

## An Industrial Application of a Method for Spatial Motion Measurement

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**Abstract:** This paper deals with 3D motion measurement of a washing unit realized on an industrial washer-extractor, which was realized on the basis of a demand from industry. The inductive standard displacement transducers were used in the measurement. With respect to the number of degrees of freedom in 3D six axial displacement transducers were used. A new displacement transducers arrangement was suggested to minimize measuring error. The basic principles of measurement are briefly described too. Acquired results were compared with results from simple displacement measurement conducted at choice points of the industrial washer-extractor. A validation measurement by the 7th displacement transducer was also realized and very good correspondence was found. The proposed measurement method makes possible relatively easy and cheap real-time capture of the current position of vibrated constructional parts.

**Keywords:** Measurement Method, Spatial Motion, Washer-Extractor

### 1. Introduction

Vibrating constructional parts appear relatively often practically in all branches of the machine industry. In many cases it is necessary to find the maximal displacement magnitude or ideally the complete limits of spatial motion under real service conditions. For the solution of this task an optical method can be used, for example the digital image correlation method [1]. The second possibility is usage of a special accelerometer [2], but in this case the evaluation of measurement is complicated by the necessity of displacement record integration to get time dependence of displacements. The third way is usage of sufficient number of axial displacement transducers. The last possibility will be discussed in this paper.

### 2. Measurement Method and Evaluation

The number of degrees of freedom of a rigid body is generally equal to six. Thus, we can suppose that at least six transducers are necessary to capture position of a rigid body. For illustration of the main ideas we can consider the arrangement of six

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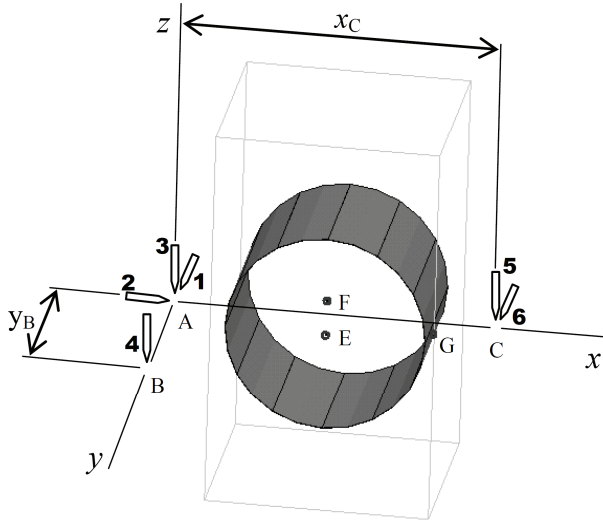
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displacement transducers shown at the Fig. 1. This arrangement makes possible the spatial motion measurement by the idea of capturing the position of three perpendicular planes which are coupled with the body.

Under assumption of the Cartesian coordinate system (Fig. 1) we can suppose that the distances  $x_C$  and  $y_B$  between transducers are known. The positions of points from investigated planes  $y_1, x_2, z_3, z_4, z_5, y_6$  correspond to the displacements measured by displacement transducers 1-6. Other necessary coordinates of currently investigated points on planes are given by the original position of transducers.



**Fig. 1.** Sample arrangement of six displacement transducers, which makes possible the measurement of spatial motion of a body.

The current position of a plane can be expressed by the general equation of plane

$$a \cdot x + b \cdot y + c \cdot z + d = 0 \quad (1)$$

All three investigated planes have own coefficients  $a, b, c$  which correspond to directional cosine of normal and the parameter  $d$ . All these parameters are dependent on the current displacements measured by transducers  $y_1, x_2, z_3, z_4, z_5, y_6$  and the distances  $x_C$  and  $y_B$ . To identify the first plane (initially plane  $x$ - $y$  at the Fig. 1) it is necessary to know the current position of three points from the transducers 3 (coordinate  $z_3$ ), 4 ( $z_4$ ) and 5 ( $z_5$ ). The equation of plane is then given by

$$\begin{vmatrix} x & y & z & 1 \\ 0 & 0 & z_3 & 1 \\ 0 & y_B & z_4 & 1 \\ x_C & 0 & z_5 & 1 \end{vmatrix} = 0. \quad (2)$$

Comparing the equation (2) and (1) we get coefficients  $a_1, b_1, c_1, d_1$ . The position of the second plane (initially  $x$ - $z$ ) can be derived from displacements obtained from two transducers only, the transducers 1 and 6, because the second plane has to be perpendicular to the first plane. It leads to

$$\begin{vmatrix} x & y - y_1 & z \\ x_C & y_6 - y_1 & 0 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0. \quad (3)$$

Using equation (3) coefficients  $a_2, b_2, c_2, d_2$  are computed. The third plane (initially  $y$ - $z$ ) has to be perpendicular to the two previous planes. At the same time the coordinate  $x_2$  is known as the displacement obtained from the transducer 2. Thus we know also one point of the third plane and it can be written

$$\begin{vmatrix} x - x_2 & y & z \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0. \quad (4)$$

All coefficients  $a_i, b_i, c_i, d_i$  should be normalized, so they are divided by

$$\alpha = \sqrt{a_i^2 + b_i^2 + c_i^2}. \quad (5)$$

Finally it is possible to find the current position of a body point (for example point  $G$ ) using the well known transformation relations

$$\begin{aligned} x_G &= a_3 \cdot x_{G0} + a_2 \cdot y_{G0} + a_1 \cdot z_{G0} + x_0, \\ y_G &= b_3 \cdot x_{G0} + b_2 \cdot y_{G0} + b_1 \cdot z_{G0} + y_0, \\ z_G &= c_3 \cdot x_{G0} + c_2 \cdot y_{G0} + c_1 \cdot z_{G0} + z_0, \end{aligned} \quad (6)$$

where  $x_0, y_0, z_0$  are current coordinates of the point of intersection of identified three planes and  $x_{G0}, y_{G0}, z_{G0}$  are initial coordinates of the investigated point  $G$ . It can be considered that the equations of all planes have to be satisfied for the point of intersection too, what leads to the system of algebraic equations

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} -d_1 \\ -d_2 \\ -d_3 \end{bmatrix} \quad (7)$$

for the calculation of coordinates  $x_0, y_0, z_0$ .

The six displacement transducers can be placed to the different positions than stated at the Fig. 1, but it is useful to be three transducers used for identification of the first plane, two transducers for measurement of the second plane and the last one for the third plane. Therefore the equations (2) to (4) have to be reformulated. In the proposed strategy it is necessary to prepare or create three measured planes on the investigated body.

### 3. Industrial Application

On the basis of commission for 3D motion measurement of washing unit of a industrial washer-extractor the proposed methodology has been verified in practice. The main task was critical extraction speed determination with absolute maximum of displacement in the case of 2kg weighting.

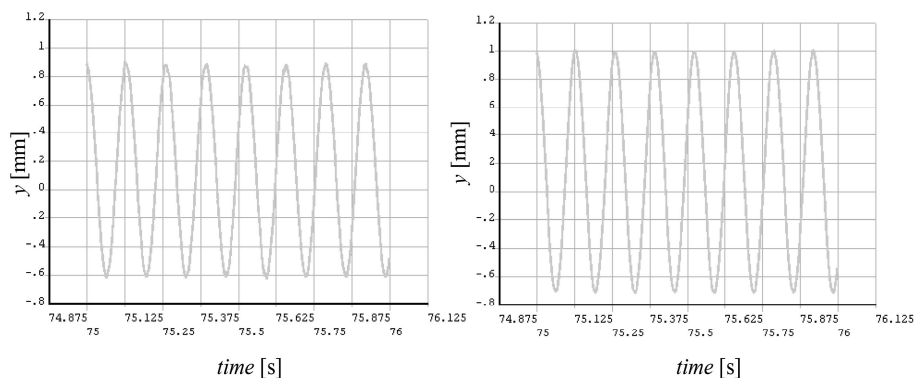
#### 3.1. Measurement Description

The inductive standard displacement transducers were used in the measurement. The suitable displacement transducers arrangement was suggested to minimize measuring error. Three light-weight aluminium-base alloy bars have been glued to the washing unit to create measuring places on three perpendicular planes. Whole washing cycle including starting and run-out of drum was measured. Data recorded using LabVIEW8.2 software were imported into the FE program Ansys 11, which was used for evaluation [3].

#### 3.2. Verification of used methodology

Proposed methodology was verified using the control measurement with the seventh displacement transducer (see Fig. 2) and the standard measurement method performed by Primus CE Company. The trajectory of point F obtained by the common measurement method and the proposed measurement method is shown at the Fig. 3. All experimental results show good agreement.

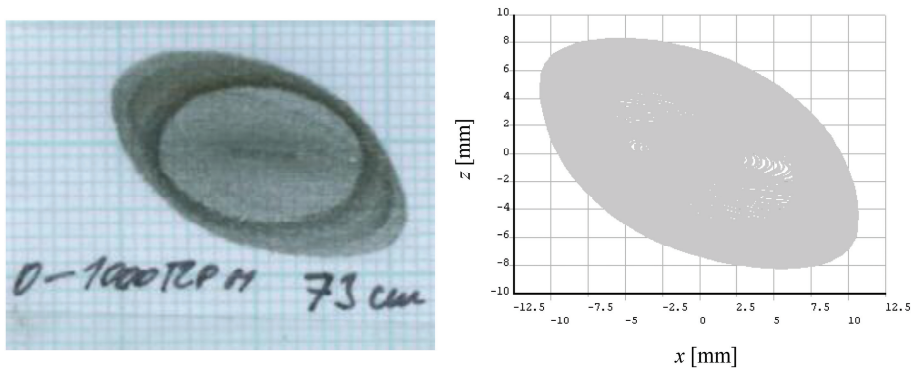
Other results from the verification measurements are reported in the full text of the paper [4] and the paper [5].



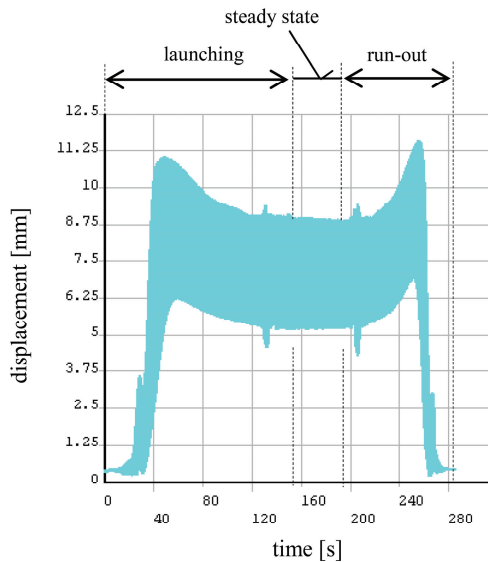
**Fig. 2.** Displacement measured by the seventh transducer (left) and evaluated using proposed method (point E, 450 revolutions per minute, steady state).

#### 3.3. Main Results

The maximal total displacement magnitude was gained in run-out stage for all evaluated points. An example can be seen at the Fig. 4. It was recognized, that critical extraction speed of the washer-extractor is about 300 revolutions per minute. The corresponding maximal value of displacement was 16 millimetres.



**Fig. 3.** Comparison of the trajectory of the point F obtained by the common (left) and the proposed measurement method (0-1000 revolutions per minute, complete washing cycle).



**Fig. 4.** A sample record of displacement (0-1000 revolutions per minute, complete washing cycle).

#### 4. Conclusions

This paper describes basic principles of the measurement method of spatial motion of a rigid body, which come from the idea of six axial displacement transducers usage. The methodology does not depend on the type of used displacement transducers. It is possible to use standard inductive, optical or ultrasonic transducers. The benefit of the proposed method is the fact that there are no simplifications made in the evaluation procedure. Described methodology leads to the real-time measurement of the current position of rigid mechanical parts or structures. The

method was verified by the measurement of spatial motion of the washing unit of the industrial washer-extractor. The interesting future application could be optimalization of dampers or development of active damping systems [6].

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