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An identification of material low cycle fatigue parameters

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Abstract: Attempts to find appropriate methods for identification of low cycle fatigue material parameters with lower efforts compared with classical ones are described in detail. All the works were done with an aim to shorten time for obtaining more precise result of experimental works. The numerical part of the process is based on an application of very effective nonlinear solver of systems of nonlinear equations for a least-squares problem rising from minimization of differences between a mathematical model and measurements.

Keywords: Fatigue, Low-cycle, Parameters, Identification, Hysteresis, Loops

1. Introduction

The phenomenon designated as a material fatigue has been known more than 170 years [1]. In spite of it, the problem has not been solved in the full complexity yet. Many areas of fatigue are solved separately. One of those areas is a low cycle fatigue (LCF). Parts of structures suffering from the low cycle fatigue are loaded in such a way that stresses originated in certain places, called critical, are higher than the yield stress limit of the material. The high level of repeating stress causes fast damage of the material in a low number of loading repetitions.

The fatigue of materials was investigated more than 60 years only experimentally with the aim to obtain reliable service lives of dynamically loaded parts of structures. The first man, who observed that the fatigue life, number of cycles to failure N_f, was an exponential function of stress amplitude σ_a, was Basquin [2]. He expressed his discovery into a simple equation

$$\sigma_a = \sigma_f' \, (2\, N_f)^b, \tag{1}$$

where σ_f' is a coefficient of cyclic fatigue strength, and b an exponent of cyclic fatigue strength. Almost half a century after Basquin, two gentlemen, Mr. Manson [3] and Mr. Coffin [4], discovered independently that a similar equation holds for a plastic strain amplitude:

$$\varepsilon_{ap} = \varepsilon_f' \, (2\, N_f)^c. \tag{2}$$

Variable ε_f' is a coefficient of cyclic fatigue ductility, and c an exponent of cyclic fatigue ductility. Thus, the set [σ_f', b, ε_f', c] of four independent LCF parameters describes the stress-strain behavior of a material under LCF loading.

2. Identification of material LCF parameters

Unknown LCF parameters gathered in column vector $\boldsymbol{p} = [\,\sigma_f', b, \varepsilon_f', c\,]^T$ can be obtained by processing histories $\boldsymbol{\sigma}(t)$ and $\boldsymbol{\varepsilon}(t)$ recorded either graphically on plotters or digitally in files during LCF tests. The mathematical procedures of identification processes are based on equations (1) and (2). The equations represent a mathematical model of the damaging process of the low cycle fatigue. For any vector of parameters $\tilde{\boldsymbol{p}}$, there are differences, residuals $\boldsymbol{r}$, between measurements and a mathematical model, to be minimized during the identification:

$$\boldsymbol{r} = \begin{bmatrix} \boldsymbol{r}_\sigma \\ \boldsymbol{r}_\varepsilon \end{bmatrix} = \begin{bmatrix} \boldsymbol{\sigma}(t) - \boldsymbol{\sigma}(t, \tilde{\boldsymbol{p}}) \\ \boldsymbol{\varepsilon}(t) - \boldsymbol{\varepsilon}(t, \tilde{\boldsymbol{p}}) \end{bmatrix}. \tag{3}$$

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2.1. Classical method

A classical method of identification of LCF parameters modifies a bit the system of equations (3) built out of n selected hysteresis loops measured on n tested specimens. The loops should be chosen from about half lifetime steady history of the specimen. The necessary data of each loop should be put into a table, columns of which create column data vectors $\boldsymbol{\sigma_a}$, $\boldsymbol{\varepsilon_{ap}}$ and $\boldsymbol{N_f}$ entering the identification procedure. Table Tab. 1 contains measured data from the LCF testing of the material ČSN 41 1523.1 performed and processed at VZÚ Plzeň [8]:

Table 1. Measured parameters of hysteresis loops of material ČSN 41 1523.1

Spec. No	σ_a [MPa]	ε_{at} [1e-3]	ε_{ap} [1e-3]	N_f
1	426	12.058	9.170	540
2	421	11.288	8.434	860
3	239	1.892	0.272	76000
4	282	3.789	1.877	14171
5	393	7.860	5.196	1443
7	246	2.128	0.461	60560
9	434	11.830	8.778	520

The exponential forms of equations (1) and (2) suggest that, after taking the logarithm of them, the resulting new equations become linear functions of logarithms:

$$\log_{10} \boldsymbol{\sigma_a} = \log_{10} \sigma_f' + b \, \log_{10} (2\, \boldsymbol{N_f}) \tag{4}$$

$$\log_{10} \boldsymbol{\varepsilon_{ap}} = \log_{10} \varepsilon_f' + c \, \log_{10} (2\, \boldsymbol{N_f}) \tag{5}$$

It is not very difficult to gather both equations into one matric equation for unknowns $\bar{\boldsymbol{p}} = [\, \log_{10} \sigma_f',\, b,\, \log_{10} \varepsilon_f',\, c]^T$ and with a vector full of ones, $\boldsymbol{j}$

$$\underbrace{\left[\ \boldsymbol{j} \quad \log_{10} (2\, \boldsymbol{N}_f)\ \right]}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \log_{10} \sigma_f' & \log_{10} \varepsilon_f' \\ b & c \end{bmatrix}}_{\boldsymbol{X}} = \underbrace{\left[\ \log_{10} \boldsymbol{\sigma}_a \quad \log_{10} \boldsymbol{\varepsilon}_{ap}\ \right]}_{\boldsymbol{B}}, \tag{6}$$

with the solution in the least squares sense, where $\boldsymbol{A}^+$ is the Moore-Penrose pseudoinverse:

$$\boldsymbol{X} = \boldsymbol{A}^+ \boldsymbol{B}\,. \tag{7}$$

Hence, the sought LCF parameters but $\boldsymbol{b}$ and $\boldsymbol{c}$ may be evaluated from the formulae

$$\sigma_f' = 10^{\log_{10} \sigma_f'} \qquad \text{and} \qquad \varepsilon_f' = 10^{\log_{10} \varepsilon_f'}\,. \tag{8}$$

2.2. New method

The advantage of the classical method is in its simplicity. It does not need anything more than measured data. The only uncertainty lies in a definition of the number of cycles to failure N_f. Unfortunately, it happens quite often that a developing crack changes stiffness of a specimen in such a way that the hysteresis loop becomes non-symmetric, and a service life of such a modified specimen becomes much higher. If such N_f be accepted, the identified LCF parameters become biased. This has been a reason why a new method not dependent on N_f has been searched.

Let us eliminate the numbers of cycles to failure N_f from equations (1) and (2) by expressing the term $(2\, N_f)$ out of both equations. After this step, a single equation is born from the identity of left-hand sides:

$$\sigma_a = \sigma_f' \left(\frac{\varepsilon_{ap}}{\varepsilon_f'} \right)^{n'} = K' \, {\varepsilon_{ap}}^{n'}, \tag{9}$$

where $K' = \frac{\sigma_f'}{{\varepsilon_f'}^{n'}}$ is a cyclic strength coefficient, and $n' = \frac{b}{c}$ a cyclic strain hardening exponent.

It is obvious that equation (9) has reduced the number of independent parameters, describing the functional bond between σ_a and ε_{ap}, to only two. Stress and total strain processes are sampled periodically in time by a sampling period T producing thus two vectors, time series $\boldsymbol{\sigma}(kT)$ and $\boldsymbol{\varepsilon_t}(kT)$ for $k = 1, 2, 3, \cdots$. Both vectors are stored in files for next processing.

It is necessary to extract amplitudes $\boldsymbol{\sigma_a}$ and $\boldsymbol{\varepsilon_{at}}$ for an application in the classical method of identification. While the vector of stress amplitudes may be found rather easily, the vector of amplitudes of plastic strain $\boldsymbol{\varepsilon_{ap}}$ is calculated as a difference between total strain and its elastic component. In this case, it is evaluated from the following formula

$$\boldsymbol{\varepsilon_{ap}} = \boldsymbol{\varepsilon_{at}} - \frac{\boldsymbol{\sigma_a}}{E'}\,. \tag{10}$$

The fraction in the formula substitutes elastic strain $\boldsymbol{\varepsilon_e}$. Quantity E' is a circular module of elasticity, which should be found from the form of a hysteresis loop. It has been used for evaluating ε_{ap} in Tab. 1.

Time series $\boldsymbol{\sigma}(kT)$ and $\boldsymbol{\varepsilon_t}(kT)$ are the coordinates of points of hysteresis curves. They may be used for the identification of parameters n' and K'. Equation (9) serves as a good origin for it. Rewritten with the use of matrices, differences between measurement and the mathematical model are residuals $\boldsymbol{r_\sigma}$

$$\boldsymbol{r_\sigma} = K'\,(\boldsymbol{\varepsilon_t} - \boldsymbol{\varepsilon_e})^{n'} - \boldsymbol{\sigma}, \tag{11}$$

where $\boldsymbol{\varepsilon_e} = \boldsymbol{\sigma}/E'$ is a column vector of elastic components of total strain $\boldsymbol{\varepsilon_t}$. However, once obtained parameters n' and K' are describing not only stresses $\boldsymbol{\sigma}$, but also strain $\boldsymbol{\varepsilon}$. Differences between measured and evaluated strains can be found from the equation

$$\boldsymbol{r_\varepsilon} = \left(\frac{\boldsymbol{\sigma}}{K'}\right)^{\frac{1}{n'}} - (\boldsymbol{\varepsilon_t} - \boldsymbol{\varepsilon_e}) \tag{12}$$

Both vectors r_σ and r_ε form vector $\boldsymbol{r}$ of all residuals

$$\boldsymbol{r} = [\,\boldsymbol{r}_\sigma^T, \boldsymbol{r}_\varepsilon^T\,]^T, \tag{13}$$

that should be minimized. Due to the method of least squares, an identification procedure should minimize the scalar function

$$S \;=\; \boldsymbol{r}^H\,\boldsymbol{r} \;=\; \sum_{\forall k} r_k^2. \tag{14}$$

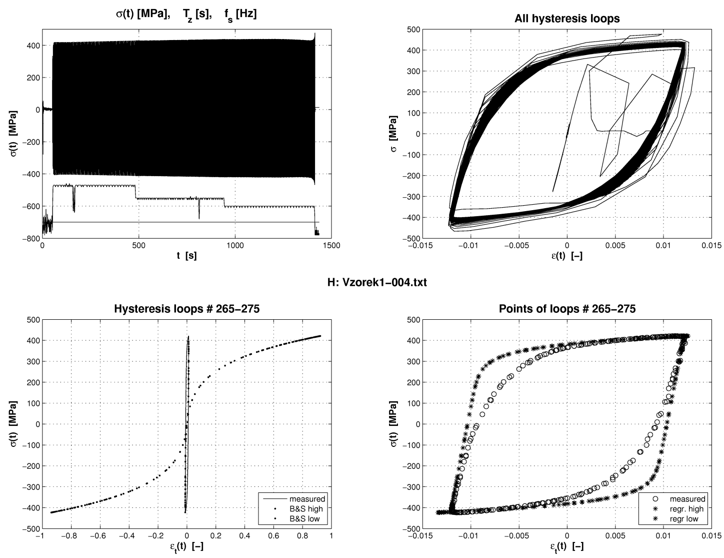

Fig. 1. Specimen No.1, B&S estimate of LCF parameters [7]

It may be done either by a straight minimizing of the scalar function $S(n',\, K')$ applying any of procedures for unconstraint minimization, or by solving an over determined system of nonlinear equations (11) and (12). However, it is not easy to solve such a complicated system. This was a reason why a simpler system of equations containing only one residual vector (12) was set up and solved by the author's Matlab function `LMFnlsq` [6]. The simplification brings a drawback in getting only a suboptimal solution of the problem.

Figure 1 gives a survey on measurement results from the specimen No.1. Its subfigure Fig. 1a displays the time history of three functions, i.e. stress $\sigma(t)$, loading period T_p, and sampling period T. The second subfigure Fig. 1b depicts the whole history of the function $\sigma(\varepsilon_t(t))$. Figure Fig. 1c shows how LCF parameters σ_f', b, ε_f', c estimated due to Bäumel and Seeger (B&S) [7] approximate a group of 10 hysteresis loops drawn in compressed horizontal scale by a solid line. It is obvious that the estimate may not be used for an approximation of hysteresis loops. The last subfigure Fig. 1d displays the same group of hysteresis loops from the mid of specimen's lifetime, samples of which are plotted as circles. The asterisks denote the samples of hysteresis loop evaluated out of LCF parameters identified from the original estimates [7]. The approximation is much better.

Young's modulus E used in formula (12) should be pointed out. Its value was obtained by a standard tension test. It is clear that a better approximation of hysteresis loop would be obtained, if another E', a cyclic modulus of elasticity, be used during the identification process. Its value can be found by processing a tangent to hysteresis loops at the beginnings of loading and unloading region of the test.

The influence of cyclic modulus E' on the identification is seen from subfigure Fig. 2d, when compared with the same subfigure from figure Fig. 1. The final approximation is a bit better than that last mentioned.

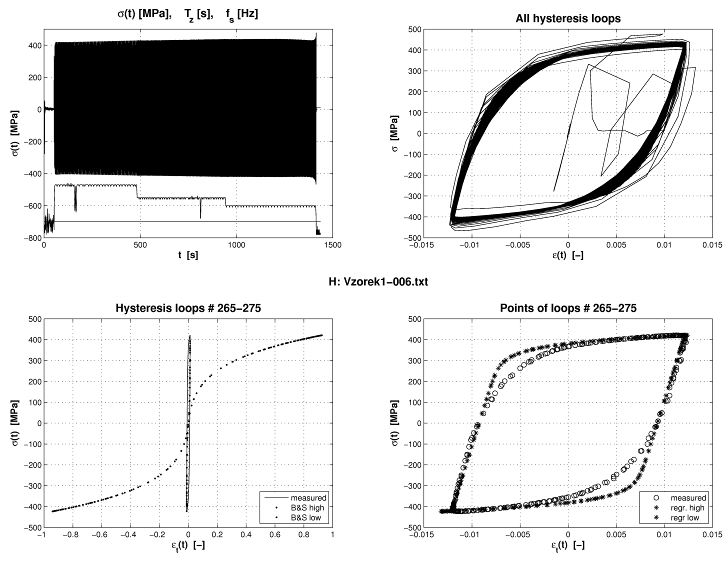

Fig. 2. Specimen No.1, B&S estimate of LCF parameters [7], cyclic E'

The same group of hysteresis loops has been processed starting from another initial estimates of LCF parameters found by the standard procedure described at the old Czech Standard ČSN 42 0368 [9]. It is seen from figure Fig. 3d that the identification process converged to the far better approximation of the hysteresis loop compared to the first case. The cause lied in much better starting estimate of the LCF parameters. Subfigure Fig. 3c proved it in a wider curve of hysteresis loops, nevertheless, the initial parameters did not provide any closed loop. The final identified parameters provided much better approximation of the set of 10 hysteresis loops with an exception in their middle parts.

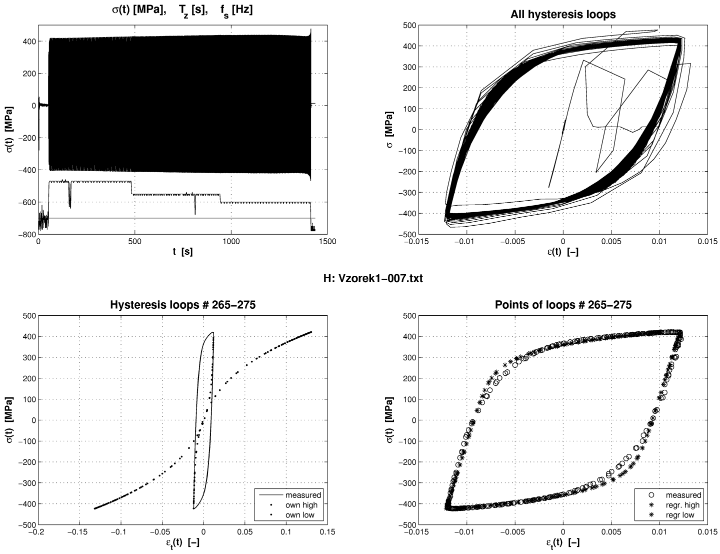

Fig. 3. Specimen No.1, Classical estimate of LCF parameters [8], cyclic E'

It is expected that inclusion of vector $\boldsymbol{r_\sigma}$ in the mathematical model according to equation (13) could improve a tightness of the fit. Table Tab. 2 gathers both initial estimates of LCF parameters σ_f', b, ε_f', c and identified parameters n' and K'. The table shows big differences between classical and new approaches to the problem of the low cycle fatigue parameters.

Table 2. A survey of LCF parameter values

Variable $\rightarrow$ $\downarrow$ Source	E' [MPa]	σ_f' [MPa]	b [-]	ε_f' [-]	c [-]	n' [-]	K' [MPa]
B&S estimate [7]	237418	787,5	-0,087	0,59	- 0,58	0,15	851
Classic method [8]	237418	1052	-0,1261	0,3305	-0,5011	0,2517	1390
Identification from [7]	237418					0,1085	1299
Identification from [7]	147518					0,04707	1017
Identification from [8]	147518					0,08541	1196

3. Conclusions

Low cycle fatigue parameters play an important role in design of many high stressed parts of machines and structures. Good knowledge of the parameters is of high interest of designers, because they determine the reliability of the whole structure. Up to now, values of the parameters are evaluated from the experimental data obtained during low cycle tests of material specimens.

A damage of parts is caused by a cumulation of energy spent in their material. The energy density of every loading cycle is defined by a hysteresis loop in coordinates $\{\varepsilon, \sigma\}$. It means that forms of hysteresis loops are good measures of a damaging process. Unfortunately, an attempt to use the standard low cycle fatigue parameters for a construction of hysteresis loop fails. There might be several reasons for the discrepancy.

At first, the mathematical model defined by equations (1) and (2) takes into account only fatigue lives N_f without any respect to hysteresis loop form. This fact may lead to unrealistic forms of hysteresis loops, because of their high sensitivity to values of LCF parameters.

Secondly, the identification is based on fitting of mathematical formulae to two sets of experimental data, fatigue lives N_f and points of hysteresis loops. Both sets include measurement deviations, which influence the values of resulting parameters.

The last reason, and may be the most important for differences between results obtained via both approaches, is an approximate description of damaging processes by Basquin and Manson-Coffin equations. Even that they describe fatigue lives quite well, the combination of both equations into a cyclic strain-stress curve might amplify the approximative nature of the both descriptions.

As a recourse to the situation appears a combination of both approaches, the classical method and the hysteresis curve fitting. It can be expected that there will be bigger differences both in fatigue life expectations and in a quality of the hysteresis loop fit. Nevertheless, this way has not been tested yet.

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