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INVERSE IDENTIFICATION METHOD FOR MATERIAL PARAMETERS ESTIMATIONS – ANALYSIS OF EXPERIMENTS

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Abstract: The article describes identifications of material parameters (Inverse Method, FEM) from set of experiments. The experiments were made with hollow cylindrical specimens {steel (11375)}. In this paper are used the data obtained from 5 experiments with different loads (axial force, torque and their combination). The article uses Multilinear isotropic material model (3. material parameters), Hill anisotropy material model (9. material parameters) and kinematic material models (3, 13. material parameters) for the problem solution. The solutions were found by FEM (Inverse algorithm, Probability algorithm) and were compared with the experimental data.

1. Introduction

The paper describes estimation material models and material parameters by inverse method and finite element method (FEM) [1]. The data from five static experiments (tension/ compression axial force, torsion and their combination) with hollow cylindrical specimen were used to estimation material parameters. The four material models were tested (multilinear isotropic, multililinear kinematics, Hill anisotropic and Chaboche material models). The prestress effects were tested with kinematics model (Chaboche). The final material model (material parameters) must correspond to set of experiments (proportional or no proportional combined loading). Every experiment was solved apart.

A simple algorithm was used to solve the problem of material parameter estimations [2], [3]. Basic part of solution is the FEM (model of the specimen with boundary conditions related to experiments). The FEM solution is repeated in cycles, every cycle has unique material parameters. The output of the FEM solution (reaction axial force, reaction torsion moment, displacement, twist angle) is compared with experiments. Base on the provided calculation there were proposed changes in material parameters (probability algorithm, gradient). The random basic algorithm [2], [3] and gradient basic algorithm was applied. Final data sets were analyzed by correlation methods [8].

Software ANSYS and DELPHI were used for programming the algorithms. The four “ANSYS” material models (MISO, MISO+HILL, KINH, MISO+CHAB) with different number of material parameters (3, 9, 13) were tested [9]. The material parameter estimation methods and basic data analysis were programmed in DELPHI.

The next step of the study will be focused on prestress (in surface layer) and anisotropy (HILL) effects.

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2. Solution procedure

Following material models implemented in software ANSYS [9] were used:

- Multilinear isotropic (MISO) – 3 material parameters.
- HILL´s anisotropy (MISO+HILL) – 9 material parameters.
- Multilinear kinematics (KINH) – 3 material parameters.
- Chaboche (MISO+CHAB) – 13 material parameters.

The basic theory of these material models are describe in ANSYS manual [9].

The constitutive equation was represented by Ramberg-Osgood approximation (1).

$$\varepsilon = \frac{\sigma}{C1} + \left(\frac{\sigma}{C2}\right)^{\frac{1}{C3}}. \tag{1}$$

There are $C1$, $C2$, $C3$ material parameters. The constitutive equation in material models was replaced by set of lines (multilinear approximation, MISO, KINH).

The basic solution algorithm is described by Fig.1.

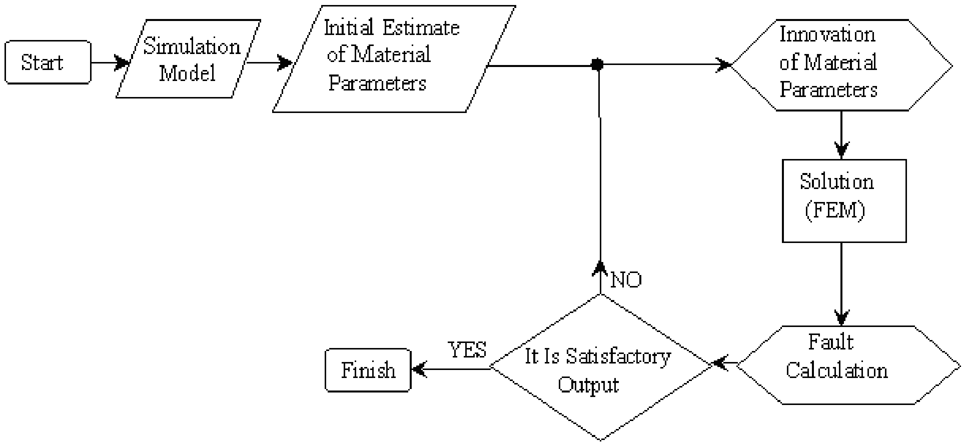

Figure 1: *Basic algorithm.*

The first point of algorithm diagram (Fig.1) is creation of Simulation Model. The simulation model includes basic design of specimen – geometric model, finite element model and boundary conditions (loads and deformations). The basis of boundary conditions are experimental data (y, φ) applied to pivot A, the measured value of loads corresponds to reaction loads from FEM (pivot A). The simulation model is showed on Fig.2. The simulation model doesn't include assignment material parameters (they are inserted later). Element type was selected with respect to appearance of buckling and large deformation effects.

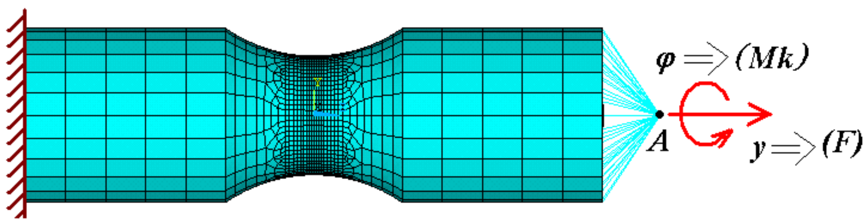

Figure 2: *The simulation model of specimen.*

The calculation begins with the initial parameters (material). The initial parameters can be estimated analytically from experiments - measured value of axial force F was recalculated to axial stress σ, measured value of elongation y was recalculated to strain ε (Tension). The results (set of points σ, ε) were smoothed by Ramberg-Osgood approximation of constitutive equation (1) in terms of probability algorithm (for more details see [2], [6] etc.). The initial parameters have little effect to speed of convergence. The initially estimated values of material parameters ($C1$, $C2$, $C3$) are showed in Tab. 1 (after half correction).

Table 1: *Initial material parameters*

	$C1$ [MPa]	$C2$ [MPa]	$C3$ [1]
Initial material parameters	210000	1000	0,2

The next step of the algorithm is correction of the material parameters. The method used for innovation – parameter modifications (material) are main part of solution. In first section there was calculated a number (e.g. 10) of FE solutions with random generated parameters ($C1$, $C2$, $C3$ etc.). These solutions (material parameters) are set of input data. The set of output data contains relevant error of solution (The calculate error is described in next paragraph).

The new values of parameters e.g. $C1$ is put together from three parts (2):

$$C1 = C10 + \Delta random + gradient\_1 . \qquad (2)$$

1. Value of parameter $C10$ corresponds to solution with minimum error.
2. Random component of $C1$ (uniform density function).
3. Movable component of $C1$ (gradient identified from m previous solutions).

- Given $\mathbf{a}_i = (Error_i, C1_i, C2_i, \ldots Ck_i)$, k is number of parameters and $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots \mathbf{a}_m\}$; $\mathbf{B} = \{[b_1, c_1], [b_2, c_2] \ldots [b_n, c_n]\}$; $p, s \in N$; $p + s \leq m$; $p \cdot s = n$.
- Let $\forall \mathbf{a}_i \in \mathbf{A} : Error_i < Error_{i+1}$; $i \in \langle 1, m \rangle$ and $\forall [b_i, c_i] \in \mathbf{B} : (c_i = b_i + d_i) \wedge ((b_i \neq b_j) \wedge (c_i \neq c_j) : i \neq j)$; $i, j \in \langle 1, n \rangle$; $d \in \langle 1, s \rangle$; $b \in \langle 1, p \rangle$.
- Then $\delta_{(i,j)} = (\delta 1_{(i,j)}, \delta 2_{(i,j)}, \ldots \delta k_{(i,j)}) = (C1_i - C1_j, C2_i - C2_j, \ldots Ck_i - Ck_j)$; $\mathbf{\Delta} = \{\delta_{(1,2)}, \delta_{(1,3)}, \ldots \delta_{(p,p+s)}\}$; $\forall \delta_{(i,j)} \in \mathbf{\Delta} : [i, j] \in \mathbf{B}$ and $gradient\_i = \frac{1}{n}(\delta i_{[1,2]} + \delta i_{[1,3]} + \ldots \delta i_{[p,p+s]})$.

The error of actual solution step was solved by compare with values of forces, moments obtained from FE solution and values of forces and moments get from experiments. The fundamental error solutions are shown in Fig. 3, the error value was calculated by equation (3).

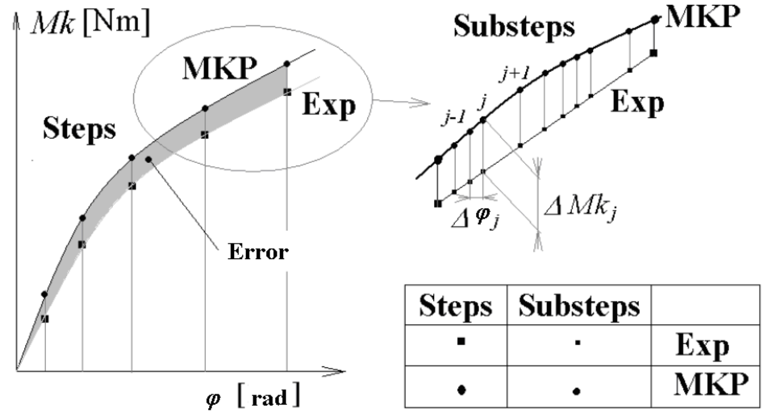

Figure 3: *Error analyze for moment.*

$$ERROR_{Exp} = \left\{ \frac{\sum_j \left\{ \left| \left(Mk_j^{Exp} - Mk_j^{FEM} \right) \cdot \left(\varphi_j - \varphi_{j-1} \right) \right| \right\}}{\sum_j \left\{ \left| Mk_j^{Exp} \cdot \left(\varphi_j - \varphi_{j-1} \right) \right| \right\}} + \frac{\sum_k \left\{ \left| \left(F_k^{Exp} - F_k^{FEM} \right) \cdot \left(y_k - y_{k-1} \right) \right| \right\}}{\sum_k \left\{ \left| F_k^{Exp} \cdot \left(y_k - y_{k-1} \right) \right| \right\}} \right\} / 2 \qquad (3)$$

If error component is equal to zero (the specimen is loaded only by axial force or torsion moment), then $\mathrm{ERROR}_{\mathrm{Exp}}$ is not divide by two.

The algorithm can be finished after obtaining prescribed value of error, prescribed number of cycles or after stopping convergence.

3. Experiments

The experiments were realized on Universal Testing Machine (Department of Mechanics of Materials) see [4], [5], [7]. The hollow cylindrical specimens (see Fig. 4) used for experiments were made from the steel (11375).

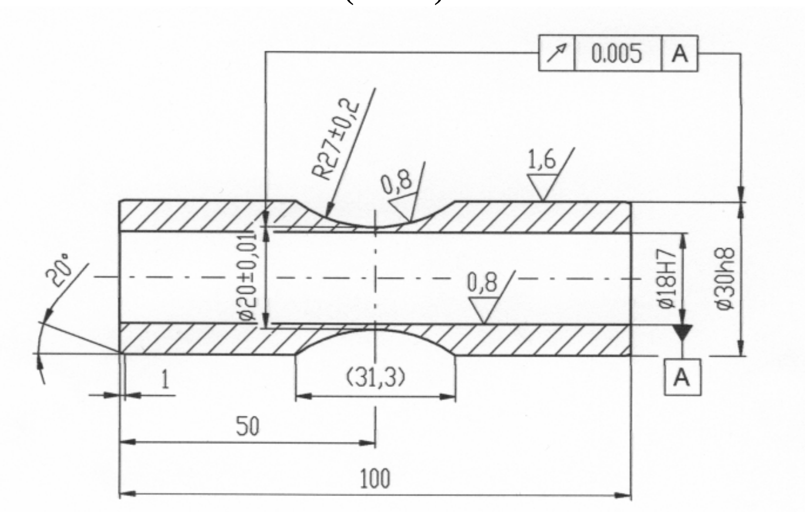

Figure 4: *The hollow cylindrical specimen.*

The applications of loads in experiments were controlled by the deformation (linear increased elongation *y*, twisting angle *φ*). The loads at every experiment were set by elongation value *y* and twisting angle value *φ* (around the combinations - the twisting angle value *φ* was calculate from the elongation value *y* in agreement with equations inside of Tab. 2). At the same time was measured torque value *Mk* and axial force value *F*. The loading variant called Tension showed null torque value *Mk* (negligible) and for calculation was ignored. The loading variant called Torsion showed low axial force value *F* (negligible), for calculation was not used.

Five experiments were performed:

- Tension – axial force, elongation,
- Torsion – torque, twisting angle,
- Combin_1 – combination of axial tension force and torque,
- Combin_2 – combination of axial tension force and torque,
- Combin_3 – combination of axial compression force and torque.

The loads are described in Tab.2.

Table 2: *Application of loads*

Type:	Torque *Mk*	Axial Force *F*	Angle φ [rad]	Elongation y_{MAX} [mm]
Tension	Measured	Measured	0	1.67
Torsion	Measured	Measured	φ_{MAX} = 1.484	0
Combin_1	Measured	Measured	y×(5×π/180)/0.25	1.36
Combin_2	Measured	Measured	y×(5×π/180)/0.1	0.86
Combin_3	Measured	Measured	-y×(5×π/180)/0.1	-1.98

For calculation was used computer with: processor Core 2 Duo E6420, 2x1024MB DDR2 800 RAM, 320GB SATAII/300 7200RPM (software ANSYS v.11.0 - MKP, Borland DELPHI for Windows 2005, 2007 – probability algorithm, value analyses etc.). One computational cycle takes about 10-15 min. – the one experiment solution takes about 24 hours.

4. Results - MISO and HILL models

The material parameters (*C*1, *C*2, *C*3 etc.), with the errors found by algorithm, are displayed on Tab. 3. The searching algorithm was finished after convergence ceased.

Table 3: *Results for MISO material model (recalculate)*

Typ:	*C*1 [MPa]	*C*2 [MPa]	*C*3 [1]	*Error* [%]
Tension	251398	745,8	0,19413	0,567
Torsion	262720	607,8	0,17485	0,544
Combin_1	423476	735,7	0,17215	5,41
Combin_2	279925	662,9	0,13239	11,2
Combin_3	43564	565,8	0,091898	7,73

Results for HILL anisotropy material model (solution from [11]*)*

	*C*1 [MPa]	*C*2 [MPa]	*C*3 [1]	*C*4 [1]	*C*5 [1]	*C*6 [1]	*C*7 [1]	*C*8 [1]	*C*9 [1]	*Error* [%]
Ini. Param.	196000	759	0.15	1	1	1	1	1	1	
Tension	192623.9	748.97	0.1944	1.00	1.01	1.001	0.97	0.99	0.98	1.05
Torsion	201865.5	746.3	0.183	1.03	1.01	0.988	1.02	1.02	0.99	0.67
Combin_1	196398.8	672.97	0.1753	1.03	1.01	0.988	1.02	0.92	0.96	1.04
Combin_2	176532.7	647.96	0.22	1.03	0.94	1.015	0.98	0.95	0.96	0.92
Combin_3	223715.8	647.62	0.200	1.10	1.01	0.836	1.04	0.96	0.96	1.986

The strong correlation between boundary conditions (displacements, angle) and material parameters were identified in MISO material model (parameter *C*2 - angle (96 %), parameter *C*2 - displacement (95 %), parameter *C*1 – angle (82 %) and parameter *C*1 – displacement (81 %)).

The “correlation” in Hill material model is low. The final value of material parameters *C*4 - *C*9 indicates that tested material has no strong anisotropy behavior, perhaps too many parameters.

The specimen behavior (experiment) can be explained by prestress effect, that is why kinematics material models was chosen (multilinear kinematic – KINH, Chaboche model – CHAB+MISO)

5. Results - KINH and Chaboche models

Only the first two experiments (tension, torsion) are defined by one graph. The material parameters related to the both experiments can be set with error up to 1% for all tested material models (alike as MISO and Hill = ANISO see fig.6, 7). Material model estimation of those two experiments is not relevant. There is buckling in the specimen loaded by combin_3. For the buckling analyses are very important imperfections (form variance – tolerance) see [1], which are not topic of this paper. Material parameter estimation of the experiment is not relevant too. The last two experiments (combin_1, combin_2) are defined by two graphs and they are critical for material parameter estimation.

The basic multilinear kinematic material model (KINH) gives bad outputs see fig.6 (errors for combin_1 and combin_2 are greater then 10 %). This model gives the worst results. The material parameters (*C*1, *C*2, *C*3), with the errors found by algorithm, are displayed on Tab.4. The searching algorithm was finished after convergence ceased.

Table 4: *Results for KINH material model*

Typ:	*C*1 [MPa]	*C*2 [MPa]	*C*3 [1]	*Error* [%]
Combin_1	448303	750,6	0,18378	10,2
Combin_2	52218	714	0.0549	17,7

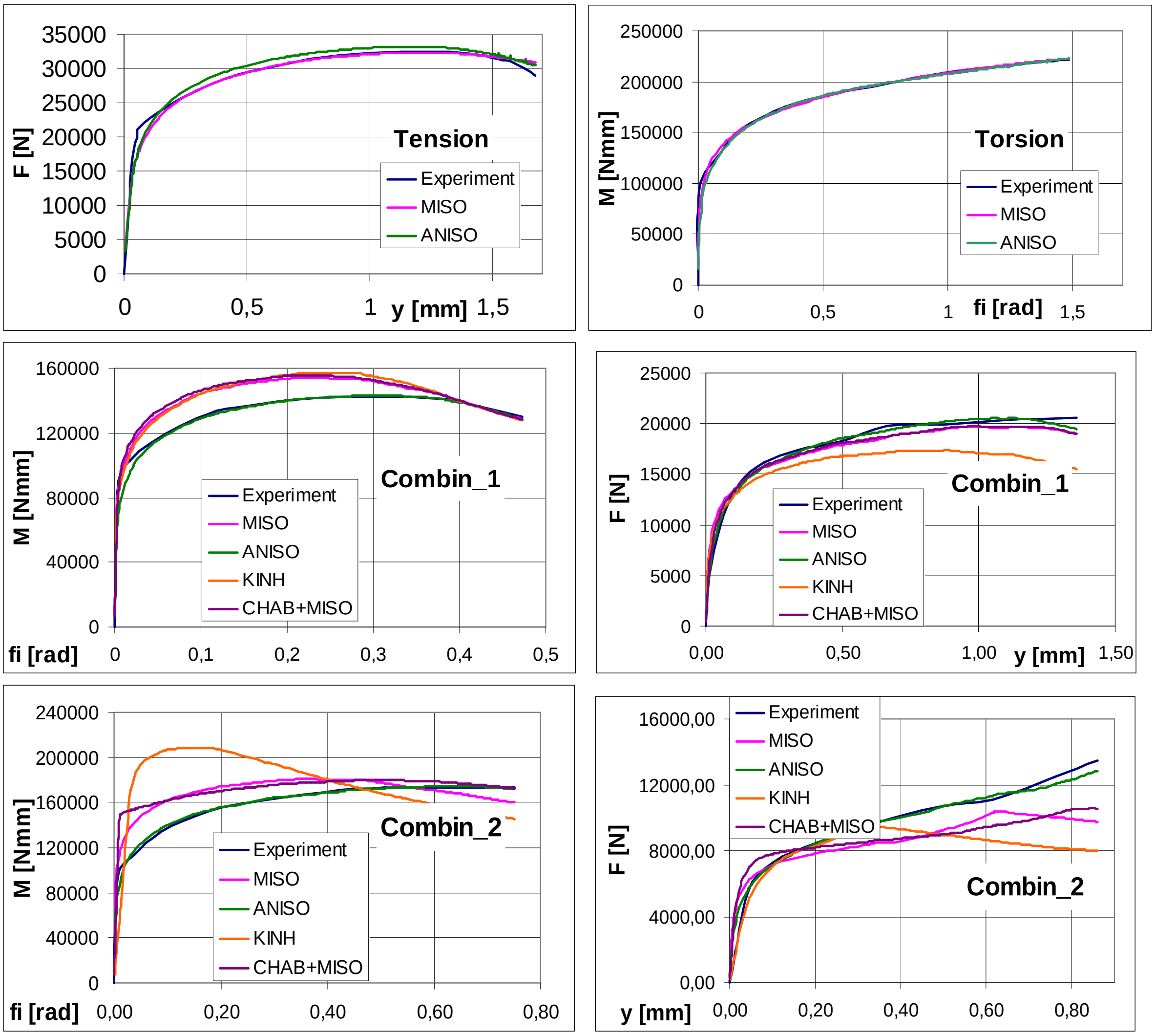

Figure 6: *Visual comparison of selected results.*

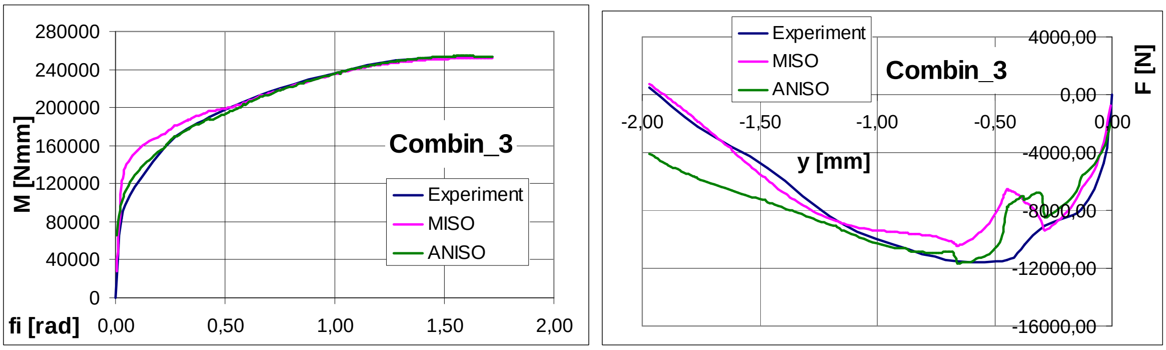

Figure 7: *Visual comparison of selected results.*

The Chaboche material model (CHAB+MISO) was loaded by two steps. In the first step the specimen was loaded by "prestress" axial force F_{pre} (4) and disburden.

$$F_{pre} = (prestress + 200) \cdot 60 \;\; for \;\; F > 0$$
$$F_{pre} = (prestress - 200) \cdot 60 \;\; for \;\; F < 0 \tag{4}$$

In the second step the displacement was corrected (move to zero) and the specimen was loaded by corresponding loads (combin_1, combin_2). The error for Chaboche material model is practically the same as the error for MISO material model. The material parameters ($C1$, $C2$, $C3$ etc.), with the errors found by algorithm, are displayed on Tab.5. The searching algorithm was finished after convergence ceased.

Table 5: *Results for Chaboche+MISO material model*

	*C*1 [MPa]	*C*2 [MPa]	*C*3 [1]	*C*4 [MPa]	*C*5 [1]	*C*6 [1]	*C*7 [1]	*Error* [%]
Combin_1	219073.2	696.47	0.1789	44.910	197.76	202052.3	20877.50	5,47
Combin_2	196340.1	717.48	0.2135	331.63	223.34	138111.8	22150.2	10,78
	*C*8 [1]	*C*9 [1]	*C*10 [1]	*C*11 [1]	*C*12 [1]	*C*13 [1]		
Combin_1	199175.6	22350.7	46171.9	4544.7	24812.1	2067.45		
Combin_2	87232.2	9375.83	47018.6	4536.5	26164.1	2583.75		

6. Results interpretation

The best results give the HILL anisotropy material model (approximately 1%). The Chaboche and Multilinear isotropic material models give practically the same error. The Multilinear isotropic material model gives better results because it has only three parameters (the Chaboche model has 13.parameters). The tested model of prestress effect (basic on kinematics material models) didn't clarify behavior of analyzed experiments. The kinematic material models (KINH, Chaboche) cannot describe material behavior for very low number of cycles (two cycles), see [10] too.

7. Conclusion

1. The material parameters determined by the inverse method reflect the reality more exactly. The error of Hill model was approximately 1% for all tested experiments.
2. The inverse method proposed in this paper is applicable to other material models (has been tested 4 material models and 3, 9, 13 material parameters).
3. For the material parameters estimation is optimal using several independent set of data (e.g. combin_1, combin_2 - defined by two graphs). The material parameters

estimated from two experiments (tension, torsion) defined by one graph were set with error up to 1% for all tested material models, but for another tested experiments (combin_1, combin_2 , KINH model) was error greater then 10%.

4. The material parameters determined from the set of different experiments (tension/ compression axial force, torsion and their combination) has great variability. For the best – HILL model (error approximately 1%) it was up to 8%, for MISO model (error approximately 5%) it was up to 50%.
5. In the next step will be analyzed HILL anisotropy model (influence of single parameters, reduction of number of parameters) and the prestress effect or hardening effect in surface layer.

Acknowledgements

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