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Experimental Stress Analysis

Theory of Stress State Identification After Hole-Drilling Method Application

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Abstract: The theory reported here increases applicability of the hole drilling principle for the stress state identification. The new theory is proposed for the stress state identification in the surface at the place of already drilled holes with a complete drilling rosette equipment already installed either centrically or even eccentrically. The method thus allows a further reusing of already installed measuring items, which were originally placed there for the residual stress state identification, for measurements of the stress states induced by any following external loading as if the hole had not been drilled at all.

Keywords: Stress, Identification, Hole, Drilling

1. Introduction

The hole-drilling experimental method for stress state identification results in a small cylindrical hole drilled into the component surface. The strain-gauge rosette is placed around the hole usually. The rosette strain-gauges placed on the component surface capture strains in vicinity of the hole and the rosette stays fully functional even after the hole-drilling process ends. Though, the hole-drilling theory principle [2, 3] is applicable only for determination of stresses existing at the place of the hole before the hole-drilling process. A repeated application of the same rosette already installed for any next measurement would be therefore useful. A theory allowing the analysis of following stress changes at the same point is desirable. We consider the stress state changes that could be induced during the hole-drilling method application as neglectable, as if the hole had not been drilled. We follow the hole-drilling theory previously published [4, 5, 6] with the hole eccentricity effect included.

2. The drilling hole theory for stress state identification

For the experimental stress state identification on surfaces of loaded components, the semi-destructive hole-drilling principle uses the similitude of a thin Kirch's plate elastic model with a small straight-through hole with radius R_0 lead perpendicularly

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to the surface. Fig. 1 shows the Kirch's plate loaded uniaxially by σ_x principal stress and defines the relative radius $r = R/R_0 \geq 1$ for the general point r, α positioning reported in the polar coordinate system. The Kirch's theory [1, 2] sets $\sigma_r, \sigma_\theta, \tau_{r\theta}$ stresses at planes through the general point according to Eq. (1).

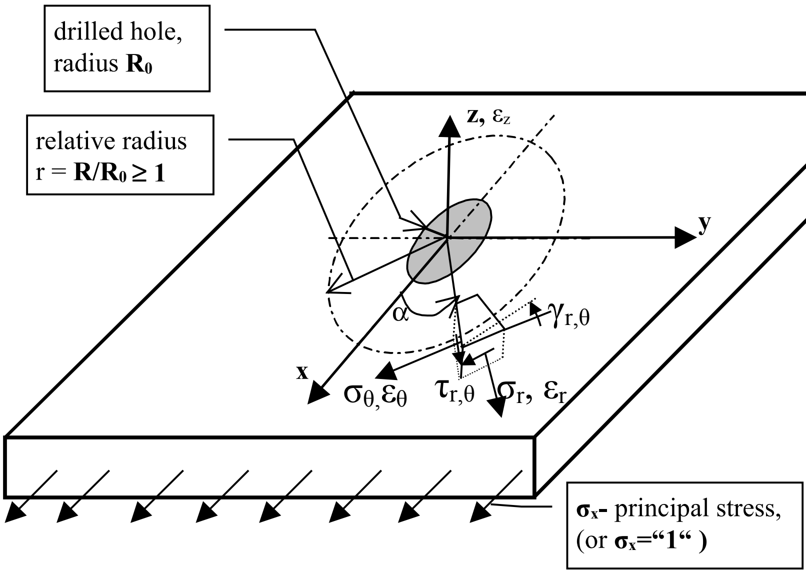

Fig. 1. Description of the stress state in the hole vicinity.

$$\begin{Bmatrix} \sigma_r = \dfrac{\sigma_x}{2}(1 - \dfrac{1}{r^2}) + \dfrac{\sigma_x}{2}(1 + \dfrac{3}{r^4} - \dfrac{4}{r^2}) \cos 2\alpha \\ \sigma_\theta = \dfrac{\sigma_x}{2}(1 + \dfrac{1}{r^2}) - \dfrac{\sigma_x}{2}(1 + \dfrac{3}{r^4}) \cos 2\alpha \\ \tau_{r\theta} = \dfrac{\sigma_x}{2}(1 - \dfrac{3}{r^4} + \dfrac{2}{r^2}) \sin 2\alpha \end{Bmatrix} \tag{1}$$

The potential stress change manifests itself in the surface layer around the drilled hole by a measurable strain change, which can be calibrated in advance. Hooke's law in Eq. (2) allows expression of $\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}$ and ε_z strains changes in the straight hole through the plate from the $\sigma_r, \sigma_\theta, \tau_{r\theta}$ plane stress changes, Young's modulus E and Poisson's ratio ν. The similitude between the plane stress state at the body surface and the stressing of the thin Kirch's plate with a straight-through hole can be used in the hole-drilling principle for stress-state analyses [5, 6] (as well as in the hole-drilling method supported by E 837 international standard [3]). The individual components reported in Eq. (1) have to be nevertheless modified by multiplication by twelve different constants $c_{21}, c_{22},..., c_{32}$ according to Eq. (3). The multipliers rectify the stress state for the real conditions of the bottom hole with a perpendicular direction to the free surface. Thus, e.g. for the thin Kirch's plate the stress state directly corresponds with the Eq. (1) thanks to all twelve constants equal to one, i.e. $c_k(r,h) = 1$. In the case of the bottom hole, the $c_{21}, c_{22},..., c_{32}$ constants depend first on the distance from the hole center described by the r radius and, second, on the h depth of the hole.

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma_{r\theta}\\ \varepsilon_z\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\\ -\nu & -\nu & 0\end{bmatrix}\cdot\begin{bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{bmatrix} =$$

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma_{r\theta}\\ \varepsilon_z\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & 0\\ -\nu & 1 & 0\\ 0 & 0 & 2(1+\nu)\\ -\nu & -\nu & 0\end{bmatrix}\cdot\begin{Bmatrix}\frac{\sigma_x}{2}(1-\frac{1}{r^2}) + \frac{\sigma_x}{2}(1+\frac{3}{r^4}-\frac{4}{r^2})\cos 2\alpha\\ \frac{\sigma_x}{2}(1+\frac{1}{r^2}) - \frac{\sigma_x}{2}(1+\frac{3}{r^4})\cos 2\alpha\\ \frac{\sigma_x}{2}(1-\frac{3}{r^4}+\frac{2}{r^2})\sin 2\alpha\end{Bmatrix} =$$

$$\begin{bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \gamma_{r\theta}\\ \varepsilon_z\end{bmatrix} = \frac{\sigma_x}{2E}\begin{bmatrix}\left\{\left[1-\nu-\frac{1}{r^2}-\frac{1}{r^2}\nu\right]+\left[1+\nu+\frac{1}{r^4}3+\frac{1}{r^4}3\nu-\frac{1}{r^2}4\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[1-\nu+\frac{1}{r^2}+\frac{1}{r^2}\nu\right]-\left[1+\nu+\frac{1}{r^4}3+\frac{1}{r^4}3\nu-\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\\ \left\{\left[1+\nu-\frac{1}{r^4}3+\frac{1}{r^2}2-\frac{1}{r^4}3\nu+\frac{1}{r^2}2\nu\right]\cdot 2\cdot\sin 2\alpha\right\}\\ \left\{\left[-2\nu\right]+\left[-2\nu+\frac{1}{r^2}4\nu\right]\cdot\cos 2\alpha\right\}\end{bmatrix} \tag{2}$$

$$\begin{cases}\sigma_r(\sigma_x) = \frac{\sigma_x}{2}(1\cdot c_{28} - \frac{1\cdot c_{21}(r,z)}{r^2}) + \frac{\sigma_x}{2}(1\cdot c_{29} + \frac{3\cdot c_{22}(r,z)}{r^4} - \frac{4\cdot c_{23}(r,z)}{r^2})\cos 2\alpha\\ \sigma_\theta(\sigma_x) = \frac{\sigma_x}{2}(1\cdot c_{30} + \frac{1\cdot c_{24}(r,z)}{r^2}) - \frac{\sigma_x}{2}(1\cdot c_{31} + \frac{3\cdot c_{25}(r,z)}{r^4})\cos 2\alpha\\ \tau_{r\theta}(\sigma_x) = \frac{\sigma_x}{2}(1\cdot c_{32} - \frac{3\cdot c_{26}(r,z)}{r^4} + \frac{2\cdot c_{27}(r,z)}{r^2})\sin 2\alpha\end{cases} \tag{3}$$

$$\begin{bmatrix} \varepsilon_r(\sigma_x) \\ \varepsilon_\theta(\sigma_x) \\ \gamma_{r\theta}(\sigma_x) \\ \varepsilon_z(\sigma_x) \end{bmatrix} = \frac{\sigma_x}{2E} \begin{bmatrix} \left\{ \left[c_{28} - c_{30}\nu - \frac{c_{21}}{r^2} - \frac{c_{24}}{r^2}\nu \right] + \left[c_{29} + c_{31}\nu + \frac{c_{22}}{r^4}3 + \frac{c_{25}}{r^4}3\nu - \frac{c_{23}}{r^2}4 \right] \cdot \cos 2\alpha \right\} \\ \left\{ \left[c_{30} - c_{28}\nu + \frac{c_{24}}{r^2} + \frac{c_{21}}{r^2}\nu \right] - \left[c_{31} + c_{29}\nu + \frac{c_{25}}{r^4}3 + \frac{c_{22}}{r^4}3\nu - \frac{c_{23}}{r^2}4\nu \right] \cdot \cos 2\alpha \right\} \\ \left\{ \left[c_{32} + c_{32}\nu - \frac{c_{26}}{r^4}3 + \frac{c_{27}}{r^2}2 - \frac{c_{26}}{r^4}3\nu + \frac{c_{27}}{r^2}2\nu \right] \cdot 2 \cdot \sin 2\alpha \right\} \\ \left\{ \left[-c_{28}\nu - c_{30}\nu + \frac{c_{21}}{r^2}\nu - \frac{c_{24}}{r^2}\nu \right] + \left[-c_{29}\nu + c_{31}\nu - \frac{c_{22}}{r^4}3\nu + \frac{c_{25}}{r^4}3\nu + \frac{c_{23}}{r^2}4\nu \right] \cdot \cos 2\alpha \right\} \end{bmatrix} \quad (4)$$

The complete components of the stress state changes in the component surface in the hole vicinity can be further transformed by Hooke's law in the analogy to Eq. (2) to the strain components given in Eq. (4). Such strains are directly measurable around the hole and the calibrated measured strain changes allow stress identification in the hole center as if the hole had not been there.

Let the strain gauge winding be oriented along the g direction (see Fig. 2) with acute φ angle from θ axis at the point described by coordinates r, θ (depicted in Fig. 1). The strain along the strain-gauge winding g, which is identified by the strain-gauge, results from ε_θ, ε_r and $\gamma_{r\theta}$ strains and is derived by the use of double 2φ angle through the Mohr's transformation (5).

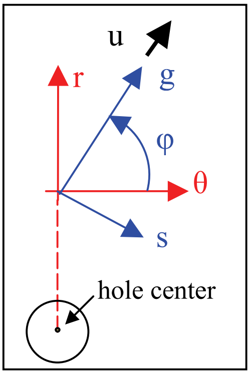

Fig. 2.

$$\varepsilon_g = \frac{\varepsilon_\theta + \varepsilon_r}{2} + \frac{\varepsilon_\theta - \varepsilon_r}{2}\cos 2\varphi + \frac{\gamma_{\theta,r}}{2}\sin 2\varphi \quad (5)$$

Fig. 1 shows the partially unit vector defined in the direction of σ_x principal stress in the angle α to the evaluated point, above which the $i-th$ strain gauge winding is positioned in the g direction (see Fig. 2). The curvilinear integral of the normalized strain transformed by Eq. (5) along the winding with total length u defines the $t_i(\alpha)$ strain-gauge sensitivity for σ_x principal stress in Eq. (6). A definition of the second sensitivity of this strain-gauge $t_i(\alpha + \pi/2)$ for σ_y principal stress rotated along the surface by $\pi/2$ from the σ_x stress direction follows a similar way. Both sensitivities of the $i-th$ strain gauge of the drilling rosette are functions of the theory constants $c_{21}, c_{22}, \ldots, c_{32}$ and particular positions and orientations r, α, g of points along the winding. The orientation and position of individual strain gauges is defined in accordance with E 837 standard [3], which postulates the hole drilled in the ideal center of the drilling rosette, defines the angle α parameter of a particular strain gauge to σ_x principal stress and derives the placing of resting strain gauges of the rosette from it. Here in this more general theory, the symbol $\overline{\alpha}$ marks the orientation of a defined base point of the $i-th$ strain gauge to σ_x principal stress. This $\overline{\alpha}$ angle of the $i-th$ strain gauge thus results from its relative position to the main strain gauge, which is inclined by α from σ_x principal stress.

$$t_i(\alpha) = \frac{\oint_u \overline{\varepsilon}_{g,j}(\alpha) \cdot du}{\oint_u du} \text{ nebo } \cdots t_i(\alpha + \pi/2) = \frac{\oint_u \overline{\varepsilon}_{g,j}(\alpha + \pi/2) \cdot du}{\oint_u du} \tag{6}$$

For unknown principal stresses σ_x, σ_y and angle α a set of at least three non-linear and independent equations can be established from the computed sensitivities of individual strain gauges in analogy to Eq. (7). The equations encompass the influence of both principal stresses on ε_i strains measured by strain gauges. A convenient way of their solution is reported in [5, 6].

$$\varepsilon_i = \sigma_x \cdot t_i(\overline{\alpha}) + \sigma_y \cdot t_i(\overline{\alpha} + \pi/2) \equiv \sigma_x \cdot t_{i,x}(\overline{\alpha}) + \sigma_y \cdot t_{i,y}(\overline{\alpha}) \tag{7}$$

A typical example of a use can be started from three signals ε_1, ε_2 and ε_3 of three independent strain gauges of the drilling rosette, we have the system of three non-linear equations (8).

$$\begin{Bmatrix} \varepsilon_1 = \left[\sigma_x \cdot t_{1,x}(\overline{\alpha}) + \sigma_y \cdot t_{1,y}(\overline{\alpha})\right] \\ \varepsilon_2 = \left[\sigma_x \cdot t_{2,x}(\overline{\alpha}) + \sigma_y \cdot t_{2,y}(\overline{\alpha})\right] \\ \varepsilon_3 = \left[\sigma_x \cdot t_{3,x}(\overline{\alpha}) + \sigma_y \cdot t_{3,y}(\overline{\alpha})\right] \end{Bmatrix} = \begin{bmatrix} t_{1,x}(\overline{\alpha}) & t_{1,y}(\overline{\alpha}) \\ t_{2,x}(\overline{\alpha}) & t_{2,y}(\overline{\alpha}) \\ t_{3,x}(\overline{\alpha}) & t_{3,y}(\overline{\alpha}) \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} \tag{8}$$

The first two equations serve for determination of unknown principal stresses σ_x and σ_y as a functions of ε_1 and ε_2 strain signals and of an unknown angular parameter $\overline{\alpha}$ defining the position of the principal stress σ_x according to Eq. (9).

$$\sigma_x = \frac{\varepsilon_1 \cdot t_{2,y}(\overline{\alpha}) - \varepsilon_2 \cdot t_{1,y}(\overline{\alpha})}{t_{1.x}(\overline{\alpha}) \cdot t_{2,y}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{1,y}(\overline{\alpha})} \quad , \quad \sigma_y = \frac{\varepsilon_2 \cdot t_{1,x}(\overline{\alpha}) - \varepsilon_1 \cdot t_{2,x}(\overline{\alpha})}{t_{1,x}(\overline{\alpha}) \cdot t_{2,y}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{1,y}(\overline{\alpha})} \tag{9}$$

The substitution of two Eqs. (9) for ε_3 to the third Eq. (8) allows the computation of $\overline{\alpha}$ parameter from Eq. (10), while the last substitution of $\overline{\alpha}$ back to Eq. (9) leads to σ_x and σ_y principal stresses.

$$\varepsilon_3 = \frac{\varepsilon_1 \cdot t_{2,y}(\overline{\alpha}) - \varepsilon_2 \cdot t_{1,y}(\overline{\alpha})}{t_{1,x}(\overline{\alpha}) \cdot t_{2,y}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{1,y}(\overline{\alpha})} \cdot t_{3,x}(\overline{\alpha}) + \frac{\varepsilon_2 \cdot t_{1,x}(\overline{\alpha}) - \varepsilon_1 \cdot t_{2,x}(\overline{\alpha})}{t_{1,x}(\overline{\alpha}) \cdot t_{2,y}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{1,y}(\overline{\alpha})} \cdot t_{3,y}(\overline{\alpha}) \cdots \Rightarrow$$

$$\left\{ \begin{aligned} &\varepsilon_1 \left(t_{2,y}(\overline{\alpha}) \cdot t_{3,x}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{3,y}(\overline{\alpha}) \right) + \\ &+ \varepsilon_2 \left(t_{1,x}(\overline{\alpha}) \cdot t_{3,y}(\overline{\alpha}) - t_{1,y}(\overline{\alpha}) \cdot t_{3,x}(\overline{\alpha}) \right) + \\ &+ \varepsilon_3 \left(t_{1,x}(\overline{\alpha}) \cdot t_{2,y}(\overline{\alpha}) - t_{2,x}(\overline{\alpha}) \cdot t_{1,y}(\overline{\alpha}) \right) = 0 \end{aligned} \right\} \tag{10}$$

Potentially, the regression model proposed here with 12 constants can be further simplified by decreasing the number of constants, if the simplified solution (e.g. with 7 constants) is objectivelly sufficient for stress state modelling in the drilled hole point or the drilled hole vicinity.

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References

[1] Timoshenko S., *Theory of Elasticity*. 1st ed. (McGraf-Hill Book Company, Inc., New York and London, 1934), pp. 52-81.

[2] Vishay Micro-Measurements, “Measurement of Residual Stresses by the Hole-Drilling Strain Gage Method,” Tech Note TN-503 (USA, 2007), Revision 09, pp. 19-33.

[3] ASTM Designation: E 837-01. *Standard Test Method for Determining Residual Stresses by the Hole-Drilling Strain Gage Method* (ASTM International, USA, updated in January 2002), pp. 1-10.

[4] Vítek K., Mareš T., Doubrava K., Sochor M. and Španiel M., “Original Experimental Method for Stress and Strain Identification Based on of the Hole Drilling Principle,” in *Proceedings of Danubia-Adria Symposium on Experimental Methods in Solid Mechanics*, Brijuni - Pula, Croatia, September 29-October 2 2004 (Croatian Society of Mechanics, Zagreb, 2004), pp. 62-63. ISBN 953-96243-6-3.

[5] Vítek K., “Stress State Identification By Numerical Simulation of the Hole Drilling Principle – Part A,” in *Proceedings of 46th International Scientific Conference Experimental Stress Analysis 2008*, Fuxa J. et al., eds., Horní Bečva, June 2008 (TU Ostrava, Ostrava, 2008), pp. 271–274. ISBN 978-80-248-1774-3.

[6] Vítek K., “Stress State Identification By Numerical Simulation of the Hole Drilling Principle – Part B”, in *Proceedings of 46th International Scientific Conference Experimental Stress Analysis 2008*, Fuxa J. et al., eds., Horní Bečva, June 2008 (TU Ostrava, Ostrava, 2008), pp. 275–278. ISBN 978-80-248-1774-3.