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Experimental Stress Analysis

Modelling of moving load effect

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Abstract: Moving load on transport structures represents the important component of real loading spectrum. The effect of this load can be analysed by numerical and experimental way. The most effective way is the combination the both mentioned advances. The certain assumptions are adopted within the theoretical analysis. It is good to verify some assumptions by experimental test. For this purpose the experiment on model beam was realised in laboratory conditions. The experimentally and numerically obtained results were mutually compared.

Keywords: Experimental test, Moving load, Numerical analysis

1. Introduction

Many institutions in the world pay attention to the following of moving load effect on the transport structures. Especially the moving load effect on bridges is frequently analysed [1], [2]. The analysis can by realized by theoretical or numerical or experimental way. But the most effective way is the combination the both mentioned advances. Department of Structural Mechanics Faculty of Civil Engineering University of Zilina deals with the solution of the problems of numerical simulation of moving loads effect on transport structures. Within theoretical approaches various assumptions are adopted. Also various numerical procedures are utilised during numerical computations. To verify some adopted assumptions and accuracy of the used numerical procedures the model of the beam subjected to moving load was created. The moving load effect on the beam vibration was experimentally tested and the results were compared with the numerically obtained results. The submitted paper is dedicated to theoretical description of the analysed problem and to the description of experimental test. The analysis of obtained results is carried out to the end.

2. Mathematical description of the problem

The object of the analysis is the effect of mass point movement on the vibration of single supported beam. The effect of the mass point movement can be modelled with the influence of inertial force – movement of the mass force or without the influence of inertial force – movement of the mass-less force. The beam can be modelled as Bernoulli-Euler prismatic bar with continuously distributed mass. According to [2] the equation of motion can be written as

$$E.I.\frac{\partial^4 y(x,t)}{\partial x^4} + \mu.\frac{\partial^2 y(x,t)}{\partial t^2} + 2.\mu.\omega_b.\frac{\partial\, y(x,t)}{\partial t} = p(x,t). \tag{1}$$

We adopt the assumption about the shape of deflection curve as

$$y(x,t) = q(t).\sin\frac{\pi.x}{l}\ . \tag{2}$$

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By substituting (2) into (1) we obtain

$$\left\{q(t).E.I.\frac{\pi^4}{l^4}+\dot{q}(t).2.\mu.\omega_b+\ddot{q}(t).\mu\right\}.\sin\frac{\pi.x}{l}=p(x,t)\ , \tag{3}$$

where

$$p(x,t)=\frac{2}{l}.\sin\frac{\pi.x}{l}.\sum_j \varepsilon_j.F_{\mathrm{int},j}(t).\sin\frac{\pi.x_j}{l}\ . \tag{4}$$

In the case of mass-less force

$$F_{\mathrm{int},j}=G=m\cdot g\ , \tag{5}$$

$$p(x,t)=\frac{2}{l}\cdot\sin\frac{\pi\cdot x}{l}\cdot G\cdot\sin\frac{\pi\cdot c\cdot t}{l}=\frac{2}{l}\cdot\sin\frac{\pi\cdot x}{l}\cdot G\cdot\sin\omega\cdot t\ . \tag{6}$$

And the equation of motion is

$$\mu\cdot\frac{l}{2}\cdot\ddot{q}(t)+\mu\cdot l\cdot\omega_b\cdot\dot{q}(t)+E\cdot I\cdot\frac{l}{2}\cdot\frac{\pi^4}{l^4}\cdot q(t)-G\cdot\sin\omega\cdot t=0\ . \tag{7}$$

In the case of mass force

$$F_{\mathrm{int},j}=G-m\cdot\ddot{v}_1=m\cdot(g-\ddot{v}_1)\ , \tag{8}$$

$$p(x,t)=\frac{2}{l}\cdot\sin\frac{\pi\cdot x}{l}\cdot m\cdot(g-\ddot{v}_1)\cdot\sin\omega\cdot t\ , \tag{9}$$

$$v(x,t)=y(x,t)=q(t).\sin\frac{\pi.x}{l}\ , \tag{10}$$

$$v_1(t)\equiv v(x_1,t)=q(t)\cdot\sin\frac{\pi\cdot x_1}{l}=q(t)\cdot\sin\frac{\pi\cdot c\cdot t}{l}=q(t)\cdot\sin\omega\cdot t\ , \tag{11}$$

$$\ddot{v}_1(t)=\ddot{q}(t)\cdot\sin\omega\cdot t+\dot{q}(t)\cdot 2\cdot\omega\cdot\cos\omega\cdot t-q(t)\cdot\omega^2\cdot\sin\omega\cdot t\ . \tag{12}$$

The equation of motion is

$$\begin{aligned}&\ddot{q}(t)\cdot[\mu\cdot\frac{l}{2}+m\cdot\sin^2\omega\cdot t]+\dot{q}(t)\cdot[\mu\cdot l\cdot\omega_b+m\cdot 2\cdot\omega\cdot\sin\omega\cdot t\cdot\cos\omega\cdot t]+\\ &+q(t)\cdot[E\cdot I\cdot\frac{l}{2}\cdot\frac{\pi^4}{l^4}-m\cdot\omega^2\cdot\sin^2\omega\cdot t]-m\cdot g\cdot\sin\omega\cdot t=0\ .\end{aligned} \tag{13}$$

The equations of motion were solved numerically. The Runge-Kutta 4th order step-by-step integration method was applied in the environment of the program system MATLAB.

The meaning of the used symbols is as follows: *E.I* bending stiffness of the beam, μ mass per unit length, ω_b damping circular frequency, x length coordinate, t time coordinate, $y(x,t)$ beam deflection, $p(x,t)$ load function, $q(t)$ Lagrange generalised coordinate having the meaning of the mid-span beam deflection. F_{int} interactive force between the load and the beam, ε = 1 if the load in on the beam, ε = 0 if the load out of the beam, G gravity firce, g = 9.81 m.s^{-2}, l span of the beam, c velocity in m.s^{-1}, $\omega = \pi.c.t/l$, v_1 deflection of the beam at the load position respecting the road profile, m mass of the load.

3. Experimental equipment

The model beam was made on the principle of model similarity. It is steel beam with the span l = 2.9 m and cross section 12 x 18 mm. The beam was made in such a way that due to dead load the beam axis is in horizontal position. Special equipment providing the movement of the load was developed (Fig. 1). In front of the beam the speeding-up path and behind the beam the braking path were built up. The inductive sensor IVR 99427 was used for the observation of mid-span vertical deflections (Fig. 2). The signal from the sensor was leaded via amplifier and the A/D interface to the computer. Then the signal was analysed by numerical way in the program system DAS16 or DISYS. The time of the load passage was registered by the use of two accelerometers BK 4508 situated at the beginning of the beam and at the beginning of the braking path, (Fig. 3). The time of the load passage was calculated as $t = t_k - t_z$, (Fig. 4). The average velocity of moving load was determined as $c = l/t$.

Fig. 1. Speeding-up path and equipment providing the movement of moving load

Fig. 2. Inductive sensor IVR 99427

Fig. 3. Accelerometer BK 4508

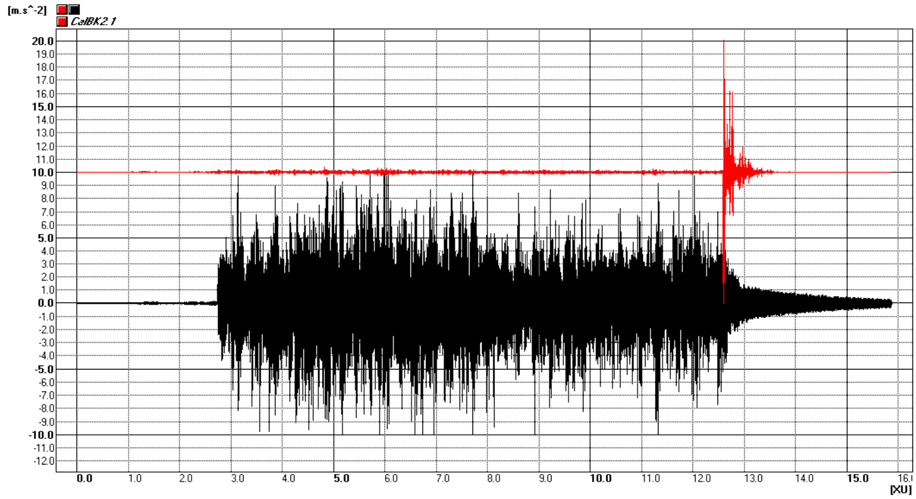

Fig. 4. Records of acceleration from 2 sensors for determination the time of load passage

4. Results of experimental tests

Thirty tests were realised altogether. The mass of the moving load was m = 220.5 g. The speed of vehicle motion was changed from 0.3620 to 2.3387 m/s. All results of experimental tests were compared with the results of numerical simulations realised for the model of mass force and for the model of mass-less force. Demonstration of mutual comparison of obtained results is in the Fig. 5, black – experiment, red – mass-less force, blue – mass force. Comparison of extreme values of vertical mid-span deflection and position of the load at this moment are put into the Table 1.

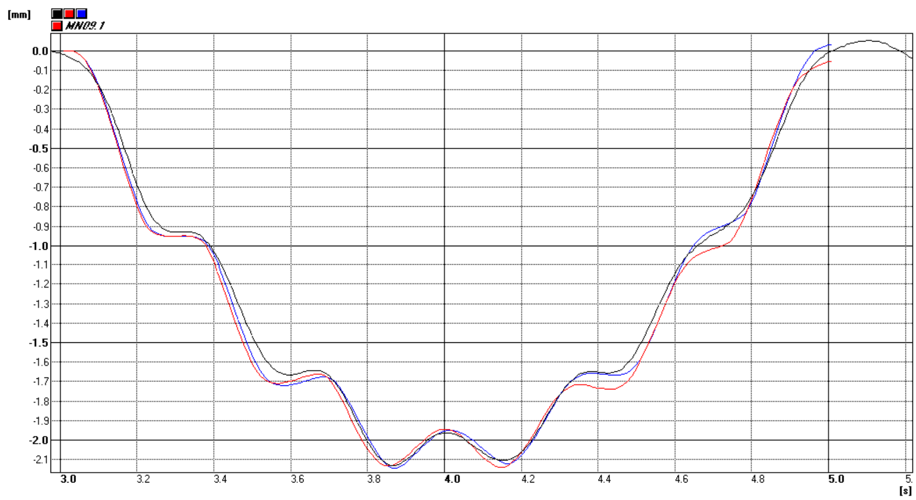

Fig. 5. Experiment versus numerical solution, No. 9, c = 1.4500 m/s

Table 1. Comparison of numerically and experimentally obtained results

No.		Extreme values of vertical mid-span deflection and position of the load						
		$V_{c,mf}$	$V_{c,mlf}$	v_e	$Dif_{mf} = v_{c,mf} - v_e$	$Dif_{mlf} = v_{c,mlf} - v_e$	Dif_{mf} in % v_e	Dif_{mlf} in % v_e
1	*t* [s]	11,6615	11,8803	12.0226	-0.3611	-0.1423	-3.0035	-1.1836
	v [mm]	2,0645	2,0633	2.0577	+0.0068	+0.0056	+0.3305	+0.2721
2	*t* [s]	6,7731	6,7421	6.7761	-0.0030	-0.0340	-0.0443	-0.5018
	v [mm]	2, 0913	2,0950	2.0918	-0.0005	+0.0032	-0.0239	+0.1530
3	*t* [s]	6,5906	6,5399	6.5737	+0.0169	-0.0338	+0.2571	-0.5142
	v [mm]	2,1318	2,1364	2.1457	-0.0139	-0.0093	-0.6478	-0.4334
4	*t* [s]	14,4722	14,4554	14.4712	+0.0010	-0.0158	+0.0069	-0.1092
	v [mm]	2,1540	2,1476	2.1468	+0.0072	+0.0008	+0.3354	+0.0373
5	*t* [s]	4,6119	4,5935	4.6067	+0.0052	-0.0132	+0.1129	-0.2865
	v [mm]	2,1996	2,1996	2.2236	-0.0240	-0.0240	-1.0793	-1.0793
6	*t* [s]	4,7253	4,6984	4.6957	+0.0296	+0.0027	+0.6304	+0.0575
	v [mm]	2,1779	2,1756	2.1529	+0.0250	+0.0227	+1.1612	+1.0544
7	*t* [s]	4,3527	4,3315	4.3406	+0.0121	-0.0091	+0.2788	-0.2096
	v [mm]	2,1622	2,1720	2.1524	+0.0098	+0.0196	+0.4553	+0.9106
8	*t* [s]	5,7572	5,7469	5.7532	+0.0040	-0.0063	+0.0695	-0.1095
	v [mm]	2,1567	2,1515	2.1518	+0.0049	-0.0003	+0.2277	-0.0139
9	*t* [s]	3,8687	3,8505	3.8642	+0.0045	-0.0137	+0.1165	-0.3545
	v [mm]	2,1413	2,1278	2.1278	+0.0135	+0.0000	+0.6345	0.0000
10	*t* [s]	4,0008	3,9719	3.9717	+0.0291	+0.0002	+0.7327	+0.0050
	v [mm]	2,1286	2,1481	2.1313	-0.0027	+0.0168	-0.1267	+0.7883
11	*t* [s]	3,6986	3,6944	3.6944	+0.0042	+0.0000	+0.1137	0.0000
	v [mm]	2,1906	2,1807	2.1758	+0.0148	+0.0049	+0.6802	+0.2252
12	*t* [s]	3,3953	3,3950	3.3950	+0.0003	+0.0000	+0.0088	+0.0000
	v [mm]	2,1622	2,1832	2.1832	-0.0210	+0.0000	-0.9619	+0.0000
13	*t* [s]	3,3481	3,3362	3.3686	-0.0205	-0.0324	-0.6086	-0.9618
	v [mm]	2,1643	2,1430	2.1536	+0.0107	-0.0106	+0.4968	-0.4922
14	*t* [s]	3,0954	3,0954	3.1058	-0.0104	-0.0104	-0.3349	-0.3349
	v [mm]	2,3253	2,3168	2.3309	-0.0056	-0.0141	-0.2403	-0.6049
15	*t* [s]	3,1037	3,0950	3.1146	-0.0109	-0.0196	-0.3500	-0.6293
	v [mm]	2,3242	2,3145	2.2852	+0.0390	+0.0293	+1.7066	+1.2822
16	*t* [s]	10,5023	10,4636	10.4644	+0.0379	-0.0008	+0.3622	-0.0076
	v [mm]	2,1108	2,1153	2.1251	-0.0143	-0.0098	-0.6729	-0.4612
17	*t* [s]	7,1044	7,0759	7.0603	+0.0441	+0.0156	+0.6246	+0.2210
	v [mm]	2,1101	2,1071	2.0983	+0.0118	+0.0088	+0.5624	+0.4194
18	*t* [s]	8,1221	8,0875	8.1221	0.0000	-0.0346	0.0000	-0.4260
	v [mm]	2,1202	2,1246	2.1125	+0.0077	+0.0121	+0.3645	+0.5728
19	*t* [s]	5.4588	5.4358	5.4478	+0.0110	-0.0120	+0.2019	-0.2203
	v [mm]	2.1338	2.1333	2.1258	+0.0080	+0.0075	+0.3763	+0.3528
20	*t* [s]	5.5719	5.5400	5.5438	+0.0281	-0.0038	+0.5069	-0.0685
	v [mm]	2.1448	2.1456	2.1367	+0.0081	+0.0089	+0.3791	+0.4165
21	*t* [s]	4.1712	4.4652	4.1752	-0.0040	+0.2900	-0.0958	+6.9458
	v [mm]	2.1170	2.1207	2.1241	-0.0071	-0.0034	-0.3343	-0.1601
22	*t* [s]	3.9085	3.9026	3.9085	0.0000	-0.0059	0.0000	-0.1510
	v [mm]	2.1569	2.1521	2.1569	0.0000	-0.0048	0.0000	-0.2225
23	*t* [s]	6.7781	6.7521	6.7433	+0.0348	+0.0088	+0.5161	+0.1305
	v [mm]	2.1111	2.1208	2.1062	+0.0049	+0.0146	+0.2326	+0.6932
24	*t* [s]	6.7875	6.7626	6.7875	0.0000	-0.0249	0.0000	-0.3669
	v [mm]	2.1341	2.1387	2.1020	+0.0321	+0.0367	+1.5271	+1.7460
25	*t* [s]	5.7257	5.7039	5.7329	-0.0072	-0.0290	-0.1256	-0.5059
	v [mm]	2.1591	2.1641	2.1343	+0.0248	+0.0298	+1.1620	+1.3962
26	*t* [s]	5.5386	5.5187	5.5306	+0.0080	-0.0119	+0.1446	-0.2152
	v [mm]	2.1551	2.1504	2.1227	+0.0324	+0.0277	+1.5264	+1.3049
27	*t* [s]	4.0306	4.0141	4.0536	-0.0230	-0.0395	-0.5674	-0.9744
	v [mm]	2.1796	2.1796	2.1374	+0.0422	+0.0422	+1.9744	+1.9744
28	*t* [s]	3.8696	3.8475	3.8641	+0.0055	-0.0166	+0.1423	-0.4296
	v [mm]	2.2036	2.2036	2.1693	+0.0343	+0.0343	+1.5812	+1.5812

29	*t* [s]	4.5624	4.4099	4.5474	+0.0150	-0.1375	+0.3299	-3.0237
	v [mm]	2.1623	2.1668	2.1078	+0.0545	+0.0590	+2.5856	+2.7991
30	*t* [s]	5.5494	5.5442	5.5546	-0.0052	-0.0104	-0.0936	-0.1872
	v [mm]	2.1354	2.1308	2.1123	+0.0231	+0.0185	+1.0936	+0.8758

5. Conclusion

On the basis of mutual comparison of extreme values of vertical mid-span deflection of the tested beam with the results obtained by numerical simulation methods (Table 1) we can say that the differences are very small. In the case of mass force the differences between experimentally and numerically obtained values are in the interval from 0.0000 mm to 0.0545 mm, average value 0.0168 mm. The difference 0.0545 mm represents 2.5856 % and difference 0.0168 mm represents 0.7970 % from the maximal deflection 2.1078 mm obtained by experimental way.

In the case of mass-less force the differences between experimentally and numerically obtained values are in the interval from 0.0000 mm to 0.0590 mm, average value 0.0160 mm. The difference 0.0590 mm represents 2.7991 % and difference 0.0160 mm represents 0.7591 % from the maximal deflection 2.1078 mm obtained by experimental way.

The statistical evaluation of differences concerning the vertical mid-span deflection gives better results for mass-less force than for mass force (ratio 16 : 11). But from the complex evaluation of time courses of mid-span beam vibration we can say that the computing model of the beam with moving mass force gives better results in relation to experimentally obtained results.

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