for an aim of the drilled hole. Strings of the strain gauge winding, which measure the strains in the vicinity of the drilled hole, do not have a uniform (e.g. radial) direction in relation to the hole. The objective distribution of strains in the measured surface under an applied strain gauge is not symmetric in its surroundings along the local g axis generally. Due to this fact, the measurement properties of the rosettes during the hole drilling according to E 837 standard are considerably dependant on the accuracy of compliance with standardized conditions of the experiment. If the hole is drilled eccentrically, then the hole drilling experiment as formulated by E 837 standard cannot be used for any more complex determination of the strain state in the vicinity of the drilled hole, which would be necessary for any eventual improving corrections.

The computing Eq. (7) of the measured signal is established for each particular strain gauge and a given measured hole eccentricity with coefficients set from the Fourier's series. The equation is created by a superposition of wanted principal stresses σ_x and σ_y. Angular position of the principal stresses is defined by angular parameter $\overline{\alpha}$.

$$\bar{\varepsilon}_r = \sigma_x \left[\sum_j c_j \cos(j\alpha) + \sum_j d_j \sin(j\alpha) \right] + \sigma_y \left[\sum_j c_j \cos(j(\alpha + \pi/2)) + \sum_j d_j \sin(j(\alpha + \pi/2)) \right] \quad (7)$$

The method of Fourier's series allows inclusion of many other effects (dimensional imperfections, stress gradients, material aspects), which can on principle affect the hole drilling experiment. It thus can be interpreted as a general and powerful calculating tool for the stress state identification based on the hole drilling principle. In order to realize the method, a great number of constants used within the numerous terms of Fourier's series have to be set by a numerical simulation.

2 CONCLUSIONS

To be continued in the part B: "STRESS STATE IDENTIFICATION BY NUMERICAL SIMULATION OF THE HOLE DRILLING PRINCIPLE – PART B" and in the full version of this article published on the CD.

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