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Integral Equation Method for Ring-Core Residual Stress Measurement

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Abstract: The ring-core method is the semi-destructive experimental method used for evaluation of the homogeneous and non-homogeneous residual stresses. In this paper, the integral equation method to quantify non-uniform residual stress fields is discussed. Therefore, correctly determined and properly used calibration factors a_{ij} and b_{ij}, necessary for the residual stress measurement by the ring-core method, are essential. The finite element method is used for the simulation of residual states of stress and to calculate relieved strains on the top face of the core, which are necessary for the subsequent calibration factors´ determination.

Keywords: Calibration factor, Relaxation factor, Integral equation method, Residual stress, Ring-core method, Strain gauge rosette

1. Introduction

The ring-core method (RCM) is a semi-destructive experimental method used for the evaluation of homogeneous and non-homogeneous residual stresses, acting over depth of drilled core. Therefore, the specimen is not totally destroyed during measurement and it could be used for further application in many cases.

In this paper, the integral equation method (IEM) to quantify non-uniform residual stress fields, acting over depth of drilled core, is discussed. This method is described by a physical theory with a proper mathematical model. The IEM overcomes typical drawbacks of the incremental strain method (ISM), which leads to incorrect results, where a steep gradient of residual state of stress occurs. The incremental strain method assumes, that the measured deformations $d\varepsilon_a$, $d\varepsilon_b$, and $d\varepsilon_c$ are functions only of the residual stresses, acting in the current depth z of the drilled groove and they do not depend on the previous increments dz, including another residual stresses. More information about the ISM could be found in papers [1, 2]. In fact, relieved strains on the top face of the core do not depend only on the stress acting within the drilled layer and its position, but also on the geometric changes of the ring groove during deepening. These two factors are taken into account by the integral equation method, which has been particularly developed for the Hole-drilling method in 1988 [3, 4].

Anyway, the integral equation method assumes, that strain relaxation on the top face of the core, for the particular depth of the drilled groove, is superposition of all deformations. These deformations are caused by a partial residual stresses, acting within every drilled layer, of all depth increments (see Figs. 1 and 2).

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Ajovalasit et al. [5] and Zuccarello [6] extended theory of the IEM into the ring-core application. They generally describe the IEM like a method, with a high sensitivity to the measurement errors, due to the numerical ill-conditioning of the equation set. This is caused by the position of the strain gauge on the top of the core, which is not enough sensitive to strains, relieved in the deeper layers of the drilled groove.

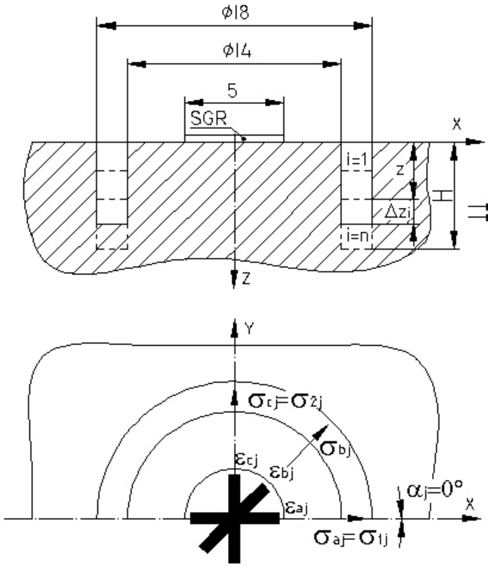

Fig. 1. The ring-core method: geometry and general notation

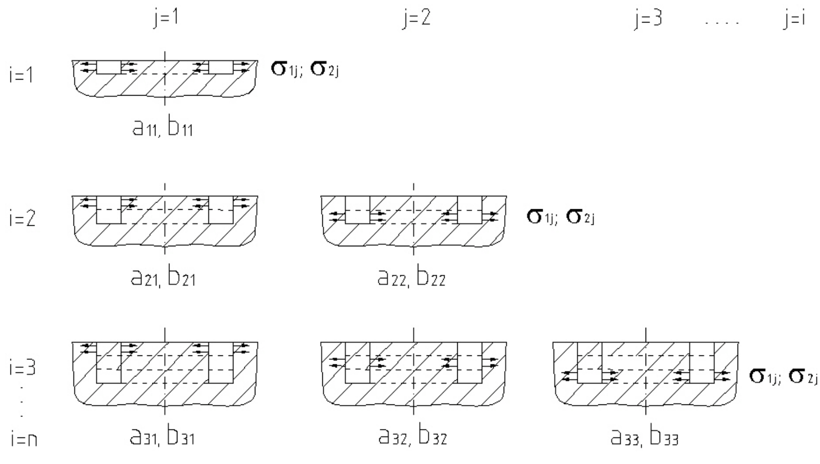

Fig. 2. Loading cases based on theory of the integral equation method

This paper describes how, application of the ring-core method with theory of the IEM and the finite element method (FEM) could be used for a numerical simulation and for determination of the uniform or non-uniform residual state of stress. The numerical simulation is used for the measurement of relieved strains on the top face of the model´s core, at real positions of the strain gauge rosette´s measuring grids. Calibration factors´ matrices ***a*** and ***b***, which are lower triangular, need to be calculated first in order to quantify the uniform or non-uniform residual state of stress, by the integral equation method.

2. Integral Equation Method

Like each method, the integral equation method has its own theoretical background to define certain relations between known and unknown parameters. Generally, total deformation measured on the top face of the core, for the ring-groove having depth H, is the integral of an infinitesimal strain relaxation components, caused by the residual stresses, acting at all depths in the range of $0 \leq z \leq H$:

$$\varepsilon_{k(H)} = \frac{1}{E}\int_0^H \left\{ \left[A_{(H,z)} \cdot \left(\sigma_{1(z)} + \sigma_{2(z)}\right) + B_{(H,z)} \cdot \left(\sigma_{1(z)} - \sigma_{2(z)}\right)\right] \cdot \cos 2\alpha_{k(z)} \right\} dz \qquad k = a, b, c \qquad (1)$$

where $\varepsilon_{a(H)}$, $\varepsilon_{b(H)}$, $\varepsilon_{c(H)}$ are the strains, measured by the strain gauge rosette on the top of the core´s surface after a milling a groove having depth H, $\sigma_{1(z)}$ and $\sigma_{2(z)}$ are the unknown residual stresses acting at current depth z, $\alpha_{k(z)}$ is the angle between the maximum principal stress $\sigma_{1(z)}$ and the direction of the strain gauge´s measuring grid $k = a$ and $A_{(H,z)}$, $B_{(H,z)}$ are the calibration functions, dependent on the shape and geometry of the ring-groove (Fig.1). Using a three-grid rosette, Eq. (1) leads to a three linear equation set, from which the principal residual stresses and their orientation can be evaluated too.

For $i = 1,..., n$ finite depth increments Eq. (1) can be written as:

$$\varepsilon_{ki} = \sum_{j=1}^{i} \varepsilon_{kij} \quad k = a, b, c \qquad (2)$$

where the strain ε_{kij} depends only on the stresses existing in the j^{th} layer by means of Eq. (3). Consequently, it is necessary to divide the maximum depth H into i intervals with the depth increment of Δz_i and to approximate the function of the principal residual stresses $\sigma_{1(z)}$ and $\sigma_{2(z)}$ in each interval by the proper uniform distribution (Fig.3). Therefore, considering i finite depth increments, Eq. (1) can be written as:

$$\varepsilon_{kij} = \frac{a_{ij}}{E}\left(\sigma_{1j} + \sigma_{2j}\right) + \frac{b_{ij}}{E}\left(\sigma_{1j} - \sigma_{2j}\right)\cos 2\alpha_{kj} \qquad (3)$$

in which ε_{kij} is the strain component, relaxed on the surface solely due to the stress acting in the j^{th} layer, when i^{th} depth increments have been achieved, σ_{1j}, σ_{2j} are the stresses within the j^{th} layer and a_{ij} ,b_{ij} are calibration factors. Knowledge of their dependence on the geometric changes of the ring-groove and on the disposition of the residual state of stress cross the depth of metallic material is essential and correspond of theirs appropriate application.

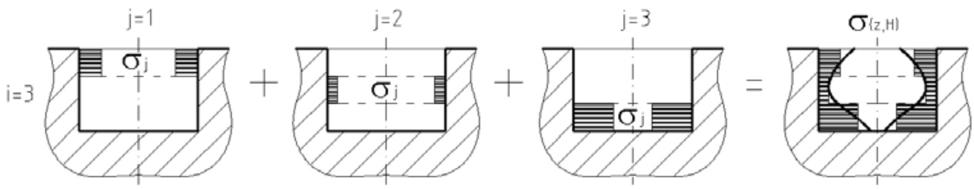

Fig. 3. Approximation of non-homogenous residual stress field by the integral equation method

Calibration factors a_{ij} and b_{ij} of the lower triangular matrices ***a*** and ***b*** cannot be determined by calibration coefficients K_1 and K_2 used for the incremental strain method [1, 2]. They can be possibly obtained by the finite element simulation.

$$\mathbf{a} = \begin{bmatrix} a_{11} & & \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33} \end{bmatrix} ; \ \mathbf{b} = \begin{bmatrix} b_{11} & & \\ b_{21} & b_{22} & \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \qquad (4)$$

2.1. Calibration factors′ determination

For the correct determination of the depth-varying principal residual stresses σ_{1j}, σ_{2j} by the IEM, it is necessary to determine calibration factors a_{ij}, b_{ij}, respective generalized calibration factors A_{ij}, B_{ij} for each type of used depth increment distribution. Generalized calibration factors A_{ij} and B_{ij} are not dependent on the material properties.

Eq. (3), followed from the basic equation of the integral equation method Eq. (1), can be rewritten in order to determine the principal strains ε_{1ij} and ε_{2ij} as follows [5]:

$$\varepsilon_{1ij} = A_{ij} \cdot (\sigma_{1j} + \sigma_{2j}) + B_{ij} \cdot (\sigma_{1j} - \sigma_{2j}) \tag{5}$$

$$\varepsilon_{2ij} = A_{ij} \cdot (\sigma_{1j} + \sigma_{2j}) - B_{ij} \cdot (\sigma_{1j} - \sigma_{2j}) \tag{6}$$

In order to determine factors a_{ij}, A_{ij} it is necessary to consider a biaxial state of uniform stress with $\sigma_{1j} = \sigma_{2j} = 1\ MPa$ and the position of the strain gauge rosette placed centric on the top face of the core. Then, by considering Eq. (3) and relieved strains $\varepsilon_{aij} = \varepsilon_{bij} = \varepsilon_{cij} = \varepsilon_{ij}$:

$$a_{ij} = \frac{E\varepsilon_{ij}}{2\sigma_{1j}} \ ; \ A_{ij} = \frac{a_{ij}}{E} = \frac{\varepsilon_{ij}}{2\sigma_{1j}} \tag{7, 8}$$

To evaluate factors b_{ij}, B_{ij}, it is necessary to consider a pure shear state of uniform stress with $\sigma_{1j} = -\sigma_{2j} = 1\ MPa$. Then, considering Eq. (3), for $\alpha_j = 0°$ are relieved strains $\varepsilon_{aij} = -\varepsilon_{cij}$:

$$b_{ij} = \frac{E\varepsilon_{aij}}{2\sigma_{1j}} \ ; \ B_{ij} = \frac{b_{ij}}{E} = \frac{\varepsilon_{aij}}{2\sigma_{1j}} \tag{9, 10}$$

Finally, particular principal residual stresses σ_{1j} ,σ_{2j} ($\alpha_j = 0°$), acting in j^{th} layer of drilled groove (Fig.1 and Fig.3), made by $i = 1, ..., n$ depth increments, can be determined as follows:

$$\sigma_{1j} = \frac{1}{4}\left[\frac{\varepsilon_{aij} + \varepsilon_{cij}}{A_{ij}} + \frac{\varepsilon_{aij} - \varepsilon_{cij}}{B_{ij}}\right] \ ; \ \sigma_{2j} = \frac{1}{4}\left[\frac{\varepsilon_{aij} + \varepsilon_{cij}}{A_{ij}} - \frac{\varepsilon_{aij} - \varepsilon_{cij}}{B_{ij}}\right] \tag{11, 12}$$

2.2. Residual stress determination

The strain components e_i, d_i, m_i [6], calculated from strains ε_{ai}, ε_{bi} and ε_{ci}, measured on the top face of the core by the three-element strain gauge rosette (Fig. 1), after milling the i^{th} step of the groove, are superposition of all strains, relieved in every j^{th} layer (see Eq. (2)):

$$e_i = \frac{2}{E} \cdot \sum_{j=1}^{i} a_{ij} \cdot s_j = \frac{\varepsilon_{ci} + \varepsilon_{ai}}{2} \tag{13}$$

$$d_i = -\frac{2}{E} \cdot \sum_{j=1}^{i} b_{ij} \cdot p_j = \frac{\varepsilon_{ci} - \varepsilon_{ai}}{2} \tag{14}$$

$$m_i = -\frac{2}{E} \cdot \sum_{j=1}^{i} b_{ij} \cdot q_j = \frac{\varepsilon_{ci} + \varepsilon_{ai} - 2\varepsilon_{bi}}{2} \tag{15}$$

The stress components s_j, p_j and q_j, represented by Eqs. (16÷18), are calculated from stresses σ_{aj}, σ_{bj} and σ_{cj}, acting in every j^{th} layer, when $i = 1, ..., n$ depth increments have been reached:

$$s_j = \frac{\sigma_{cj} + \sigma_{aj}}{2} \ ; \ p_j = \frac{\sigma_{cj} - \sigma_{aj}}{2} \ ; \ q_j = \frac{2\sigma_{bj} - \sigma_{aj} - \sigma_{cj}}{2} \tag{16, 17, 18}$$

Then, desired residual stresses s_i, p_i and q_i, calculated from the strain components e_i, d_i, m_i and stress components s_j, p_j and q_j, after every drilled groove´s depth, can be found by following equations:

$$s_i = \frac{1}{a_{ii}} \cdot \left(\frac{E}{2} \cdot e_i - \sum_{j=1}^{i-1} a_{ij} \cdot s_j \right) \tag{19}$$

$$p_i = -\frac{1}{b_{ii}} \cdot \left(\frac{E}{2} \cdot d_i - \sum_{j=1}^{i-1} b_{ij} \cdot p_j \right) \tag{20}$$

$$q_i = -\frac{1}{b_{ii}} \cdot \left(\frac{E}{2} \cdot m_i - \sum_{j=1}^{i-1} b_{ij} \cdot q_j \right) \tag{21}$$

The basis of the integral equation method is described by Eqs. (19÷21). These equations consider stresses, acting in every j^{th} layer, when $i = 1,\dots, n$ depth increments have been reached, and therefore strains, measured on the top face of core after milling another dept increment. Furthermore, correctly determined and properly used calibration factors a_{ij}, b_{ij}, are required too.

Finally, the principal residual stresses σ_{1i} and σ_{2i}, corresponding to the i^{th} layer of drilled groove, can be re-calculated by Eq. (22):

$$\sigma_{1i,}\sigma_{2i} = s_i \pm \sqrt{\left(p_i^2 + q_i^2\right)} \tag{22}$$

2.3. Step optimization

Papers [5, 6] made by Ajovalasit et al. and Zuccarello are focused on the step optimization, which is important for the IEM too. The strain measurement errors´ influence, which occurs in practice, affects accuracy of the residual state of stress determination. Influence of the strain measurement errors depends particularly on the number and magnitude of the depth increment distributions Δz_i and consequently on the maximum depth H of the drilled groove.

Study of the step distribution´s influence on the determination of calibration factors a_{ij}, b_{ij} and on the subsequent residual stress state determination is not in the scope of this paper. Therefore, total depth of drilled groove $H = 5$ *mm* has been made by $n = 8$ optimized depth increments (see Table 1), as recommended in [5, 6]. Optimum depth increment distribution minimizes error sensitivity of the experimental measurement and considerably improves the numerical conditioning.

Table 1. Distribution of the depth n = 8 increments Δz_i for a total depth of H = 5 mm

Depth increment Δz_i [mm]:							
Δz_1	Δz_2	Δz_3	Δz_4	Δz_5	Δz_6	Δz_7	Δz_8
0.6	0.45	0.4	0.4	0.45	0.5	0.7	1.5

3. FE-simulation

A prerequisite for correct and accurate measurement of residual strains on the top of the core is to use the finite element simulation. Therefore, the ANSYS analysis system is used for the subsequent FE-simulation.

FE-analysis is based on a specimen volume with dimensions of a x $a = 50$ *mm* and thickness of $t = 50$ *mm*. Due to symmetry, only a quarter of the model has been modelled with centre of the core on the surface as the origin. The shape of the model is simply represented by a block with planar faces with a quarter of the annular groove drilled away (Figs. 4 and 5). The annular groove has been made by $n = 8$ increments with the different step´s size Δz_i (Table 1). The maximum depth of drilled groove is $H = 5\ mm$. Dimension of the outer diameter is D = $2r_i = 18\ mm$ and groove width is $h = 2\ mm$.

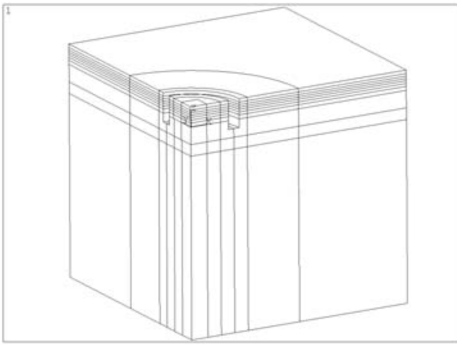

Fig. 4. Quarter of global solid model

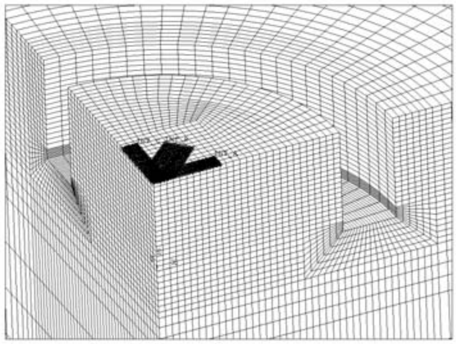

Fig. 5. Detail of the core with finite element mesh

Linear, elastic and isotropic material model is used with material properties of Young´s modulus $E = 210\ GPa$ and Poisson´s ratio $\mu = 0.3$. Length and width of each measuring of grid of FR-5-11-3LT strain gauge rosette is $l = 5\ mm$ and $w = 1.9\ mm$ respectively [7]. In case of known directions of principal residual stresses, placing of the three-element strain gauge rosette on the top of the ring-core is shown in Fig. 1. Strain measurement on the top of the core is made by the integration across rosettes´ measuring grid surface.

In case of the biaxial state of uniform stress simulation, applied surface´s pressure $p = p_r = 1\ MPa$ is demonstrated in Fig. 6a and pressure $p_r = p{\cdot}cos(2\alpha)$, $p_t = p{\cdot}sin(2\alpha)$ (load´s magnitude is varying with the angle $0° \le \alpha \le 90°$) in order to simulate the pure shear state of uniform stress is demonstrated in Fig. 6b. All loading conditions are applied to the both inner sides of the ring-groove, in the range of the each depth increment.

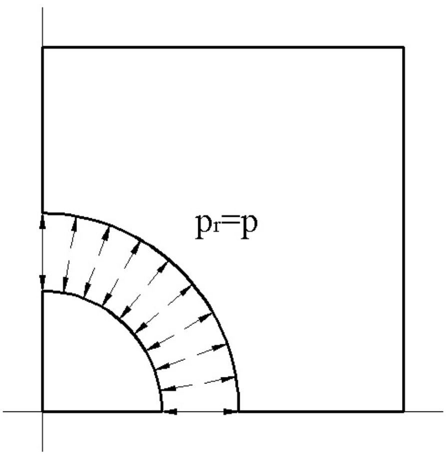

Fig. 6a. Loading conditions for the biaxial state of uniform stress simulation

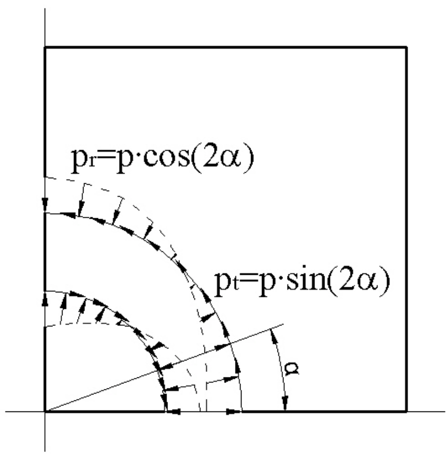

Fig. 6b. Loading conditions for the pure shear state of uniform stress simulation

4. Results

Process of the ring-core method´s FE-simulation has been carried out. Strains, relieved on the top face of the core, have been measured for two types of loading boundary conditions, in order to determine calibration factors. Table 2 contains magnitudes of strains, measured in case of the biaxial state of uniform stress ($\varepsilon_{aij} = \varepsilon_{bij} = \varepsilon_{cij} = \varepsilon_{ij}$), see Fig. 6a. Appropriate calibration factors a_{ij}, calculated from measured strains by the Eq. (7), are shown in Table 3a. The last column of the Table 2 represents the sum of all strain increments, caused by the unit stress $\sigma_j = 1\ MPa$, acting in every drilled layer (see Fig. 2). This sum $\sum\varepsilon_{ij} = e_i$ is a necessary part of residual stress determination by the Eq. (19).

Table 2. Relieved strains measured by the FE-simulation of the biaxial state of uniform residual stress

ε_{ij} [1]	j=1	2	3	4	5	6	7	8	$e_i = \sum\varepsilon_{ij}$
i=1	-3.03E-07								-3.03E-07
2	-4.94E-07	-2.44E-07							-7.39E-07
3	-6.28E-07	-3.66E-07	-2.12E-07						-1.21E-06
4	-7.33E-07	-4.52E-07	-3.13E-07	-1.99E-07					-1.70E-06
5	-8.23E-07	-5.22E-07	-3.84E-07	-2.97E-07	-2.03E-07				-2.23E-06
6	-8.92E-07	-5.76E-07	-4.36E-07	-3.58E-07	-3.02E-07	-1.90E-07			-2.75E-06
7	-9.50E-07	-6.21E-07	-4.79E-07	-4.06E-07	-3.66E-07	-2.95E-07	-2.12E-07		-3.33E-06
8	-9.93E-07	-6.55E-07	-5.11E-07	-4.41E-07	-4.10E-07	-3.54E-07	-3.35E-07	-2.48E-07	-3.95E-06

Table of strains, measured by the simulation of pure shear state of uniform stress is not included in this scope. Corresponding calibration factors b_{ij}, calculated by the Eq. (9) and necessary for solving Eqs. (20, 21), are shown in Table 3b.

Represented results are valid only for dimension of the groove´s outer diameter D = *18 mm*, groove´s width $h = 2\ mm$ and total depth of drilled ring-groove $H = 5\ mm$. Furthermore, results are valid for the groove made by $n = 8$ increments, with the depth distribution according to Table 1 and for the FR-5-11-3LT strain gauge rosette.

Table 3a. Calibration factors a_{ij} for optimized depth increment

a_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0.0318							
2	-0.0519	-0.0257						
3	-0.0659	-0.0384	-0.0222					
4	-0.0770	-0.0474	-0.0329	-0.0208				
5	-0.0864	-0.0548	-0.0403	-0.0312	-0.0213			
6	-0.0937	-0.0605	-0.0458	-0.0376	-0.0317	-0.0200		
7	-0.0998	-0.0652	-0.0503	-0.0426	-0.0384	-0.0310	-0.0223	
8	-0.1042	-0.0688	-0.0537	-0.0463	-0.0431	-0.0372	-0.0352	-0.0261

Table 3b. Calibration factors b_{ij} for optimized depth increment

b_{ij} [1]	j=1	2	3	4	5	6	7	8
i=1	-0.0300							
2	-0.0520	-0.0263						
3	-0.0670	-0.0410	-0.0244					
4	-0.0793	-0.0515	-0.0374	-0.0247				
5	-0.0903	-0.0605	-0.0469	-0.0383	-0.0280			
6	-0.0998	-0.0681	-0.0544	-0.0473	-0.0429	-0.0301		
7	-0.1092	-0.0756	-0.0616	-0.0554	-0.0539	-0.0481	-0.0414	
8	-0.1198	-0.0839	-0.0693	-0.0636	-0.0641	-0.0617	-0.0696	-0.0830

The residual stress´ magnitudes $s_i = \sigma_{1i} = \sigma_{2i}$, corresponding to the every drilled i^{th} layer, in case of considered biaxial state of uniform stress, are shown in Table 4. They have been calculated according to Eq. (19), by using sum of strains $\sum\varepsilon_{ij} = e_i$ from Table 2 and by using calibration factors a_{ij} shown in Table 3a.

Table 4. Re-calculated stress components for the biaxial state of uniform residual stress

s_i [MPa]	j=1	2	3	4	5	6	7	8
i=1	**1.00**							
2	1.49	**1.00**						
3	1.92	1.58	**1.00**					
4	2.31	2.13	1.63	**1.00**				
5	2.71	2.69	2.30	1.68	**1.00**			
6	3.09	3.23	2.95	2.37	1.63	**1.00**		
7	3.50	3.83	3.67	3.15	2.38	1.72	**1.00**	
8	3.98	4.51	4.50	4.06	3.29	2.65	1.74	**1.00**

The residual stresses´ results, located on the diagonal in Table 4, proove the correctness of used equations. In case of non-uniform residual state of stress distribution, Eqs. (19÷21) in order to determine the principal residual stresses σ_{1i} and σ_{2i} by Eq. (22), must be calculated all together.

5. Conclusion

This paper described how application of the ring-core method with theory of the integral equation method and the finite element method could be used for the numerical simulation and subsequent determination of uniform or non-uniform residual state of stress. In order to describe the non-uniform residual state of stress by the integral equation method a numerical calibration factor´s matrices ***a*** and ***b***, which are lower triangular, have been determined. For this reason, two types of loading conditions have been applied to the FE-model. Appropriate equations to evaluate particular principal residual stresses σ_{1j} ,σ_{2j} ($\alpha_j = 0°$), acting in j^{th} layer, when groove has $i = 1,\dots, n$ depth increments and the principal residual stresses σ_{1i} and σ_{2i}, corresponding to the i^{th} layer of drilled groove, have been demonstrated for optimized step distribution.

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