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STRESS STATE IDENTIFICATION BY NUMERICAL SIMULATION OF THE HOLE DRILLING PRINCIPLE – PART B

IDENTIFIKACE STAVU NAPJATOSTI NUMERICKOU SIMULACÍ ODVRTÁVACÍHO PRINCIPU – ČÁST B

Abstract

Continuation of the part A from the theme called: "STRESS STATE IDENTIFICATION BY NUMERICAL SIMULATION OF THE HOLE DRILLING PRINCIPLE – PART A" and of the entire article published on the CD.

Abstrakt

Pokračování první části tématu pod názvem: "IDENTIFIKACE STAVU NAPJATOSTI NUMERICKOU SIMULACÍ ODVRTÁVACÍHO PRINCIPU - ČÁST A" a kompletního článku uveřejněného na CD.

2 ELIMINATION OF DRILLED HOLE ECCENTRICITY EFFECTS BY THE USE OF HOLE DRILLING PRINCIPLE

Although the nowadays approach to the hole drilling theory supported by E 837 standard simplifies the theory for specific applications, it also demands wholly accurate aiming of the hole drilling experiment by elimination of the hole eccentricity. If the hole is generated in an eccentric position, then the interpretation of the stress state is unreliable. Although the hole drilling experiment is quite sensitive to the eccentricity of the hole, its potential effect on the stress state is not included in the standardized mathematical model.

The method based on the hole drilling principle should therefore break away from the specific designs of drilling rosettes and take the effect of drilled holes eccentricity onto the stress state evaluation into account. The modification of the complete Kirsch's theory in this direction would allow significant extension of the usage of the hole drilling principle in engineering practice. The elaborated method could be applied to other kinds of materials and should decrease the demands concerning the accuracy of the hole drilling process.

We expect that the real position of the hole can deviate from the ideal centric position $\bar{x}=0\,,\bar{y}=0$ to a new position with eccentricity components x_0, y_0 (see Fig. 5). The position of the strain gauge i winding is described by its local coordinates s, g with the origin in O_i. A set of j points marked M_j is defined in s, g coordinates on the centerline of the strain gauge winding. In the case that the particular strain gauge stays in the ideal central position, its response can be formulated for the real radius ratio $r = R / R_0$ by the help of the nowadays standardized theory (6). The position of the origin of i-th strain gauge in Fig. 5 defined by polar coordinates $\bar{\rho}_i, \bar{\omega}_i$ is in the Cartesian coordinates described by the formula (8). The position of a point O_i is defined in relation to the real center of the

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drilled hole shifted by the eccentricity components x_0, y_0 from the ideal position by Cartesian coordinates x_i, y_i and polar coordinates ρ_i, ω_i by the formula (9).

$$\bar{x}_i = \bar{\rho}_i \cdot \cos\bar{\omega}_i \ , \ \bar{y}_i = \bar{\rho}_i \cdot \sin\bar{\omega}_i \tag{8}$$

$$\left\{\begin{array}{l} x_i = \bar{x}_i - x_O = \bar{\rho}_i \cdot \cos\bar{\omega}_i - x_O = \rho_i \cdot \cos\omega_i \\ y_i = \bar{y}_i - y_O = \bar{\rho}_i \cdot \sin\bar{\omega}_i - y_O = \rho_i \cdot \sin\omega_i \ , \qquad \rho_i = \sqrt{x_i^{\,2} + y_i^{\,2}} \ , \\ \text{where} \quad \omega_i = \arcsin(y_i/\rho_i) \quad \text{for} \quad x_i \geq 0 \quad \text{or} \quad \omega_i = \pi - \arcsin(y_i/\rho_i) \ \ pro \ \ x_i \prec 0 \end{array}\right\} \tag{9}$$

M_j point on the centerline of the winding of strain gauge i then has coordinates x_j, y_j by the Eq. (10), derived from Cartesian coordinates x_i, y_i and local coordinates s, g of the i-th strain gauge.

$$\left\{\begin{array}{l} x_j = x_i + s_j \cdot \sin\psi_i + g_j \cdot \cos\psi_i = \rho_i \cdot \cos\omega_i + s_j \cdot \sin\psi_i + g_j \cdot \cos\psi_i = r_j \cdot \cos\omega_j \\ y_j = y_i - s_j \cdot \cos\psi_i + g_j \cdot \sin\psi_i = \rho_i \cdot \sin\omega_i - s_j \cdot \cos\psi_i + g_j \cdot \sin\psi_i = r_j \cdot \sin\omega_j \end{array}\right\} \tag{10}$$

The polar coordinates r_j, ω_j of the M_j point in Eq. (11) are set from Eq. (10) in analogy with Eq. (9). The angle φ of g axis from θ axis is described by the Eq. (12) in the local coordinate system

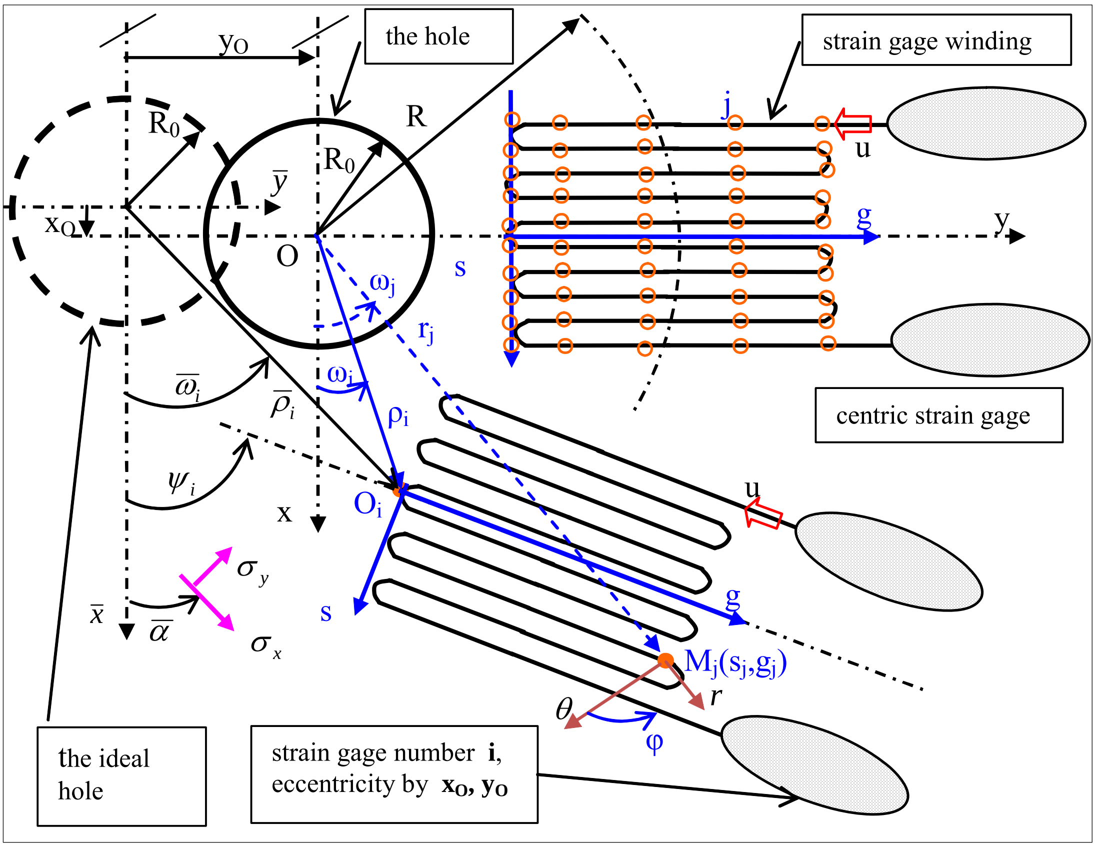

Fig. 5 Position of the strain gage winding against the drill hole

r, θ (Fig. 5), which here also represents the direction of the dominant part of the strain gauge i winding.

$$\left\{ \begin{array}{l} r_j = \sqrt{x_j{}^2 + y_j{}^2} \quad kde \quad \omega_j = \arcsin(y_j / r_j) \quad pro \quad x_j \geq 0 \\ nebo \quad \omega_j = \pi - \arcsin(y_j / r_j) \quad pro \quad x_j \prec 0 \end{array} \right\} \tag{11}$$

$$\varphi = (\psi_i - \omega_j) + \pi/2 \tag{12}$$

$$\left\{ \begin{array}{l} \sigma_r = \dfrac{\sigma_x}{2}(-\dfrac{1\cdot c_1(r,z)}{r^2}) + \dfrac{\sigma_x}{2}(\dfrac{3\cdot c_2(r,z)}{r^4} - \dfrac{4\cdot c_3(r,z)}{r^2})\cos 2\alpha \\ \sigma_\Theta = \dfrac{\sigma_x}{2}(\dfrac{1\cdot c_4(r,z)}{r^2}) - \dfrac{\sigma_x}{2}(\dfrac{3\cdot c_5(r,z)}{r^4})\cos 2\alpha \\ \tau_{r\Theta} = \dfrac{\sigma_x}{2}(-\dfrac{3\cdot c_6(r,z)}{r^4} + \dfrac{2\cdot c_7(r,z)}{r^2})\sin 2\alpha \end{array} \right\} \tag{13}$$

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\Theta \\ \gamma_{r\Theta} \\ \varepsilon_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \\ -\nu & -\nu & 0 \end{bmatrix} \cdot \begin{bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\Theta} \end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix} \left\{\left[-\dfrac{c_1}{r^2} - \dfrac{c_4}{r^2}\nu\right] + \left[\dfrac{c_2}{r^4}3 + \dfrac{c_5}{r^4}3\nu - \dfrac{c_3}{r^2}4\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[+\dfrac{c_4}{r^2} + \dfrac{c_1}{r^2}\nu\right] - \left[\dfrac{c_5}{r^4}3 + \dfrac{c_2}{r^4}3\nu - \dfrac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \\ \left\{\left[-\dfrac{c_6}{r^4}3 + \dfrac{c_7}{r^2}2 - \dfrac{c_6}{r^4}3\nu + \dfrac{c_7}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\} \\ \left\{\left[\dfrac{c_1}{r^2}\nu - \dfrac{c_4}{r^2}\nu\right] + \left[-\dfrac{c_2}{r^4}3\nu + \dfrac{c_5}{r^4}3\nu + \dfrac{c_3}{r^2}4\nu\right]\cdot\cos 2\alpha\right\} \end{bmatrix} \tag{14}$$

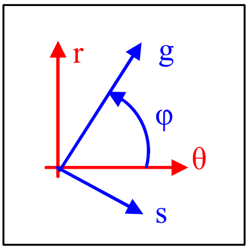

Fig. 6

We expect, that the stress state components in the surroundings of the blind drilled hole as written in Eqs. (13) are analogous to Eq. (3) of the straight-through hole. Let we also modify all seven polytropic terms of the complete Kirsch's theory by constants c_k (r, z), which are dependant on the distance from the center of the drilled hole, for the blind hole. The distance is described by the relative radius r and the depth z of the drilled hole. By the way, a similar approach is also used by E 837 standard for radial strain. These complete components of the stress state change induced by drilling the hole can be transformed to the strain components. If the isotropic Hooke's material is evaluated, then the strain components can be computed by Eq. (14) in analogy to Eq. (5). A strain state on planes perpendicular to the surface can be set by an angular transformation, where the use of the first three components ε_r, ε_Θ, $\gamma_{r\Theta}$ in Eq. (14) is sufficient, because of the principal strain ε_z does not have any effect on it. Fig. 6 defines the position of g axis towards Θ axis for an acute angle φ. The strain in the g direction is derived from ε_r, ε_Θ, $\gamma_{r\Theta}$ strains according to the Mohr's transformation Eq. (15) by the use of goniometric functions of a double angle 2φ.

$$\varepsilon_g = \frac{\varepsilon_\Theta + \varepsilon_r}{2} + \frac{\varepsilon_\Theta - \varepsilon_r}{2}\cos 2\varphi + \frac{\gamma_{\Theta,r}}{2}\sin 2\varphi \tag{15}$$

We expect the direction of the principal stress σ_x given by the angular parameter $\overline{\alpha}$ measured from either x or $\overline{x}$ axis (see Fig. 5). The bonded strain gauge reads the strain field of the contact surface and we suppose that the strains in points and direction of the conductive winding with a reflection of the weight related to the length of winding correspond to the strain values measured by the strain gauge. The strains ε_r, ε_Θ, $\gamma_{r\Theta}$ are normed by a unit load vector introduced in the direction of principal stress and transformed by the Eq. 15 to the $\bar{\varepsilon}_r, \bar{\varepsilon}_\theta, \bar{\tau}_g$ of the winding direction. An analogy to Eqs. (6) and (7) (see part A) allows assembly of a system of three independent Eqs. (25) of strain gauge signals ε_i read in the vicinity of the drilled hole for unknown principal stresses σ_x, σ_y and the angle of their position $\bar{\alpha}$.

$$\left.\begin{cases} \varepsilon_1 = \dfrac{1}{4E}\left[\sigma_x \cdot K_{1,1}(\bar{\alpha}) + \sigma_y \cdot K_{1,2}(\bar{\alpha})\right] \\ \varepsilon_2 = \dfrac{1}{4E}\left[\sigma_x \cdot K_{2,1}(\bar{\alpha}) + \sigma_y \cdot K_{2,2}(\bar{\alpha})\right] \\ \varepsilon_3 = \dfrac{1}{4E}\left[\sigma_x \cdot K_{3,1}(\bar{\alpha}) + \sigma_y \cdot K_{3,2}(\bar{\alpha})\right] \end{cases}\right\} = \frac{1}{4E}\begin{bmatrix} K_{1,1}(\bar{\alpha}) & K_{1,2}(\bar{\alpha}) \\ K_{2,1}(\bar{\alpha}) & K_{2,2}(\bar{\alpha}) \\ K_{3,1}(\bar{\alpha}) & K_{3,2}(\bar{\alpha}) \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} \tag{25}$$

The first two equations (25) serve for determination of unknown principal stresses σ_x and σ_y as a functions of ε_1 and ε_2 strain signals and of an unknown angular parameter $\bar{\alpha}$ - defining the position of the principal stress σ_x. Then the third equation for ε_3 allows the computation of $\bar{\alpha}$ parameter.

3 CONCLUSIONS

The theory shown here uses only seven constants for a given depth of the drilled hole and a distance of the measured point from the drilled hole. The constants allow formulation of the strain state on the surface in the vicinity of the hole. The constants are independent from any examined isotropic Hooke’s material as well as from the design of the measuring members. The new method is thus more universal than the international E 837 hole-drilling standard. The new method takes any eventual eccentricity of the drilled hole objectively into account. Because of the hole drilling experiments are quite sensitive to such an eccentricity in the interpretation of the stress state, the new method allows significant simplification and price reduction of their realization. It should therefore enable a much broader use of the hole drilling experiment in the common engineering practice.

ACKNOWLEDGEMENT

This research was supported by „ Josef Bozek Research Centre II, 1M6840770002 MSMT“.

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