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Adaptive Control of TITO System Using Delta Model

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Abstract: This paper presents the design of an adaptive controller for a two input – two output (TITO) system using delta models. This controller has been verified by simulation and real time control of a non-linear laboratory model CE108 - coupled drives apparatus. The recursive least squares method is used in identification part of this controller. The synthesis is based on a polynomial approach. The results of the simulation and the real-time experiments are also given.

Keywords: Adaptive control, Multivariable control, Delta model, Polynomial methods, Real time control

1. Introduction

Many technological processes require that several variables relating to one system are controlled simultaneously. Each input may influence all system outputs. The design of a controller able to cope with such a system must be quite sophisticated. There are many different methods of controlling multivariable systems. Several of these use decentralized PID controllers [1], others apply single input-single output (SISO) methods extended to cover multiple inputs [2]. All these methods gave satisfactory results but controller tuning is difficult. For example, Ziegler-Nichols method can be used to set controller parameters. This method is very simple but the disadvantage of this method is that the system has to bring up into unstable state. Here polynomial theory approach is used to control a multivariable system [3]. The polynomial theory approach is the way how to make controller tuning easier.

2. Delta models and their identification

If *G(s)* is the transfer function of a continuous-time dynamic system (*s* is a complex variable), then the following expression for the discrete transfer function with the zero - order holder is valid

$$G(z) = \frac{z-1}{z} Z\left\{ L^{-1} \frac{G(s)}{s} \right\} \tag{1}$$

This step transfer function (1) is a rational polynomial function with variable *z*. The simple model structure, easy recursive identification using measurable data, suitability for the synthesis of the discrete control loop as well as for the description and expression of different types of stochastic process, including disturbance modelling, are all advantages of the *z* – transform function.

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The step z - transfer functions have some disadvantages when the sampling period decreases. The disadvantages of the discrete models can be avoided by introducing a more suitable discrete model. The δ - model, where operator δ converges with decreased sampling period T_0 to a differential operator p is best suited to this purposes.

$$\lim_{T_0 \to 0} \delta = p \tag{2}$$

One of approaches to the design of these new discrete δ - models were publised in [4, 5]. If the new variable γ is introduced then it is possible to prove [6], that equality

$$\gamma = \frac{z-1}{\alpha T_0 z + (1-\alpha) T_0} \tag{3}$$

holds for interval $0 \le \alpha \le 1$. By substituting α in equation (3) we obtain an infinite number of new δ - models. This paper will only be concerned with the forward δ - model (α = 0). The δ - models will be used in process modelling for adaptive control based on the self - tuning controller (STC). The main idea of an STC is based on a recursive identification procedure and selected control synthesis. For this reason it is necessary to apply suitable recursive identification algorithm to this model. To parameters estimates of the δ - model, the recursive least squares method (RLSM) with directional forgetting is applied.

3. Description of TITO system

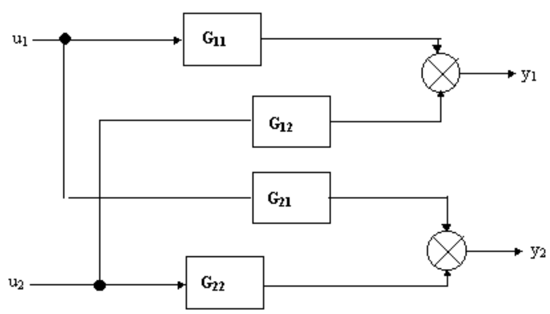

Fig. 1. A two input – output system - the "P" structure

The transfer matrix of the system is

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{4}$$

It is possible to assume that the system is described by the matrix fraction

$$\mathbf{G}(\gamma) = \mathbf{A}^{-1}(\gamma)\mathbf{B}(\gamma) = \mathbf{B}_1(\gamma)\mathbf{A}_1^{-1}(\gamma) \tag{5}$$

Where polynomial matrices $\mathbf{A} \in R_{nn}[\gamma]$, $\mathbf{B} \in R_{nm}[\gamma]$ are the left indivisible decomposition of matrix $\mathbf{G}(\gamma)$ and matrices $\mathbf{A_1} \in R_{mm}[\gamma]$, $\mathbf{B_1} \in R_{nm}[\gamma]$ are the right indivisible decomposition.

The matrices of the discrete model are

$$\mathbf{A}(\gamma)=\begin{bmatrix}\gamma^2+\alpha_1\gamma+\alpha_2 & \alpha_3\gamma+\alpha_4\\ \alpha_5\gamma+\alpha_6 & \gamma^2+\alpha_7\gamma+\alpha_8\end{bmatrix},\ \mathbf{B}(\gamma)=\begin{bmatrix}\beta_1\gamma+\beta_2 & \beta_3\gamma+\beta_4\\ \beta_5\gamma+\beta_6 & \beta_7\gamma+\beta_8\end{bmatrix} \tag{6}$$

and the differential equations of the model are

$$\begin{aligned}
y_{1\delta}(k) &= -\alpha_1 y_{1\delta}(k-1)-\alpha_2 y_{1\delta}(k-2)-\alpha_3 y_{2\delta}(k-1)-\\
&\quad -\alpha_4 y_{2\delta}(k-2)+\beta_1 u_{1\delta}(k-1)+\\
&\quad +\beta_2 u_{1\delta}(k-2)+\beta_3 u_{2\delta}(k-1)+\alpha_4 u_{2\delta}(k-2)\\
y_{2\delta}(k) &= -\alpha_5 y_{1\delta}(k-1)-\alpha_6 y_{1\delta}(k-2)-\alpha_7 y_{2\delta}(k-1)-\\
&\quad -\alpha_8 y_{2\delta}(k-2)+\beta_5 u_{1\delta}(k-1)+\\
&\quad +\beta_6 u_{1\delta}(k-2)+\beta_7 u_{2\delta}(k-1)+\beta_8 u_{2\delta}(k-2)
\end{aligned} \tag{7}$$

4. Designing 2DOF control

The 2DOF control structure contains also a feedforward part.

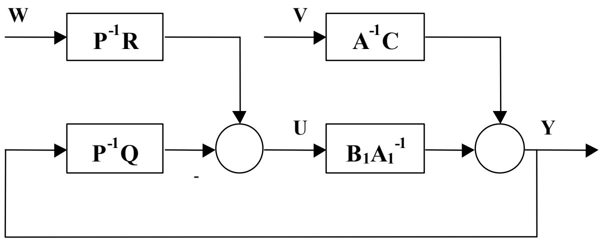

Fig. 2. Block diagram of the 2DOF control system

In the same way as the controlled system, the transfer matrices of controllers take the form of matrix fractions

$$\mathbf{G}_{FB}(\gamma)=\mathbf{P}^{-1}(\gamma)\mathbf{Q}(\gamma)=\mathbf{Q}_1(\gamma)\mathbf{P}_1^{-1}(\gamma) \tag{8}$$

$$\mathbf{G}_{FW}(\gamma)=\mathbf{P}^{-1}(\gamma)\mathbf{R}(\gamma)=\mathbf{R}_1(\gamma)\mathbf{P}_1^{-1}(\gamma) \tag{9}$$

Generally, the vector of input reference signals $\mathbf{W}$ is given by

$$\mathbf{W}(\gamma)=\mathbf{F}_w^{-1}(\gamma)\mathbf{h}_w(\gamma) \tag{10}$$

and also, the vector of input disturbances is given as

$$\mathbf{V}(\gamma)=\mathbf{F}_v^{-1}(\gamma)\mathbf{h}_v(\gamma) \tag{11}$$

Here, the reference signals and disturbances are considered from a class of step functions. In this case $\mathbf{h}_w(\gamma),\mathbf{h}_v(\gamma)$ are vectors of constants. $\mathbf{F}_w(\gamma),\mathbf{F}_v(\gamma)$ taking the following form

$$\mathbf{F}_w(\gamma)=\mathbf{F}_v(\gamma)=\begin{bmatrix}\gamma & 0\\ 0 & \gamma\end{bmatrix} \tag{12}$$

The block diagram leads to an equation for the control law (γ will be omitted from some operations for the sake of simplification)

$$\mathbf{U} = \mathbf{P}^{-1}\mathbf{F}^{-1}\mathbf{RW} - \mathbf{P}^{-1}\mathbf{F}^{-1}\mathbf{QY} \tag{13}$$

and the system output

$$\mathbf{Y} = \mathbf{B}_1\mathbf{A}_1^{-1}\mathbf{U} + \mathbf{A}^{-1}\mathbf{CV} \tag{14}$$

From (13) and (14) is possible to derive following equation

$$\mathbf{U} = \mathbf{A}_1\left(\mathbf{PA}_1 + \mathbf{QB}_1\right)^{-1} - \mathbf{A}_1\left(\mathbf{PA}_1 + \mathbf{QB}_1\right)^{-1}\mathbf{QA}^{-1}\mathbf{CV} \tag{15}$$

where

$$\left(\mathbf{PA}_1 + \mathbf{QB}_1\right) = \mathbf{D}_m \tag{16}$$

From (15) and (14) is possible to derive following equation for the system output

$$\mathbf{Y} = \mathbf{B}_1\mathbf{D}_m^{-1}\left(\mathbf{RW} - \mathbf{QA}_1\mathbf{CV}\right) + \mathbf{A}^{-1}\mathbf{VC} \tag{17}$$

The control error is given as

$$\mathbf{E} = \mathbf{W} - \mathbf{Y} \tag{18}$$

From (17) and (18) we obtain finite equation for the control error

$$\mathbf{E} = \left(\mathbf{I}_n - \mathbf{B}_1\mathbf{D}_m^{-1}\mathbf{R}\right)\mathbf{W} - \left(\mathbf{B}_1\mathbf{D}_m^{-1}\mathbf{Q} - \mathbf{I}_n\right)\mathbf{A}^{-1}\mathbf{CV} \tag{19}$$

All signal have to contain $\mathbf{D}_m^{-1}$ (determinant of this matrix is characteristic polynomial), so following modification must be used.

$$\mathbf{B}_1\mathbf{D}_m^{-1} = \tilde{\mathbf{D}}_n^{-1}\mathbf{B}_2 \tag{20}$$

$$\mathbf{E} = \tilde{\mathbf{D}}_n^{-1}\left(\tilde{\mathbf{D}}_n - \mathbf{B}_2\mathbf{R}\right)\mathbf{W} - \tilde{\mathbf{D}}_n^{-1}\left(\mathbf{B}_2\mathbf{Q} - \tilde{\mathbf{D}}_n\right)\mathbf{A}^{-1}\mathbf{CV} \tag{21}$$

Requirement for disturbance rejection will be fulfilled if all numerators in vector (11) are eliminated.

$$\mathbf{P} = \mathbf{F}_1\tilde{\mathbf{P}} \tag{22}$$

where

$$\mathbf{F}_1 = f_1\mathbf{I}_n \tag{23}$$

f_1 is a polynomial. It is divisible by all the elements of matrix $\mathbf{F}_v$.

Requirement for asymptotic tracking will be fulfilled if exist following polynomial matrix

$$\mathbf{F}_2 = f_2\mathbf{I}_n \tag{24}$$

and following equation is fulfilled

$$\tilde{\mathbf{D}}_n - \mathbf{B}_2\mathbf{R} = \mathbf{TF}_2 \tag{25}$$

Where f_2 is polynomial. It is divisible by all the elements of matrix $\mathbf{F}_w$.

It is used this modification for reducing the number of necessary operations

$$\mathbf{D}_m = \mathbf{N}\mathbf{A}_1 \Rightarrow \mathbf{D}_m^{-1} = \mathbf{A}_1^{-1}\mathbf{N}^{-1} \tag{26}$$

where matrix $\mathbf{N}(\gamma) \in R_{22}[\gamma]$ is stable diagonal polynomial matrix and takes the form

$$\mathbf{N} = \begin{bmatrix} (\gamma+\alpha_1)(\gamma+\alpha_2) & 0 \\ 0 & (\gamma+\alpha_1)(\gamma+\alpha_2) \end{bmatrix} \tag{27}$$

The roots of this polynomial matrix are the ruling factor in the behaviour of the closed loop system. They must be inside the circle with the centre point at point $-1/T_0$ with the radius $1/T_0$ if the system is to be stable.

Substitute (26) to (20)

$$\mathbf{B}_1\mathbf{A}_1^{-1}\mathbf{N}^{-1} = \tilde{\mathbf{D}}_n^{-1}\mathbf{B}_2 \tag{28}$$

Equation (5) is valid

$$\mathbf{A}^{-1}\mathbf{N}^{-1}\mathbf{B} = \tilde{\mathbf{D}}_n^{-1}\mathbf{B}_2 \Rightarrow \mathbf{D}_n = \mathbf{N}\mathbf{A}, \mathbf{B}_2 = \mathbf{B} \tag{29}$$

Then both diophantine equations are given as

$$\begin{gathered} \mathbf{P}\mathbf{F}_1\mathbf{A} + \mathbf{Q}\mathbf{B}_1 = \mathbf{N}\mathbf{A}_1 \\ \mathbf{T}\mathbf{F}_2 + \mathbf{B}\mathbf{R} = \mathbf{N}\mathbf{A} \end{gathered} \tag{30}$$

Polynomial matrices of left matrix fraction of the system are defined in the form

$$\mathbf{A}_1(\gamma) = \begin{bmatrix} \gamma^2+\alpha_9\gamma+\alpha_{10} & \alpha_{11}\gamma+\alpha_{12} \\ \alpha_{13}\gamma+\alpha_{14} & \gamma^2+\alpha_{15}\gamma+\alpha_{16} \end{bmatrix}, \mathbf{B}_1(\gamma) = \begin{bmatrix} \beta_9\gamma+\beta_{10} & \beta_{11}\gamma+\beta_{12} \\ \beta_{13}\gamma+\beta_{14} & \beta_{15}\gamma+\beta_{16} \end{bmatrix} \tag{31}$$

The coefficients of matrices are given by solving matrix equation

$$\mathbf{B}\mathbf{A}_1 - \mathbf{A}\mathbf{B}_1 = 0 \tag{32}$$

The structure of polynomial matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{T}$ were chosen so that the number of algebraic equation resulting from the solution of the diophantine equation using the uncertain coefficients method. The matrices $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are matrices of controller. The matrix $\mathbf{T}$ resulting from the solution of diophantine equation (30) is not useful.

$$\begin{gathered} \mathbf{P}(\gamma) = \begin{bmatrix} \gamma+p_1 & \gamma+p_2 \\ \gamma+p_3 & \gamma+p_4 \end{bmatrix} \quad \mathbf{R}(\gamma) = \begin{bmatrix} r_1 & r_2 \\ r_3 & r_4 \end{bmatrix} \\ \mathbf{Q}(\gamma) = \begin{bmatrix} q_1\gamma^2+q_2\gamma+q_3 & q_4\gamma^2+q_5\gamma+q_6 \\ q_7\gamma^2+q_8\gamma+q_9 & q_{10}\gamma^2+q_{11}\gamma+q_{12} \end{bmatrix} \\ \mathbf{T}(\gamma) = \begin{bmatrix} \gamma^3+t_1\gamma^2+t_2\gamma+t_3 & t_4\gamma^2+t_5\gamma+t_6 \\ t_7\gamma^2+t_8\gamma+t_9 & \gamma^3+t_{10}\gamma^2+t_{11}\gamma+t_{12} \end{bmatrix} \end{gathered} \tag{33}$$

Solving the diophantine equations (30) defines a set of algebraic equations which we use to obtain the unknown controller parameters.

5. Recursive identification

The algorithms designed here were incorporated into an adaptive control system with recursive identification. The recursive least squares method proved effective for self-tuning controllers and was used as the basis for this algorithm.

The parameter vector is completed as shown below:

$$\mathbf{\Theta}_{\delta}^{T}(k) = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{bmatrix} \tag{34}$$

The data vector is

$$\begin{aligned} \mathbf{\Phi}_{\delta}^{T}(k) = [&-y_{1\delta}(k-1), -y_{1\delta}(k-2), -y_{2\delta}(k-1), -y_{2\delta}(k-2), \\ &u_{1\delta}(k-1), u_{1\delta}(k-2), u_{2\delta}(k-1), u_{2\delta}(k-1)] \end{aligned} \tag{35}$$

The parameter estimates are actualized using the recursive least squares method plus directional forgetting.

6. Laboratory experiment

The verification of designed TITO controllers in laboratory conditions operating in real time has been realized using experimental laboratory model CE 108 - couples drives apparatus. This apparatus is based on experience with authentic industrial control applications. It allows us to investigate the ever-present difficulty of controlling the tension and speed of material in a continuous process. The process may require the material speed and tension to be controlled to within defined limits. Examples of this occur in the paper-making industry, strip metal and wire manufacture and, indeed, any process where the product is manufactured in a continuous strip.

A continuous flexible belt replaces the industrial type material strip. The principle scheme of the model is shown in the Fig. 3. It consists of three pulleys, mounted on a vertical panel so that they form a triangle resting on its base. The two base pulleys are directly mounted on the shafts of two nominally identical servomotors and the apparatus is controlled by manipulating the drive torques to these servomotors. The third pulley, the jockey, is free to rotate and is mounted on a pivoted arm. The jockey pulley assembly, which simulates a material workstation, is equipped with a special sensor and tension measuring equipment. It is the jockey pulley speed and tension which form the principle system outputs. The belt tension is measured indirectly by monitoring the angular deflection of the pivoted tension arm to which the jockey pulley is attached.

The controller output variables are the inputs to the servomotors and the process output variables are the tension and speed at the workstation. There are interactions between the control loops.

The task was to apply the methods we designed for the adaptive control of a model representing a non-linear system with variable parameters which is, therefore, impossible to control deterministically. Adaptive control using recursive identification with 2DOF controller was performed.

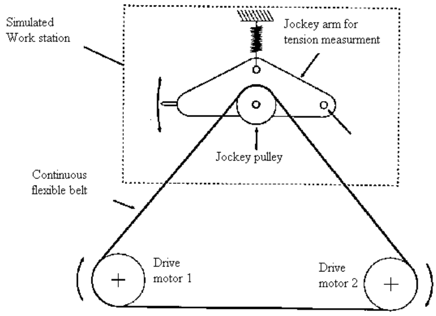

Fig. 3. Principal scheme of CE 108.

The right side control matrix $\boldsymbol{N}$ was chosen as follows

$$\mathbf{N}(\gamma)=\begin{bmatrix}\gamma^2+8.750\gamma+19.125 & 0\\ 0 & \gamma^2+8.750\gamma+19.125\end{bmatrix} \tag{36}$$

The sampling period was chosen T_0 = 0.2 sec. Process output variable y_1 is the speed and process output variable y_2 is the tension. The variables u_1 and u_2 are the controller outputs–inputs to the servomotor.

The time responses of the control are shown in Fig. 4, Fig. 5.

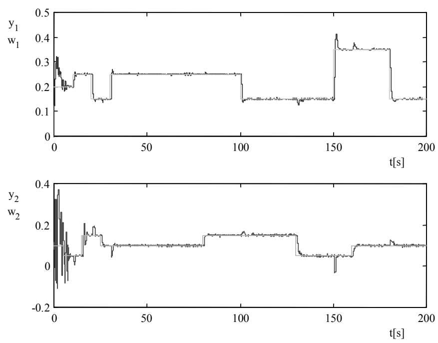

Fig. 4. The adaptive control with 2DOF controller

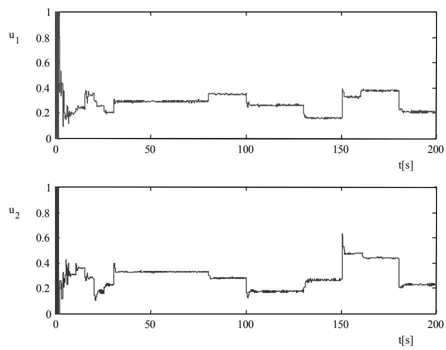

Fig. 5. The adaptive control with 2DOF controller − controller output

7. Conclusion

The adaptive control of two-variable system based on polynomial theory was designed. The designs were simulated and used to control a laboratory model. The simulation results proved that these methods are suitable for control of linear system. The control tests on the laboratory model gave satisfactory results despite the fact that the non-linear dynamics was described by a linear model.

Acknowledgements

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