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Computer simulation of the displacement of the speckle field due to 2D translation of the object and its evaluation by speckle correlation method

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Abstract: This contribution contains a computer simulation of the displacement of the speckle field due to both in-plane and normal translation of an object under investigation. The description of speckle field propagation is based on the Fresnel-Kirchhoff diffraction theory. The presented numerical model involves detection of speckle pattern at any observation angles. Some results of numerical correlation of two mutually displaced intensity sets are shown and compared with the results obtained by theoretical relation.

Keywords: Computer simulation, Speckle field, In-plane translation, Normal translation, Correlation of speckle field

1. Introduction

It is known that the state of deformation of an elementary area of object's surface under investigation is expressed by means of a small deformation tensor (translation, rotation and elastic deformation components). The analysis of deformed state of an object is a basic tool of both classical and modern mechanics. Optical noncontact measuring methods in this analysis and their applications have many advantages, such as the contactless principle or high measurement resolution. One of these methods uses well-known effect called speckle [1]. The next text is focused on measurement of the chosen components of the small deformation tensor - in-plane and normal translation.

The method [2] described in this contribution is based on the fact, that developed speckle field detected in front of an object with a rough surface moves in dependence on movement of the object under investigation. The intensity distribution of speckle pattern is gradually recorded and data corresponding to change of object location before and after its motion are mutually correlated. A position of the global maximum of cross-correlation function of intensities determines extent of the movement. In previous works [2-4] the speckle correlation method for detection of the object translation has been discussed and some experiments, which have verified presented theoretical relations have been performed [5].

The aim of this contribution is to propose the numerical model enabling to simulate the object displacement, which is a result of two translation components. The object is translated by 20 µm, 40 µm and 60 µm along both *x*-axis and *z*-axis direction. Obtained

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speckle patterns are then numerically analyzed by the speckle correlation method object translation to evaluate.

2. Theory description

Let us adopt a geometrical arrangement according to Fig. 1. The rough object's surface placed in the plane (x, y) is illuminated with a Gaussian beam [6] of wavelength λ diverging from its waist of diameter $2\omega_o$. Diameter of the Gaussian beam on the object's surface at a distance L_s from its waist is denoted as $2\omega\,(L_s)$. The complex amplitude $U\,(x', y')$ of developed speckle field in the observation plane (x', y') at a distance L_o away from the object plane (x, y) can be expressed via Fresnel-Kirchhoff diffraction integral [6]

$$U(x',y') = \frac{\exp(-ikL_o)}{i\lambda L_o} \times \\ \times \int_{-\infty}^{\infty}\!\!\int U(x,y,\Delta(x,y)) \exp\left[-ik\frac{(x'-x)^2+(y'-y)^2}{2(L_o+\Delta(x,y))} - ik\Delta(x,y)\right] dxdy, \tag{1}$$

where $\Delta\,(x, y)$ is a random variable surface roughness, $k = 2\pi/\lambda$ is wave number and $U\,(x, y, \Delta(x, y))$ is a complex amplitude of the light after reflection from the object plane.

Considering a rigid body moving along the coordinate axes x and z (in-plane direction and normal direction with respect to the object) simultaneously, the following relation between the translation components a_x, a_z and displacement $A_{x'}$ of speckle pattern in the observation plane along x'-axis direction can be derived [5]

$$A_{x'} = a_x\left(\frac{L_o}{L_s\cos\theta_o} + \cos\theta_o\right) - a_z\sin\theta_o\,, \tag{2}$$

where θ_o is the angle of the observation direction. We assume that the illumination direction is collinear with the normal to object plane (Fig. 1).

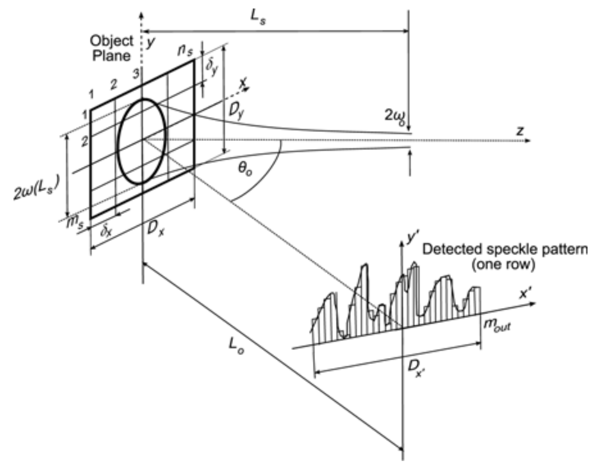

Fig. 1. Coordinate system for observation of the speckle pattern

The Eq. (2) was derived initially under the condition of an illuminating wave with a spherical wave of finite size and the propagation of the wave reflected from the object is described by means of the Fresnel approximation. It has been shown [5] that the Eq. (2) is valid in the case of illumination with the Gaussian beam as well.

3. The simulation procedure

A simple method of numerical simulation of propagation of the speckle field is based on the diffraction of light on the optical rough surface. The submatrix ($1{:}m_s, 1{:}n_s$) of $m_s \times n_s$ elements (Fig. 1) represents distribution of illuminated points situated on the object plane by the equal distances $\delta_x = D_x/(n_s-1)$ and $\delta_y = D_y/(m_s-1)$, where D_x and D_y are dimensions of illuminated area on the object plane. The elements substitute for random variable surface roughness $\Delta(x, y)$ in Eq. (1) [7].

Let us appreciate, that Eq. (1) is valid only if the object plane and the detection plane are parallel. However, according to the geometrical arrangement in Fig. 1 the detection plane (x', y') is rotated by the angle θ_o toward the object plane (x, y), hence this rotation is included within numerical simulation in the following procedure. We gradually create planes containing in sequence all x-axis coordinates of the object plane (x, y). These planes are parallel to the inclined detection plane (x', y') [7]. More detail view on the case of propagation at the angle of observation θ_o is illustrated in Fig. 2.

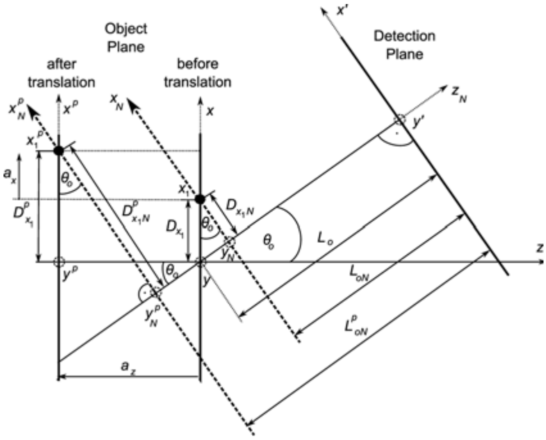

Fig. 2. Coordinate system for observation of the speckle pattern in the detection plane (x', y') rotated by the angle θ_o toward the object plane (x, y). Point x_1 under study is depicted by black mark.

Let us choose one point x_1 in the object plane. Its x-axis coordinate is denoted as D_{x_1} in the coordinate system (x, y, z) and D_{x_1N} in the new rotated coordinate system (x_N, y_N, z_N), respectively. It is obvious, that

$$D_{x_1N} = D_{x_1} \cos\theta_o . \tag{3}$$

The distance between the point x_1 and detection plane (x', y') is given by

$$L_{oN} = L_o - D_{x_1N} \tan\theta_o . \tag{4}$$

Equation (3) and Eq. (4), respectively, is then substituted into Eq. (1) for all coordinates x and distances L_o, which are changing from point to point during computing of the complex amplitude U.

In order to simulate in-plane movement of the object along x-axis direction, the position of illuminated points is now represented by the shifted submatrix ($1{:}m_s, 1+p{:}n_s+p$), whereas p is an integer corresponding to column translation. Real translation is then $a_x = p\delta_x$. To simulate simultaneous normal movement a_z of the object along z-axis direction, the relations (3) and (4) have to be rewritten as

$$D^p{}_{x_1N} = \left(D^p{}_{x_1} + a_z \tan\theta_o\right)\cos\theta_o\,, \qquad (5)$$

$$L^p{}_{oN} = L_o + \frac{a_z}{\cos\theta_o} - D^p{}_{x_1N}\tan\theta_o\,, \qquad (6)$$

where $D^p{}_{x_1}$ and $D^p{}_{x_1N}$ represent shifted x-axis coordinates of the point $x_1{}^p$ in the coordinate system $(x, y, z)^p$ and $(x_N, y_N, z_N)^p$.

The resulting complex amplitude $U(x', y')$ of light, Eq. (1), at the distance L_o from the object plane (x, y) is calculated only at points on a selected row (Fig. 1). The number of elements of the row of size $D_{x'}$ is denoted as m_{out}. By means of relation $I(x', y') = |U(x', y')|^2$ mutually shifted intensity sets $I_1(x', y')$ and $I_2(x', y')$ are determined and consequently correlated.

4. Achieved numerical results

In this section, the results of computer simulation are summarized into the table (Table 1). The various combination of both in-plane and normal displacements a_x = 20 µm, 40 µm, 60 µm and a_z = 20 µm, 40 µm, 60 µm are noticed. The method of correlation of speckle fields [2-4] is applied and the position $A_{x'}{}^{stated}$ (the third column) of the maximum of the normalized cross-correlation function of two intensity sets obtained from the computer simulation is compared with displacement $A_{x'}$ (the second column) of speckle pattern computed theoretically by means of Eq. (2). The last column contains the values of maximum of normalized cross-correlation function r_{12}.

Table 1. Displacement $A_{x'}$ of speckle pattern in consequence of translation a_x and a_z of the object and the corresponding maximum r_{12max} of normalized cross-correlation function of intensities I_1 and I_2 [a]

a_x = 20 [µm]			
a_z [µm]	$A_{x'}$ [µm]	$A_{x'}{}^{stated}$ [µm]	r_{12max} [%]
20	53.3	53.5	99.9
40	43.5	43.3	99.9
60	33.5	33.3	99.9
a_x = 40 [µm]			
a_z [µm]	$A_{x'}$ [µm]	$A_{x'}{}^{stated}$ [µm]	r_{12max} [%]
20	117.0	117.5	99.6
40	107.0	107.4	99.5
60	97.0	97.4	99.5
a_x = 60 [µm]			
a_z [µm]	$A_{x'}$ [µm]	$A_{x'}{}^{stated}$ [µm]	r_{12max} [%]
20	180.4	180.5	99.6
40	170.4	170.4	99.7
60	160.4	160.4	99.6

[a] Initial parameters of the computer simulation: $m_s = n_s = 201$, $m_{out} = 501$, $\delta_x = \delta_y = 20$ µm, $D_x = D_y = 4$ mm, $D_{x'} = 8$ mm, $L_s = 0.3$ m, $L_o = 0.6$ m, $\lambda = 632.8$ nm. The diameter of laser beam at its waist and at the distance L_s from its waist is $2\omega_o = 60$ µm and $2\omega(L_s) = 4.0$ mm. The angle of observation is $\theta_o = 30^o$.

5. Conclusion

The contribution concerns with the numerical model of propagation of speckle fields and uses this model to simulate the 2D translation of the object under investigation. The results obtained through the method of correlation of speckle fields are compared with the theoretical value. Good accordance with theory is achieved.

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