Experimentální Analýza Napětí 2010
Experimental Stress Analysis

Assessment of Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method

Adam Civín,[1] Miloš Vlk[2]

Abstract: This paper follows up an article Analysis of Calibration Coefficients of Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method [1], where the values of the calibration coefficients K_1 and K_2 have been determined in dependence on the depth of drilled hole and on the disposition of the homogenous residual state of stress. In this paper, the previously calculated values of the calibration coefficients are compared with the experimental measurements published by other authors and interposed by curve of an appropriate polynomial order. Confrontation of differential approach $d\varepsilon/dz$ vs. using differences $\Delta\varepsilon/\Delta dz$ to calculate relaxation coefficients A, B for homogenous stress state is shown and compared with results published by other authors too.

Keywords: Ring-core method, Incremental strain method, Residual stress analysis, Calibration coefficients

1. Introduction

The ring-core method is the semi-destructive experimental method used for the evaluation of homogeneous and non-homogeneous residual stresses, acting over depth of drilled core. Therefore, the specimen is not totally destroyed during measurement and in many cases could be used for another application.

One of the applicable theory, which is based on the procedure of evaluating magnitude of the residual stress is called the incremental strain method. On the one hand, despite its great theoretical shortcoming which assumes that the measured deformations $d\varepsilon_x$ and $d\varepsilon_y$ are functions only of the residual stresses acting in the current depth z of drilled hole and do not depend on the previous increments dz and residual stresses, this method is still often used. On the other hand, relieved strains do not depend only on the stress acting within drilled layer but also on the geometric changes of the ring groove during deepening. In consequence of this, relaxations of strains are still continuing and grooving with drilled depth even if next step increment is stress free. For this reason, proposed theory purveys only approximate information about real state of stress and this method is not suitable for measurements, where the steep gradient of residual stress state occurs.

[1] *Ing. Adam Civín; Brno University of Technology, Institute of Solid Mechanics, Mechatronics and Biomechanics; Technická 2896/2, 616 69 Brno, Czech Republic; civin.adam@seznam.cz*
[2] *doc. Ing. Miloš Vlk, CSc.; Brno University of Technology, Institute of Solid Mechanics, Mechatronics and Biomechanics; Technická 2896/2, 616 69 Brno, Czech Republic; vlk@fme.vutbr.cz*

This paper follows up an article Analysis of Calibration Coefficients of Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method [1], where the values of the calibration coefficients K_1 and K_2 have been determined from magnitudes of released strains ε_x a ε_y on the surface of drilled core in dependence on the depth of hole and on the disposition of applied uniaxial and biaxial residual stress state. After then, calculated and plotted points have been interposed by curve of an appropriate polynomial order of the sixth degree. Have been confirmed too, that results based on the proposed theory give inaccurate information about the residual stress state acting in the deepest layers (i.e. for depth $z > 5\ mm$), caused by smaller sensitivity to relieved strains measured on the surface.

Determination of the calibration coefficients´ magnitude affected by the progressively changing or constant geometry of the annular groove´s bottom caused by cutting tool´s blunting is examined in publication [2]. Next, investigation of influence on magnitude of relieved strains and subsequent determination of the calibration coefficients K_1, K_2 for various thickness of FE-model has been considered. Differences between results obtained by calculation of relieved strains in the middle point and across the strain gauge´s measuring grid are discussed in publication [2] too.

2. Problem description

Like the integral method, the incremental strain method needs a set of depth-independent calibration coefficients, which are necessary for further residual stress determination by the ring-core method in this case. This paper deals with results of relieved strains and consequently determined calibration coefficients K_1, K_2 obtained under homogenous uniaxial residual stress state conditions, applied to the proper finite element model (Fig. 4 and 5). Previously calculated values of the calibration coefficients [1] are now compared with results published by other authors and used for new calculations of the relaxation coefficients A, B by method using differences in three different ways:

a) Relieved strains are measured every i^{th} step of drilled depth z_i and size of step´s difference Δz is always referred to the zero magnitude of previous step´s size ($z_{i-1} = 0\ mm$), described by Eq. (1) and demonstrated in Fig. 1:

$$\Delta z = z_i - z_{i-1} = z_i, \quad \text{for } i = 0 \div 40 \tag{1}$$

b) Relieved strains are measured every i^{th} step of drilled depth z_i and size of step´s difference Δz is always referred to the previous step´s size (z_{i-1}), described by Eq. (2) and demonstrated in Fig. 2:

$$\Delta z = z_i - z_{i-1} = 0.2\ mm, \quad \text{for } i = 0 \div 40 \tag{2}$$

c) Relieved strains are measured only at two different depths, therefore step´s difference Δz consist of two particular depths z_i and $2z_i$, described by Eq. (3) and demonstrated in Fig. 3:

$$\Delta z = 2z_i - z_i = z_i, \quad \text{for } z_i = 1;\ 2;\ 3;\ 4\ mm \tag{3}$$

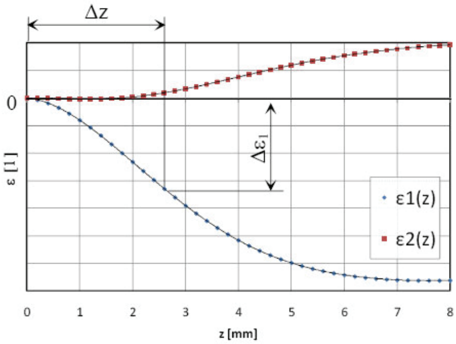


Fig. 1. Size of step´s difference *Δz* used for Eq. (1).

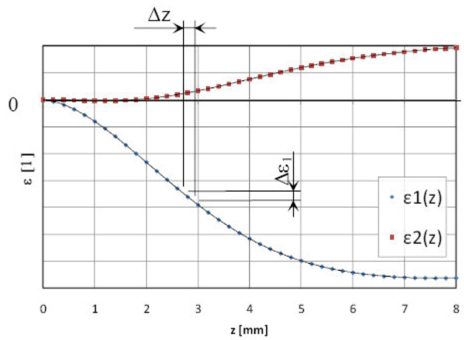


Fig. 2. Size of step´s difference *Δz* used for Eq. (2).

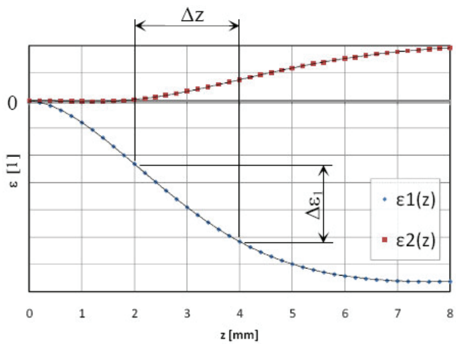


Fig. 3. Size of step´s difference *Δz* used for Eq. (3).

2.1. Basic equations

In general, differential (4, 5) and difference Eq. (6÷9) are used to express determination of the principal stresses σ_1 and σ_2 by the calibration coefficients K_1, K_2 and relaxation coefficients A and B, calculated from measured strains ε_1 and ε_2 on the top surface of the core, where the three-element ring-core rosette is placed.

With known magnitude of the calibration coefficient K_1, K_2 and numerical derivation of relaxed strains $d\varepsilon_1/dz$ and $d\varepsilon_2/dz$ in dependence on the specific magnitude of step´s increment dz could by residual stresses obtained by following equations:

$$\sigma_1 = \frac{E}{K_1^2 - \mu^2 K_2^2} \cdot \left(K_1 \frac{d\varepsilon_1}{dz} + \mu \cdot K_2 \frac{d\varepsilon_2}{dz} \right) \tag{4}$$

$$\sigma_2 = \frac{E}{K_1^2 - \mu^2 K_2^2} \cdot \left(K_1 \frac{d\varepsilon_2}{dz} + \mu \cdot K_2 \frac{d\varepsilon_1}{dz} \right) \tag{5}$$

assuming that $\frac{d\varepsilon}{dz} \approx \frac{\Delta\varepsilon}{\Delta z}$ and $\Delta z = z_i - z_{i-1}$, then:

$$\sigma_1 = \frac{E}{K_1^2 - \mu^2 K_2^2} \cdot \frac{1}{\Delta z} \cdot \left(K_1 \Delta\varepsilon_1 + \mu \cdot K_2 \Delta\varepsilon_2\right) \tag{6}$$

$$\sigma_2 = \frac{E}{K_1^2 - \mu^2 K_2^2} \cdot \frac{1}{\Delta z} \cdot \left(K_1 \Delta\varepsilon_2 + \mu \cdot K_2 \Delta\varepsilon_1\right) \tag{7}$$

$$\Delta\varepsilon_1 = (\varepsilon_1)_{zi} - (\varepsilon_1)_{zi-1} \,;\ \Delta\varepsilon_2 = (\varepsilon_2)_{zi} - (\varepsilon_2)_{zi-1} \tag{8, 9}$$

where E is Young´s modulus, μ is Poisson´s ratio and z_i is depth of the i^{th} drilled step.

Attention should be paid to formulations suggested by Eq. (4÷7). Because, if the denominator $K_1^2 - \mu^2 K_2^2$ becomes zero, for certain values of K_1 and K_2, the stress will become infinite. Further, Eq. (4÷9) are used to derive valid equations for uniaxial and biaxial state of stress.

In case of uniaxial state of stress are equations for the calibration coefficients K_1, K_2 ($\sigma_1 \neq 0$, $\sigma_2 = 0$) described by:

$$K_1 = \frac{E}{\sigma_1} \cdot \frac{d\varepsilon_1}{dz} \,;\ K_2 = -\frac{E}{\mu \cdot \sigma_1} \cdot \frac{d\varepsilon_2}{dz} \tag{10, 11}$$

Confrontation of the calibration coefficients K_1, K_2 and relaxation coefficients A, B:

$$A = \frac{E \cdot K_1}{K_1^2 - \mu^2 K_2^2} \cdot \frac{1}{\Delta z} \,;\ B = \frac{E \cdot \mu \cdot K_2}{K_1^2 - \mu^2 K_2^2} \cdot \frac{1}{\Delta z} = \mathrm{A} \cdot \mu \cdot \frac{\mathrm{K}_2}{\mathrm{K}_1} \tag{12, 13}$$

if $\overline{\varepsilon_1} = \frac{\sigma_1}{E}$ and $\Delta\varepsilon_1^* = \frac{\Delta\varepsilon_1}{\overline{\varepsilon_1}}$; $\Delta\varepsilon_2^* = \frac{\Delta\varepsilon_2}{\mu \cdot \overline{\varepsilon_1}}$ then:

$$K_1 = \frac{\Delta\varepsilon_1^*}{\Delta z} \,;\ K_2 = -\frac{\Delta\varepsilon_2^*}{\Delta z} \tag{14, 15}$$

Now, combining Eq. (14, 15) with Eq. (12, 13) we obtain another way how to determine relaxation coefficients A, B:

$$A = \frac{E \cdot \frac{\Delta\varepsilon_1^*}{\Delta z}}{\frac{1}{(\Delta z)^2} \cdot \left[\left(\Delta\varepsilon_1^*\right)^2 - \left(\mu \cdot \Delta\varepsilon_2^*\right)^2\right] \cdot \Delta z} = \frac{\mathrm{E} \cdot \Delta\varepsilon_1^*}{\left(\Delta\varepsilon_1^*\right)^2 - \left(\mu \cdot \Delta\varepsilon_2^*\right)^2} \tag{16}$$

$$B = -\frac{E \cdot \mu \cdot \frac{\Delta\varepsilon_2^*}{\Delta z}}{\frac{1}{(\Delta z)^2} \cdot \left[\left(\Delta\varepsilon_1^*\right)^2 - \left(\mu \cdot \Delta\varepsilon_2^*\right)^2\right] \cdot \Delta z} = -\frac{\mathrm{E} \cdot \mu \cdot \Delta\varepsilon_2^*}{\left(\Delta\varepsilon_1^*\right)^2 - \left(\mu \cdot \Delta\varepsilon_2^*\right)^2} \tag{17}$$

Finally, residual stresses determination by using relaxation coefficients A, B:

$$\sigma_1 = A \cdot \Delta\varepsilon_1 + B \cdot \Delta\varepsilon_2 \,;\ \sigma_2 = A \cdot \Delta\varepsilon_2 + B \cdot \Delta\varepsilon_1 \tag{18, 19}$$

3. Results

Simulation by FEM is the only reasonable way how to obtain desired information or how to simulate real experiment. Analysis system called ANSYS is used for FE-simulation. FE-analysis is based on a specimen volume with dimensions of *50 x 50 mm* and thickness of *50 mm*. Due to symmetry, only a quarter has been modeled with centre of the core on the surface as the origin. Shape of the model is simply represented by block with planar faces with quarter off drilled annular groove (Fig. 4 and 5). The annular groove has been made by *40* increments with step´s size of $\Delta z = 0.2\ mm$ in case of approach described by Eq. (1, 2) and by two particular increments in specific depth z_i and $2z_i$, Eq. (3). Maximum or the full depth of drilled groove is $z = 8\ mm$. Dimension of outer diameter is 18 mm and width of groove is *2 mm*.

Linear, elastic and isotropic material model is used with material properties of Young's modulus *210 GPa* and Poisson´s ratio $\mu = 0.3$. Relaxed strains ε_1 and ε_2 have been measured at real positions of strain gauge rosettes´ measuring grids by integration across its surface. Length and width of each measuring grid is *5 mm* and *1.9 mm* respectively.

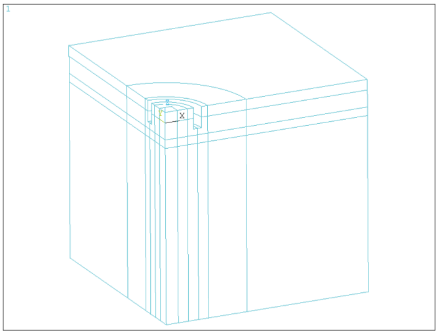


Fig. 4. Quarter of global model.

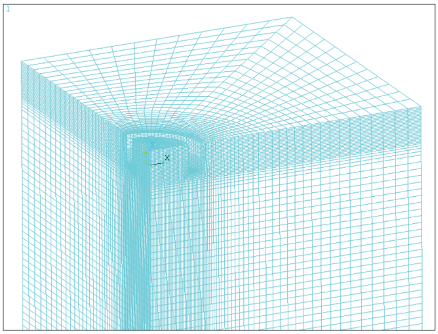


Fig. 5. Model´s global finite element mesh.

3.1. Uniaxial state of stress [1]

Released strains on the top of the core are obtained by the FE-analysis. Application of the general-purposed finite element model in order to simulate and evaluate both calibration coefficients K_1 and K_2 has been made. The uniaxial state of stress ($\sigma_1 = 60\ MPa$, $\sigma_2 = 0\ MPa$) in order to verify basic equations and theoretical approach proposed by this method is considered too.

In Fig. 6 through 8 are plotted graphs of relaxed strains calculated across strain gauge´s measuring grid, their numerical derivation and determined coefficients K_1 and K_2. Curves made by points in Fig. 8 represent values of coefficients K_1 and K_2 in dependence on the drilled depth and interposed by polynomial function of the sixth degree (Table 1). Entire hole was made by *40* increments of step´s size $\Delta z = 0.2\ mm$.

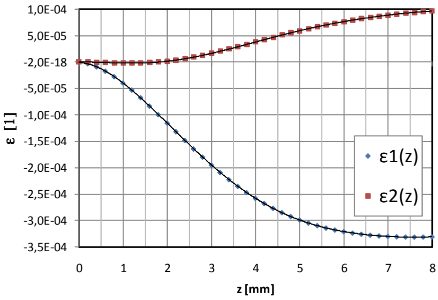


Fig. 6. Relieved strains for uniaxial stress state.

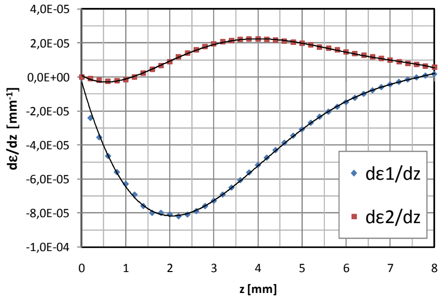


Fig. 7. Numerical derivation of relieved strains.

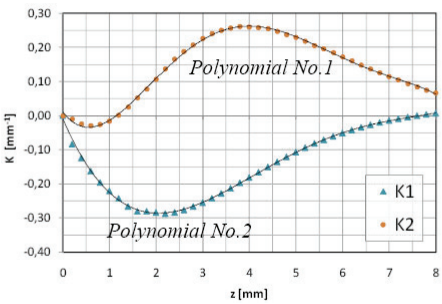


Fig. 8. Calibration coefficients K_1 and K_2.

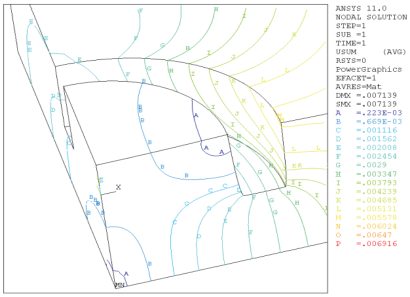


Fig. 9. Displacement vector sum calculated by FEM for σ_1 = *60 MPa*, σ_2 = *0 MPa*.

Typical behavior of both polynomial functions still remains the same for various magnitude of uniaxial stress [3-5]. Full release of strains is obtained approximately in depth about $z = 5\ mm$ (Fig. 6), but during next deepening, relieved strains are still growing. For this reason, proposed theory purveys only approximate information about real state of stress for measurements deeper than *5 mm*.

Table 1. Coefficients of polynomial equations

Polynomial No.:	**Coefficients [1]**						
	a_0	**a_1**	**a_2**	**a_3**	**a_4**	**a_5**	**a_6**
1	0.0099725	-0.1754683	0.2010887	-0.0541444	0.0051261	-0.0000723	-0.0000088
2	-0.010818	-0.3173851	0.1148700	-0.0131261	0.0002691	0.0000356	-0.0000013

$$K = a_0 + a_1 z^1 + a_2 z^2 + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 + a_6 z^6 \tag{20}$$

3.2. Relaxation functions

Values used for determination of relaxation coefficients *A, B* for homogenous uniaxial stress state ($\sigma_1 = 60\ MPa$, $\sigma_2 = 0\ MPa$), by method using differences $\Delta\varepsilon/\Delta z$

are measured across strain gauge´s measuring grid and demonstrated in Table 2. Relieved strains are measured at two different depths, therefore step´s difference Δz consist of two particular depths z_i and $2z_i$ (Fig. 3), described by Eq. (3).

Table 2. Constants for the relaxation coefficients´ determination

z_i [mm]	$\Delta\varepsilon_1$ [1]	$\Delta\varepsilon_2$ [1]	$\Delta\varepsilon_1$* [1]	$\Delta\varepsilon_2$* [1]	A [MPa]	B [MPa]	σ [MPa]
1 **2**	-7.560E-05	3.350E-06	-2.646E-01	3.908E-02	**-7.952E+05**	**-1.175E+05**	59.72
2 **4**	-1.421E-04	3.645E-05	-4.973E-01	4.253E-01	**-4.520E+05**	**-1.160E+05**	60.00
3 **6**	-1.258E-04	6.016E-05	-4.404E-01	7.018E-01	**-6.180E+05**	**-2.954E+05**	60.00
4 **8**	-7.291E-05	5.853E-05	-2.552E-01	6.829E-01	**-2.314E+06**	**-1.858E+06**	60.00

Magnitudes of the principal residual stresses are computed by Eq. (18, 19), relaxation functions are determined by Eq. (16, 17). Values of relieved strains measured by the incremental strain method only in two specific depths give very accurate results. Magnitudes of the relaxation coefficients *A, B* can be compared with results published by Bohdan [3], Fig. 13.

3.3. Results published by other autors

Graphs and tables describing magnitudes of the calibration coefficients K_1, K_2 and relaxation functions *A, B* published by authors [3-5] are shown in Figures 10 through 17. Keil [4] and Wolf [5] used Poisson´s ratio $\mu = 0.283$ which can cause mean differences between plotted values. In many cases are results consistent with curves plotted in Fig. 8.

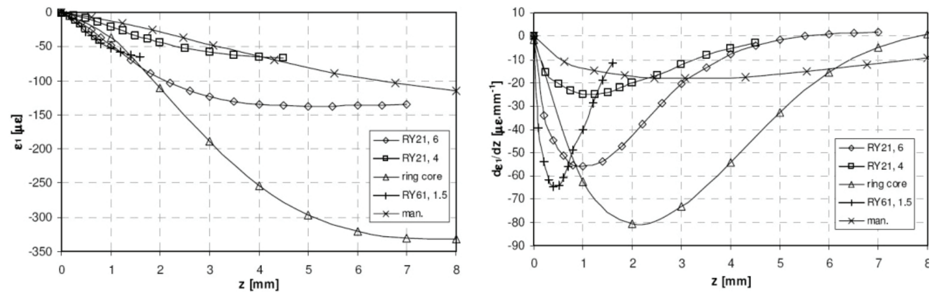


Fig. 10 and 11. Comparison of relieved strains and their derivatives for uniaxial stress $\sigma_1 = 60\ MPa$ Bohdan [3].

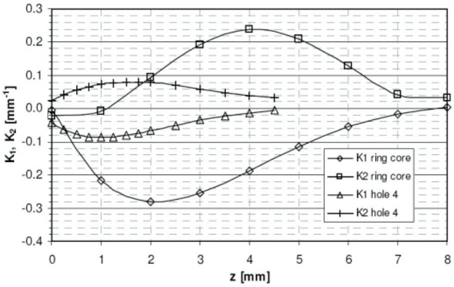
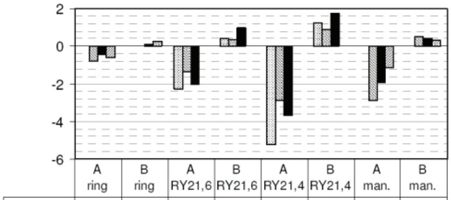


Fig. 12. Calculated calibration coefficients K_1 and K_2: Bohdan [3].



	A ring	B ring	A RY21,6	B RY21,6	A RY21,4	B RY21,4	A man.	B man.
0.5-1 mm			-2.2783	0.4386	-5.2464	1.2171		
1-2 mm	-0.8216	0.0291	-1.3418	0.3477	-2.8920	0.8579	-2.9035	0.5280
2-4 mm	-0.4444	0.1074	-2.0368	0.9575	-3.6742	1.7686	-1.9299	0.4249
3-6 mm	-0.5814	0.2669					-1.1644	0.3113

Fig. 13. Calculated relaxation coefficients *A, B*: Bohdan [3].

4. Conclusion

This paper provides basic information about semi-destructive ring-core method. By using incremental strain method for residual state of stress determination, this article gave global view about residual stress determination of homogenous and uniaxial state of stress.

Theoretical background described by basic equations and plotted values of calibration coefficients K_1, K_2 and relaxation functions *A, B* is presented. Calculated values are compared with other authors´ results too.

By concentrating the research on the observed weaknesses and the ambiguous details the ring-core method can be made an accurate and reliable method for residual stress measurement.

References

[1] Civín A. and Vlk M., "Analysis of Calibration Coefficients for Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method," in *Applied Mechanics 2010*, Jablonec nad Nisou, April 2010.

[2] Civín A. and Vlk M., "New Information about Incremental Strain Method Used for Residual Stress Measurement by Ring-Core Method," in *Engineering Mechanics 2010*, Svratka, May 2010.

[3] Bohdan P., Holý S., Jankovec J., Jaroš P., Václavík J. and Weinberg O., "Residual Stress Measurement Using Ring-Core Method," in *46th International Scietific Conference on Experimental Stress Analysis 2008*, Horní Bečva, June 2008.

[4] Keil S., "On-line evaluation of measurement results during the determination of residual stress using strain gages," *HBM – Reports in Applied Measurement,* **9**(1), pp. 15-20 (1995). ISSN 1614-9912.

[5] Wolf H., Borgmann H. and Stücker E., "Weiterentwicklung des Ring-Kern-Verfahrens zur Messung von Eigenspannungen," *Archiv für das Eisenhüttenwesen*, Nr.5, May, pp. 369-373 (1973).

Full version of this article is published on CD-ROM.